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QUARTZ CRYSTAL RESONATORS AND OSCILLATORS

For Frequency Control and Timing Applications

A Tutorial

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NOTICES

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Report Documentation Page

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13. ABSTRACT(Maximum 200 words)

Subjects covered include: applications of oscillators and clocks; types of oscillators (quartz and atomic); quartz resonator properties; quartz growing, sweeping, and material characteristics; Q and its significance; resonator and oscillator stability, including aging, short-term stability, frequency vs. temperature characteristics, oscillator circuit caused instabilities, frequency vs. drive level effects, acceleration effects, the effect of shock, and radiation effects; emerging technologies; atomic frequency standards; comparison of the major oscillator types; oscillator specifications and selection guidelines; time and timekeeping; clock errors; relativistic time; time transfer; time and frequency subsystem; and other applications of quartz resonators.

The goal of this document is to assist in presenting to the nonspecialist the most frequently encountered concepts in frequency control and timing. The document originated as a set of "hard copies" of presentation visuals (i.e., vu-graphs).

14. SUBJECT TERMS
Quartz crystal, quartz resonator, quartz oscillator, oscillator, stability, phase noise, frequency control, frequency standard, clock, atomic clock, atomic standard, rubidium standard, cesium standard, vibration effects, radiation effects, time, timekeeping, GPS.

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Preface

Why This Tutorial?

"Everything should be made as simple as possible - but not simpler," said Einstein. The main goal of this "tutorial" is to assist with presenting the most frequently encountered concepts in frequency control and timing, as simply as possible.

I have often been called upon to brief visitors, management, and potential users of precision oscillators, and have also been invited to present seminars, tutorials, and review papers before university, IEEE, and other professional groups. In the beginning, I spent a great deal of time preparing these presentations. Much of the time was spent on preparing the slides. As I accumulated more and more slides, it became easier and easier to prepare successive presentations.

I was frequently asked for "hard-copies" of the slides, so I started organizing, adding some text, and filling the gaps in the slide collection. As the collection grew, I began receiving favorable comments and requests for additional copies. Apparently, others, too, found this collection to be useful. Eventually, I assembled this document, the "Tutorial".

This is a work in progress. I plan to include new material, including additional notes. Comments, corrections, and suggestions for future revisions will be welcome.

John R. Vig

Don’t Forget the Notes!

In the PowerPoint version of this document, notes and references can be found in the “Notes” of most of the pages. To view the notes, use the “Notes Page View” icon (near the lower left corner of the screen), or select “Notes Page” in the View menu. In PowerPoint 2000 (and, presumably, later versions), the notes also appear in the “Normal view”.

To print a page so that it includes the notes, select Print in the File menu, and, near the bottom, at “Print what:,” select “Notes Pages”.

The HTML version can be viewed with a web browser (best viewed at 1024 x 768 screen size). The notes then appear in the lower pane on the right.

CHAPTER 1
Applications and Requirements

Electronics Applications of Quartz Crystals

Military & Aerospace	Industrial	Consumer
Communications Navigation IFF Radar Sensors Guidance systems Fuzes Electronic warfare Sonobouys	Communications Telecommunications Mobile/cellular/portable radio, telephone & pager Aviation Marine Navigation Instrumentation Computers Digital systems CRT displays Disk drives Modems Tagging/identification Utilities Sensors	Watches & clocks Cellular & cordless phones, pagers Radio & hi-fi equipment Color TV Cable TV systems Home computers VCR & video camera CB & amateur radio Toys & games Pacemakers Other medical devices
Research & Metrology		**Automotive**
Atomic clocks Instruments Astronomy & geodesy Space tracking Celestial navigation		Engine control, stereo, clock Trip computer, GPS

Frequency Control Device Market

(as of ~1997)

Technology	Units per year	Unit price, typical	Worldwide market, $/year
Crystal	~ 2 x 10^9	~$1 ($0.1 to 3,000)	~$1.2B
Atomic Frequency Standards (see chapter 6)			
Hydrogen maser	~ 10	$200,000	$2M
Cesium beam frequency standard	~ 300	$50,000	$15M
Rubidium cell frequency standard	~ 20,000	$2,000	$40M

Navigation

Precise time is essential to precise navigation. Historically, navigation has been a principal motivator in man's search for better clocks. Even in ancient times, one could measure latitude by observing the stars' positions. However, to determine longitude, the problem became one of timing. Since the earth makes one revolution in 24 hours, one can determine longitude form the time difference between local time (which was determined from the sun's position) and the time at the Greenwich meridian (which was determined by a clock):

Longitude in degrees = (360 degrees/24 hours) x t in hours.

In 1714, the British government offered a reward of 20,000 pounds to the first person to produce a clock that allowed the determination of a ship's longitude to 30 nautical miles at the end of a six week voyage (i.e., a clock accuracy of three seconds per day). The Englishman John Harrison won the competition in 1735 for his chronometer invention.

Today's electronic navigation systems still require ever greater accuracies. As electromagnetic waves travel 300 meters per microsecond, e.g., if a vessel's timing was in error by one millisecond, a navigational error of 300 kilometers would result. In the Global Positioning System (GPS), atomic clocks in the satellites and quartz oscillators in the receivers provide nanosecond-level accuracies. The resulting (worldwide) navigational accuracies are about ten meters (see chapter 8 for further details about GPS).

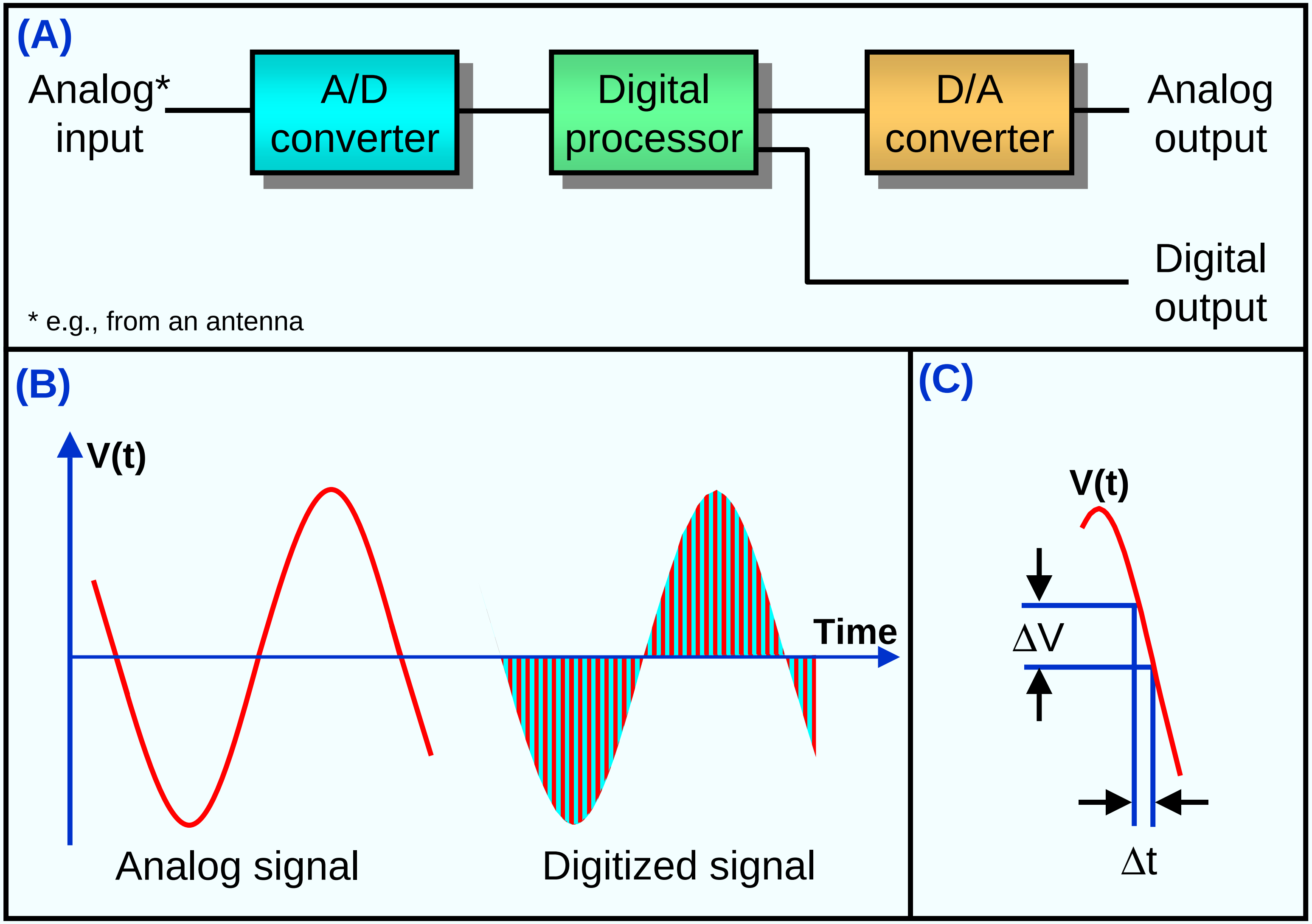
Commercial Two-way Radio

Historically, as the number of users of commercial two-way radios have grown, channel spacings have been narrowed, and higher-frequency spectra have had to be allocated to accommodate the demand. Narrower channel spacings and higher operating frequencies necessitate tighter frequency tolerances for both the transmitters and the receivers. In 1940, when only a few thousand commercial broadcast transmitters were in use, a 500 ppm tolerance was adequate. Today, the oscillators in the many millions of cellular telephones (which operate at frequency bands above 800 MHz) must maintain a frequency tolerance of 2.5 ppm and better. The 896-901 MHz and 935-940 MHz mobile radio bands require frequency tolerances of 0.1 ppm at the base station and 1.5 ppm at the mobile station.

The need to accommodate more users will continue to require higher and higher frequency accuracies. For example, a NASA concept for a personal satellite communication system would use walkie-talkie-like hand-held terminals, a 30 GHz uplink, a 20 GHz downlink, and a 10 kHz channel spacing. The terminals' frequency accuracy requirement is a few parts in 10^8.

Digital Processing of Analog Signals

The Effect of Timing Jitter



Digital Network Synchronization

- Synchronization plays a critical role in digital telecommunication systems. It ensures that information transfer is performed with minimal buffer overflow or underflow events, i.e., with an acceptable level of "slips." Slips cause problems, e.g., missing lines in FAX transmission, clicks in voice transmission, loss of encryption key in secure voice transmission, and data retransmission.

- In AT&T's network, for example, timing is distributed down a hierarchy of nodes. A timing source-receiver relationship is established between pairs of nodes containing clocks. The clocks are of four types, in four "stratum levels."

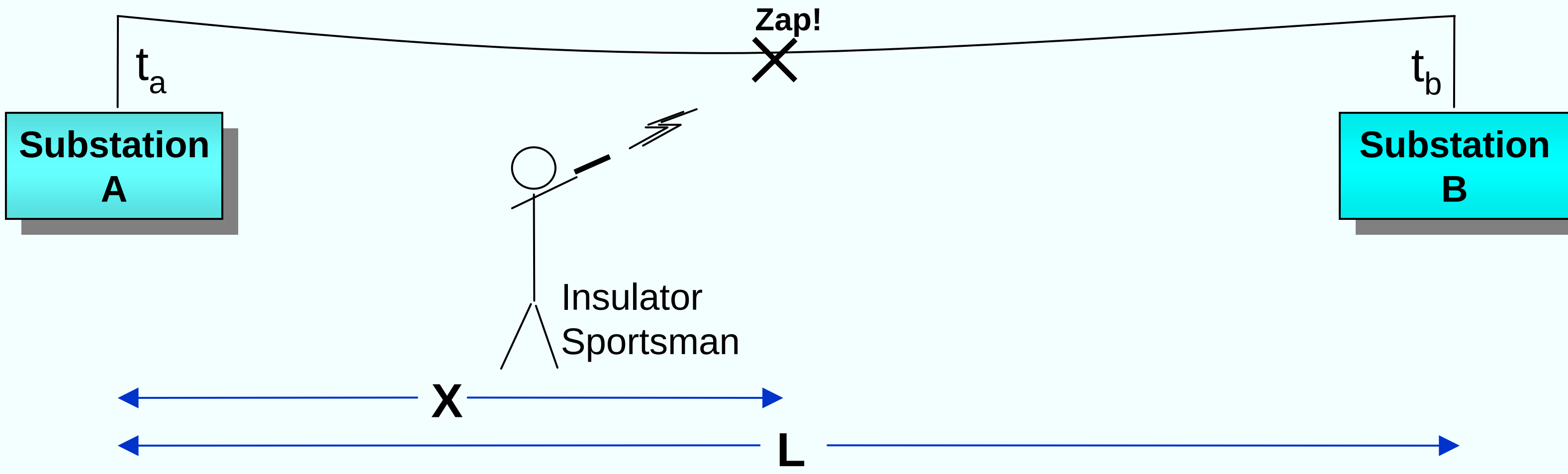
	Accuracy (Free Running)			
Stratum	Long Term	Per 1st Day	Clock Type	Number Used
1	1×10^{-11}	N.A.	GPS W/Two Rb	16
2	1.6×10^{-8}	1×10^{-10}	Rb Or OCXO	~200
3	4.6×10^{-6}	3.7×10^{-7}	OCXO Or TCXO	1000’s
4	3.2×10^{-5}	N.A.	XO	~1 million

Phase Noise in PLL and PSK Systems

The phase noise of oscillators can lead to erroneous detection of phase transitions, i.e., to bit errors, when phase shift keyed (PSK) digital modulation is used. In digital communications, for example, where 8-phase PSK is used, the maximum phase tolerance is $\pm 22.5^{o}$, of which $\pm 7.5^{o}$ is the typical allowable carrier noise contribution. Due to the statistical nature of phase deviations, if the RMS phase deviation is 1.5^{o}, for example, the probability of exceeding the $\pm 7.5^{o}$ phase deviation is 6×10^{-7}, which can result in a bit error rate that is significant in some applications.

Shock and vibration can produce large phase deviations even in "low noise" oscillators. Moreover, when the frequency of an oscillator is multiplied by N, the phase deviations are also multiplied by N. For example, a phase deviation of 10^{-3} radian at 10 MHz becomes 1 radian at 10 GHz. Such large phase excursions can be catastrophic to the performance of systems, e.g., of those which rely on phase locked loops (PLL) or phase shift keying (PSK). Low noise, acceleration insensitive oscillators are essential in such applications.

Utility Fault Location



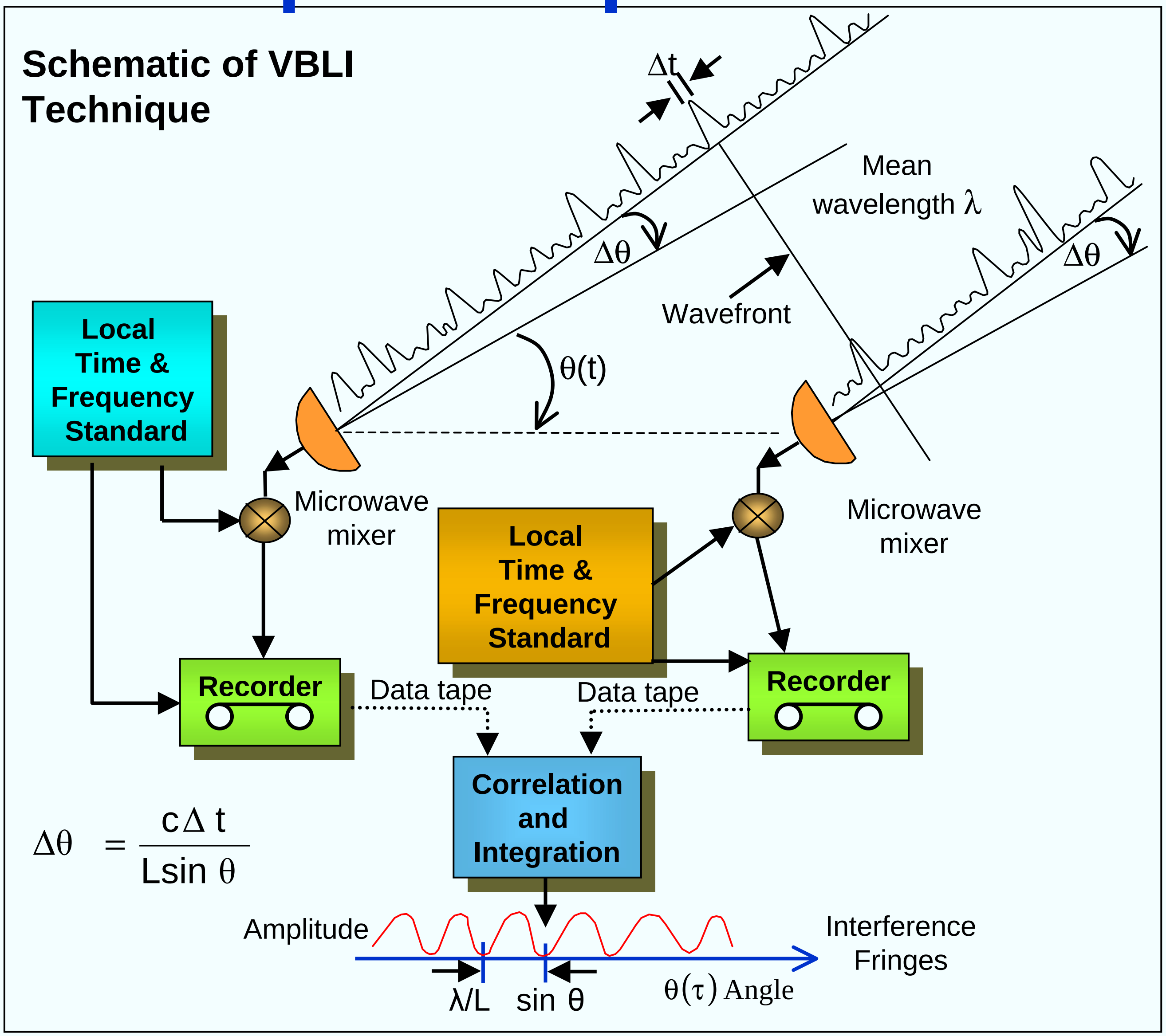
When a fault occurs, e.g., when a "sportsman" shoots out an insulator, a disturbance propagates down the line. The location of the fault can be determined from the differences in the times of arrival at the nearest substations:

$$x = 1/2[L - c(t_b - t_a)] = 1/2[L - c\Delta t]$$

where x = distance of the fault from substation A, L = A to B line length, c = speed of light, and t_a and t_b = time of arrival of disturbance at A and B, respectively.

Fault locator error = $x_{error} = 1/2(c\Delta t_{error})$; therefore, if $\Delta t_{error} \leq 1$ microsecond, then $x_{error} \leq 150$ meters $\leq 1/2$ of high voltage tower spacings, so, the utility company can send a repair crew directly to the tower that is nearest to the fault.

Space Exploration



Military Requirements

Military needs are a prime driver of frequency control technology. Modern military systems require oscillators/clocks that are:

- Stable over a wide range of parameters (time, temperature, acceleration, radiation, etc.)
- Low noise
- Low power
- Small size
- Fast warmup
- Low life-cycle cost

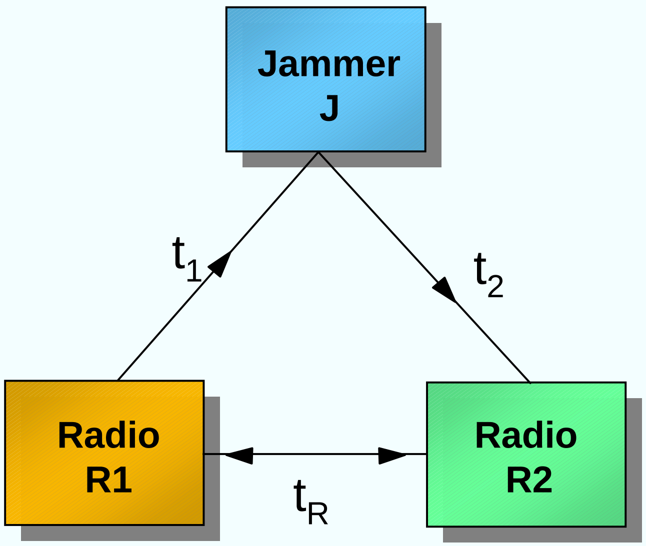
Impacts of Oscillator Technology Improvements

- Higher jamming resistance & improved ability to hide signals
- Improved ability to deny use of systems to unauthorized users
- Longer autonomy period (radio silence interval)
- Fast signal acquisition (net entry)
- Lower power for reduced battery consumption
- Improved spectrum utilization
- Improved surveillance capability (e.g., slow-moving target detection, bistatic radar)
- Improved missile guidance (e.g., on-board radar vs. ground radar)
- Improved identification-friend-or-foe (IFF) capability
- Improved electronic warfare capability (e.g., emitter location via TOA)
- Lower error rates in digital communications
- Improved navigation capability
- Improved survivability and performance in radiation environment
- Improved survivability and performance in high shock applications
- Longer life, and smaller size, weight, and cost
- Longer recalibration interval (lower logistics costs)

Spread Spectrum Systems

- In a spread spectrum system, the transmitted signal is spread over a bandwidth that is much wider than the bandwidth required to transmit the information being sent (e.g., a voice channel of a few kHz bandwidth is spread over many MHz). This is accomplished by modulating a carrier signal with the information being sent, using a wideband pseudonoise (PN) encoding signal. A spread spectrum receiver with the appropriate PN code can demodulate and extract the information being sent. Those without the PN code may completely miss the signal, or if they detect the signal, it appears to them as noise.
- Two of the spread spectrum modulation types are: 1. direct sequence, in which the carrier is modulated by a digital code sequence, and 2. frequency hopping, in which the carrier frequency jumps from frequency to frequency, within some predetermined set, the order of frequencies being determined by a code sequence.
- Transmitter and receiver contain ***clocks*** which must be synchronized; e.g., in a frequency hopping system, the transmitter and receiver must hop to the same frequency at the same time. The faster the hopping rate, the higher the jamming resistance, and the more accurate the clocks must be (see the next page for an example).
- Advantages of spread spectrum systems include the following capabilities: 1. rejection of intentional and unintentional jamming, 2. low probability of intercept (LPI), 3. selective addressing, 4. multiple access, and 5. high accuracy navigation and ranging.

Clock for Very Fast Frequency Hopping Radio



To defeat a "perfect" follower jammer, one needs a hop-rate given by:

$$\mathbf{t_m < (t_1 + t_2) - t_R}$$

where $t_m \approx$ message duration/hop
$\approx$ 1/hop-rate

Example

Let R1 to R2 = 1 km, R1 to J =5 km, and J to R2 = 5 km. Then, since propagation delay =3.3 μs/km,
$t_1 = t_2 = 16.5$ μs,
$t_R = 3.3$ μs, and $t_m < 30$ μs.
Allowed clock error $\approx 0.2\ t_m$
≈ 6 μs.

For a 4 hour resynch interval, clock accuracy requirement is:
4×10^{-10}

Clocks and Frequency Hopping C^3 Systems

Slow hopping ‹-------------------------------›Good clock

Fast hopping ‹-----------------------------› Better clock

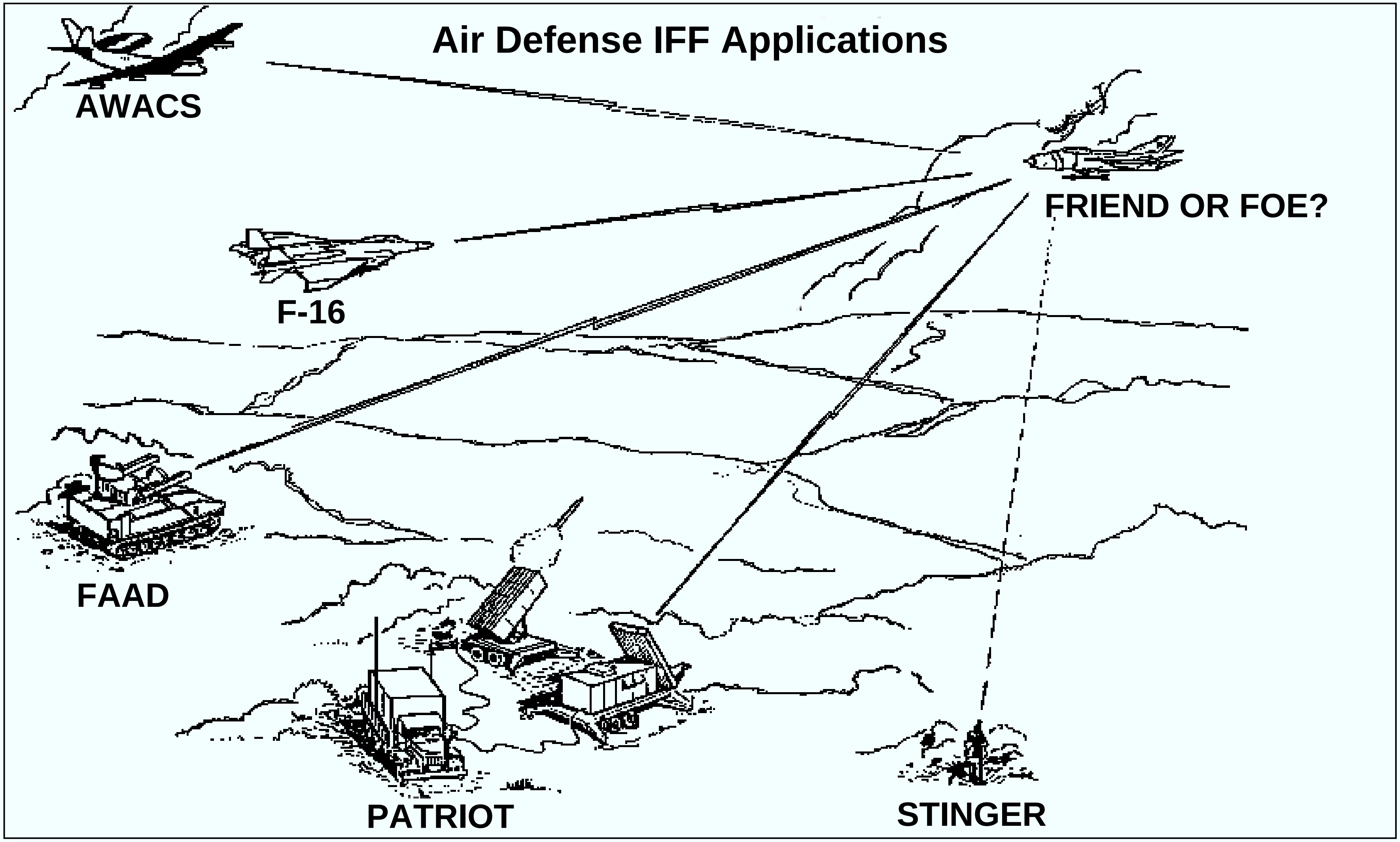
Extended radio silence ‹-----------------› Better clock

Extended calibration interval ‹----------› Better clock

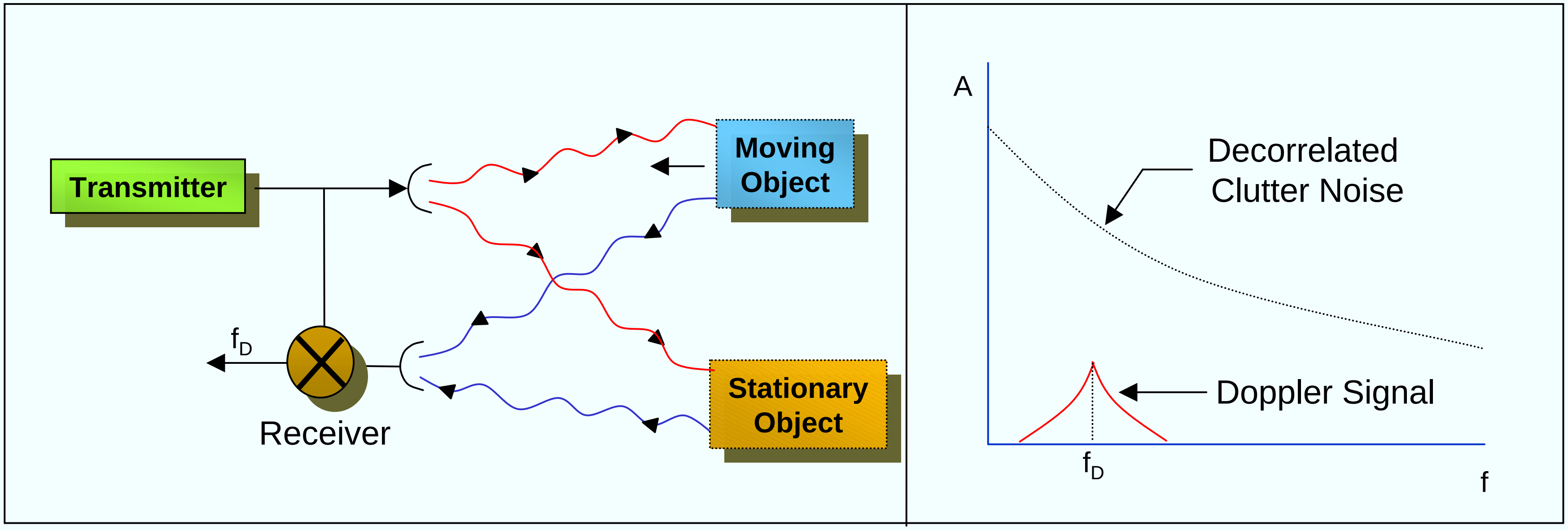
Othogonality ‹-------------------------------› Better clock

Interoperability ‹----------------------------› Better clock

Identification-Friend-Or-Foe (IFF)



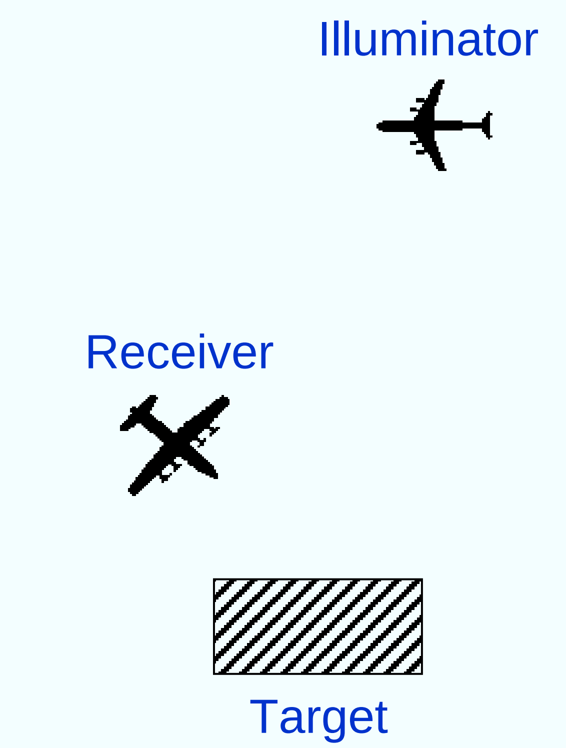
Effect of Noise in Doppler Radar System



- Echo = Doppler-shifted echo from moving target + large "clutter" signal
- (Echo signal) - (reference signal) --› Doppler shifted signal from target
- Phase noise of the local oscillator modulates (decorrelates) the clutter signal, generates higher frequency clutter components, and thereby degrades the radar's ability to separate the target signal from the clutter signal.

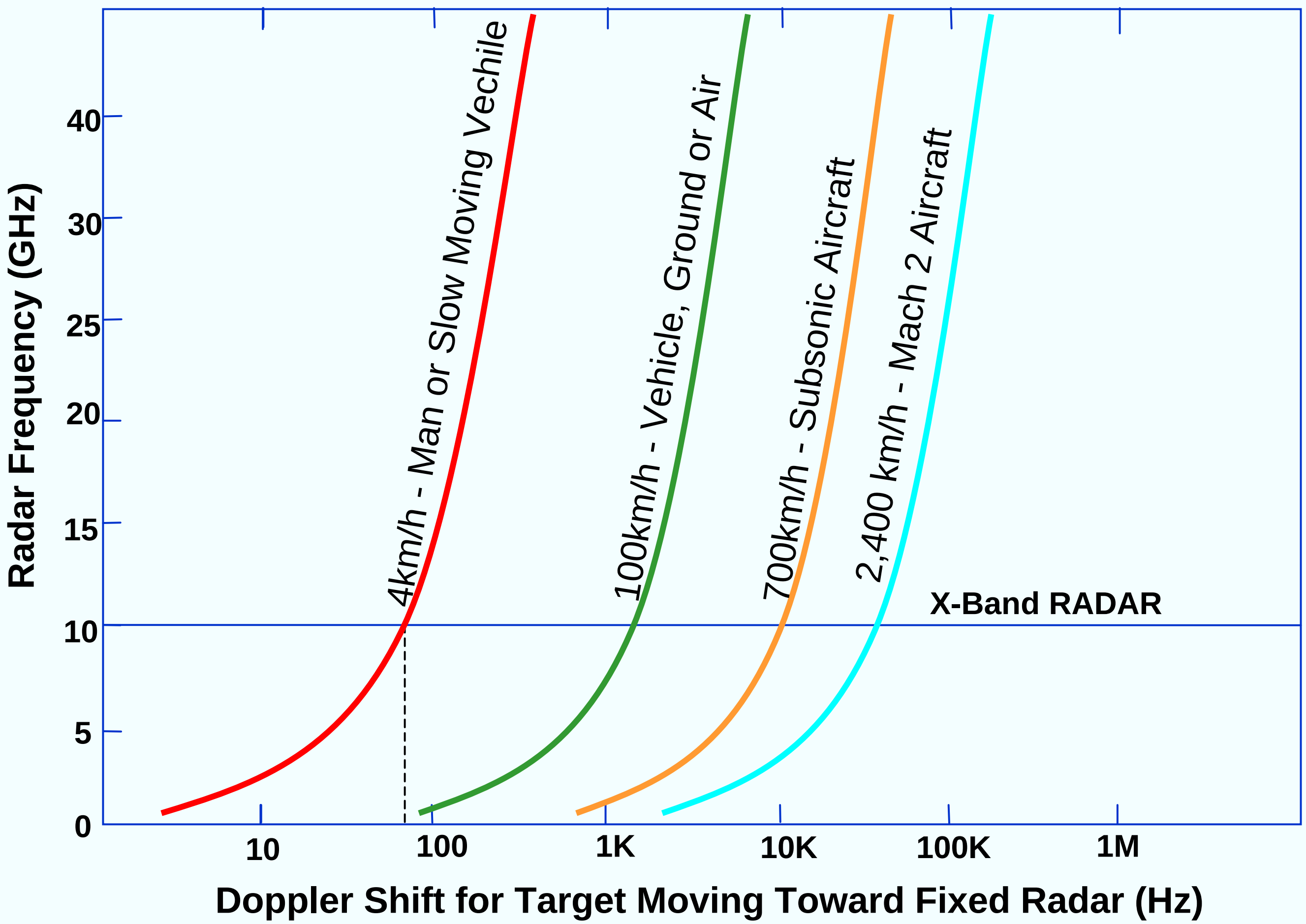
Bistatic Radar

Conventional (i.e., "monostatic") radar, in which the illuminator and receiver are on the same platform, is vulnerable to a variety of countermeasures. Bistatic radar, in which the illuminator and receiver are widely separated, can greatly reduce the vulnerability to countermeasures such as jamming and antiradiation weapons, and can increase slow moving target detection and identification capability via "clutter tuning" (receiver maneuvers so that its motion compensates for the motion of the illuminator; creates zero Doppler shift for the area being searched). The transmitter can remain far from the battle area, in a "sanctuary." The receiver can remain "quiet."



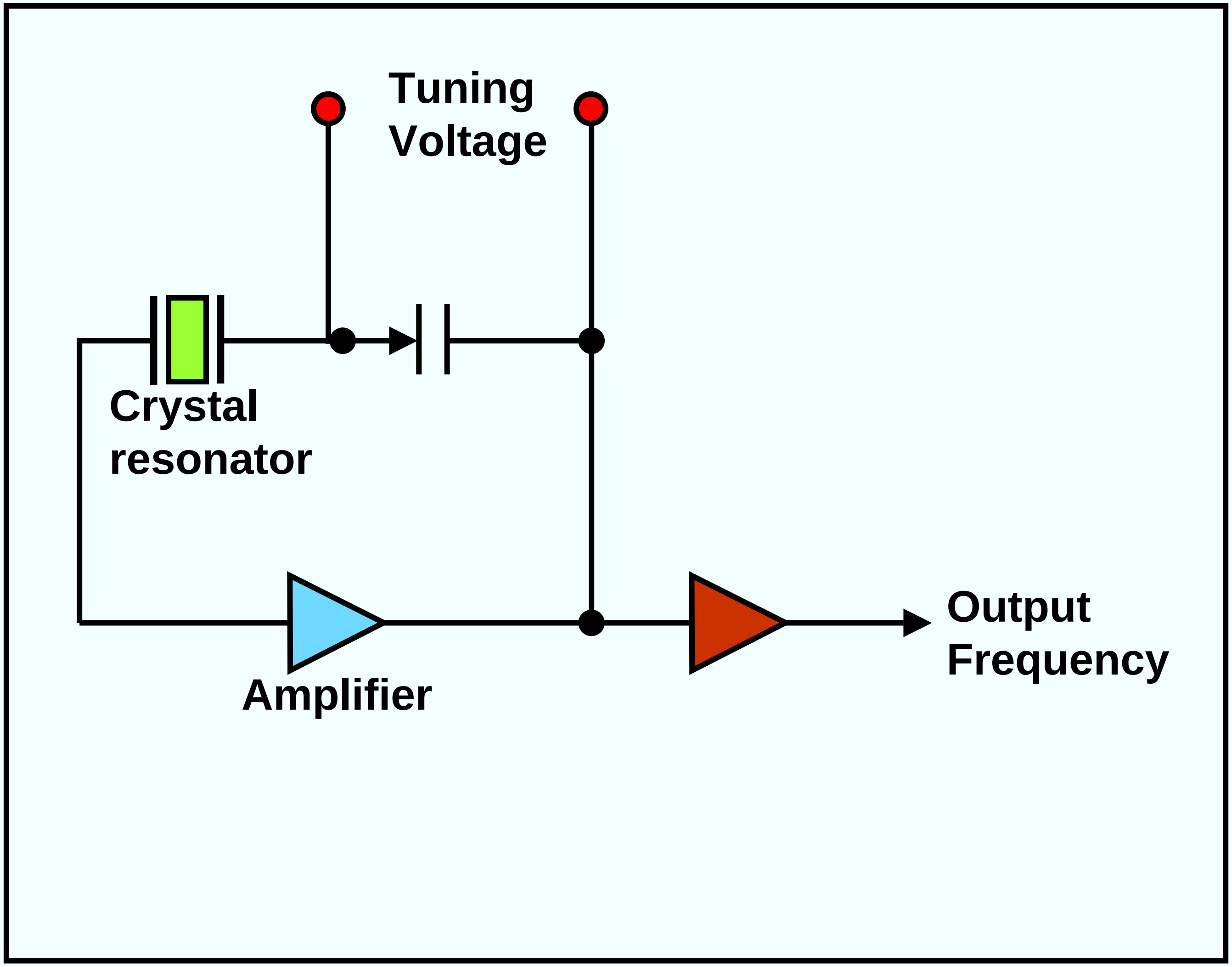
The timing and phase coherence problems can be orders of magnitude more severe in bistatic than in monostatic radar, especially when the platforms are moving. The reference oscillators must remain synchronized and syntonized during a mission so that the receiver knows when the transmitter emits each pulse, and the phase variations will be small enough to allow a satisfactory image to be formed. Low noise crystal oscillators are required for short term stability; atomic frequency standards are often required for long term stability.

Doppler Shifts



CHAPTER 2
Quartz Crystal Oscillators

Crystal Oscillator



Oscillation

- At the frequency of oscillation, the closed loop phase shift $= 2n\pi$.
- When initially energized, the only signal in the circuit is noise. That component of noise, the frequency of which satisfies the phase condition for oscillation, is propagated around the loop with increasing amplitude. The rate of increase depends on the excess; i.e., small-signal, loop gain and on the BW of the crystal in the network.
- The amplitude continues to increase until the amplifier gain is reduced either by nonlinearities of the active elements ("self limiting") or by some automatic level control.
- At steady state, the closed-loop gain = 1.

Oscillation and Stability

- If a phase perturbation $\Delta\phi$ occurs, the frequency must shift Δf to maintain the $2n\pi$ phase condition, where $\Delta f/f=-\Delta\phi/2Q_L$ for a series-resonance oscillator, and Q_L is loaded Q of the crystal in the network. The "phase slope," $d\phi/df$ is proportional to Q_L in the vicinity of the series resonance frequency (also see "Equivalent Circuit" and "Frequency vs. Reactance" in Chapt. 3).

- Most oscillators operate at "parallel resonance," where the reactance vs. frequency slope, dX/df, i.e., the "stiffness," is inversely proportional to C_1**,** the motional capacitance of the crystal unit.

- For maximum frequency stability with respect to phase (or reactance) perturbations in the oscillator loop, the phase slope (or reactance slope) must be maximum, i.e., C_1 should be minimum and Q_L should be maximum. A quartz crystal unit's high Q and high stiffness makes it the primary frequency (and frequency stability) determining element in oscillators.

Tunability and Stability

Making an oscillator tunable over a wide frequency range degrades its stability because making an oscillator susceptible to intentional tuning also makes it susceptible to factors that result in unintentional tuning. The wider the tuning range, the more difficult it is to maintain a high stability. For example, if an OCXO is designed to have a short term stability of 1×10^{-12} for some averaging time and a tunability of 1×10^{-7}, then the crystal's load reactance must be stable to 1×10^{-5} for that averaging time. Achieving such stability is difficult because the load reactance is affected by stray capacitances and inductances, by the stability of the varactor's capacitance vs. voltage characteristic, and by the stability of the voltage on the varactor. Moreover, the 1×10^{-5} load reactance stability must be maintained not only under benign conditions, but also under changing environmental conditions (temperature, vibration, radiation, etc.).

Whereas a high stability, ovenized 10 MHz voltage controlled oscillator may have a frequency adjustment range of 5×10^{-7} and an aging rate of 2×10^{-8} per year, a wide tuning range 10 MHz VCXO may have a tuning range of 50 ppm and an aging rate of 2 ppm per year.

Oscillator Acronyms

- **XO…………..**Crystal Oscillator
- **VCXO………**Voltage Controlled Crystal Oscillator
- **OCXO………**Oven Controlled Crystal Oscillator
- **TCXO………**Temperature Compensated Crystal Oscillator
- **TCVCXO..…**Temperature Compensated/Voltage Controlled Crystal Oscillator
- **OCVCXO.….**Oven Controlled/Voltage Controlled Crystal Oscillator
- **MCXO………**Microcomputer Compensated Crystal Oscillator
- **RbXO……….**Rubidium-Crystal Oscillator

Crystal Oscillator Categories

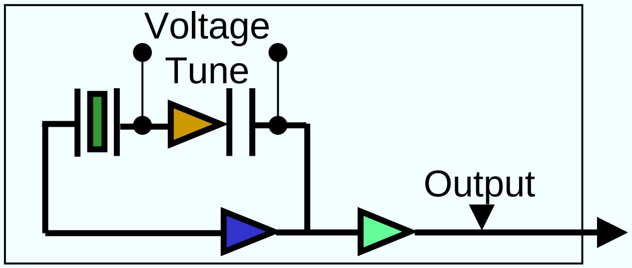
The three categories, based on the method of dealing with the crystal unit's frequency vs. temperature (f vs. T) characteristic, are:

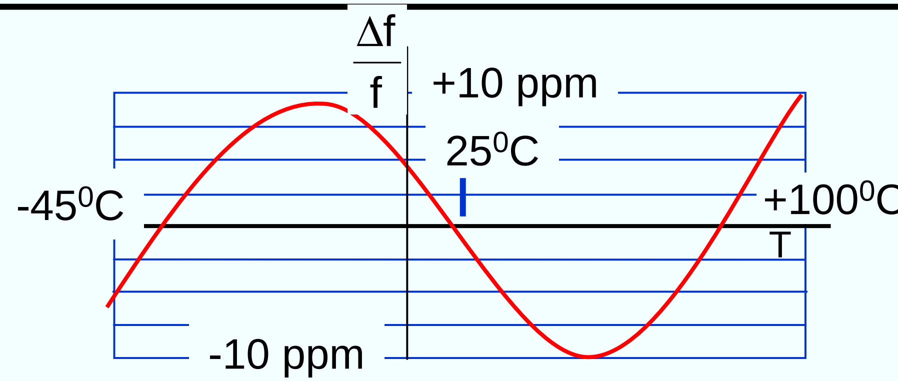
- **XO, crystal oscillator,** does not contain means for reducing the crystal's f vs. T characteristic (also called PXO-packaged crystal oscillator).

- **TCXO, temperature compensated crystal oscillator,** in which, e.g., the output signal from a temperature sensor (e.g., a thermistor) is used to generate a correction voltage that is applied to a variable reactance (e.g., a varactor) in the crystal network. The reactance variations compensate for the crystal's f vs. T characteristic. Analog TCXO's can provide about a 20X improvement over the crystal's f vs. T variation.

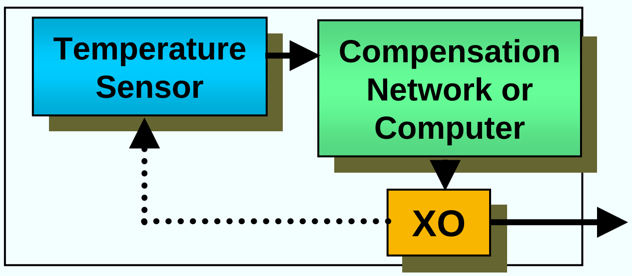
- **OCXO, oven controlled crystal oscillator,** in which the crystal and other temperature sensitive components are in a stable oven which is adjusted to the temperature where the crystal's f vs. T has zero slope. OCXO's can provide a >1000X improvement over the crystal's f vs. T variation.

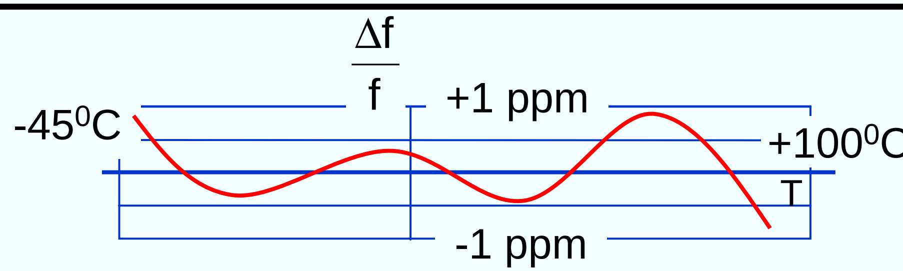
Crystal Oscillator Categories



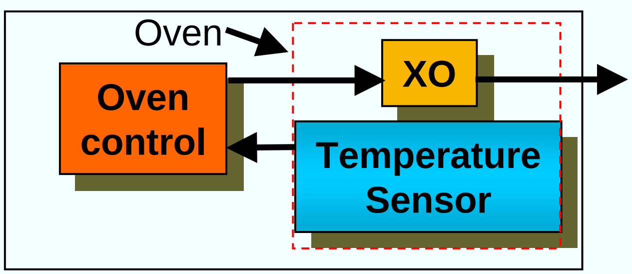


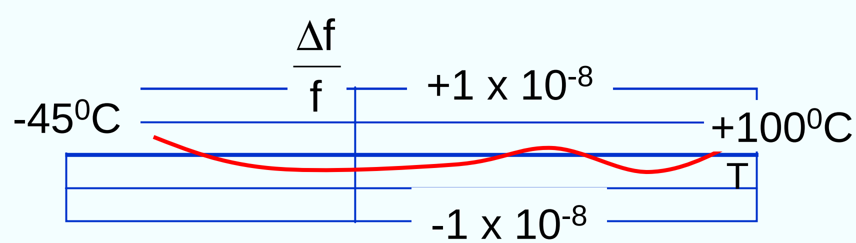
- Crystal Oscillator (XO)





- Temperature Compensated (TCXO)





- Oven Controlled (OCXO)

Hierarchy of Oscillators

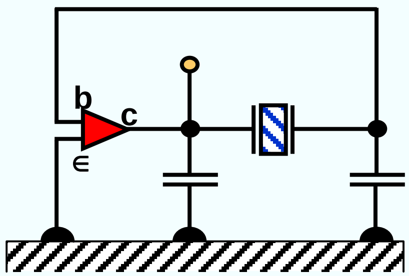
Oscillator Type*	Accuracy**	Typical Applications
• Crystal oscillator (XO)	10^{-5} to 10^{-4}	Computer timing
• Temperature compensated crystal oscillator (TCXO)	10^{-6}	Frequency control in tactical radios
• Microcomputer compensated crystal oscillator (MCXO)	10^{-8} to 10^{-7}	Spread spectrum system clock
• Oven controlled crystal oscillator (OCXO)	10^{-8} (with 10^{-10} per g option)	Navigation system clock & frequency standard, MTI radar
• Small atomic frequency standard (Rb, RbXO)	10^{-9}	C^3 satellite terminals, bistatic, & multistatic radar
• High performance atomic standard (Cs)	10^{-12} to 10^{-11}	Strategic C^3, EW

* Sizes range from <5cm^3 for clock oscillators to > 30 liters for Cs standards
Costs range from <$5 for clock oscillators to > $50,000 for Cs standards.

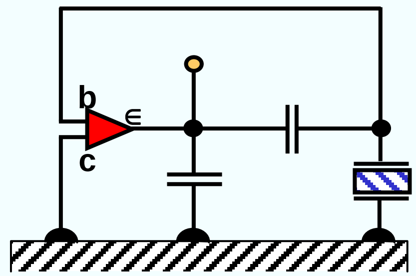
** Including environmental effects (e.g., -40^{o}C to +75^{o}C) and one year of aging.

Oscillator Circuit Types

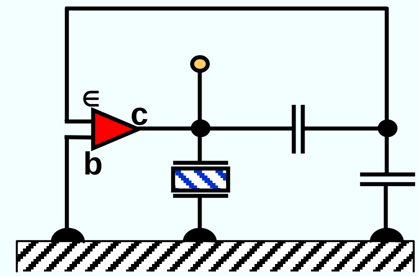
Of the numerous oscillator circuit types, three of the more common ones, the Pierce, the Colpitts and the Clapp, consist of the same circuit except that the rf ground points are at different locations. The Butler and modified Butler are also similar to each other; in each, the emitter current is the crystal current. The gate oscillator is a Pierce-type that uses a logic gate plus a resistor in place of the transistor in the Pierce oscillator. (Some gate oscillators use more than one gate).



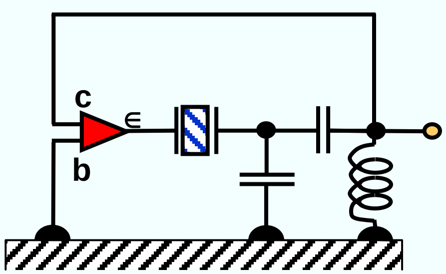
Pierce



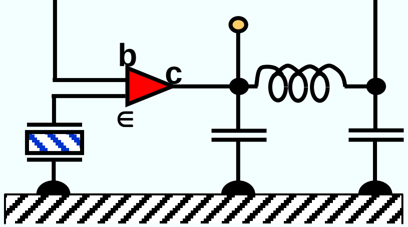
Colpitts



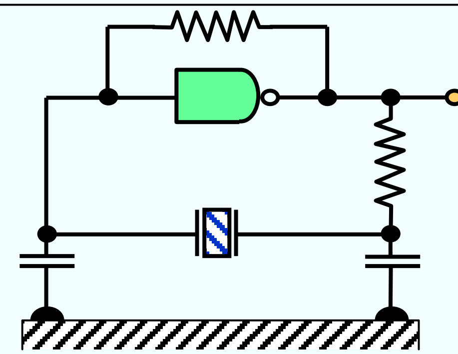
Clapp



Butler

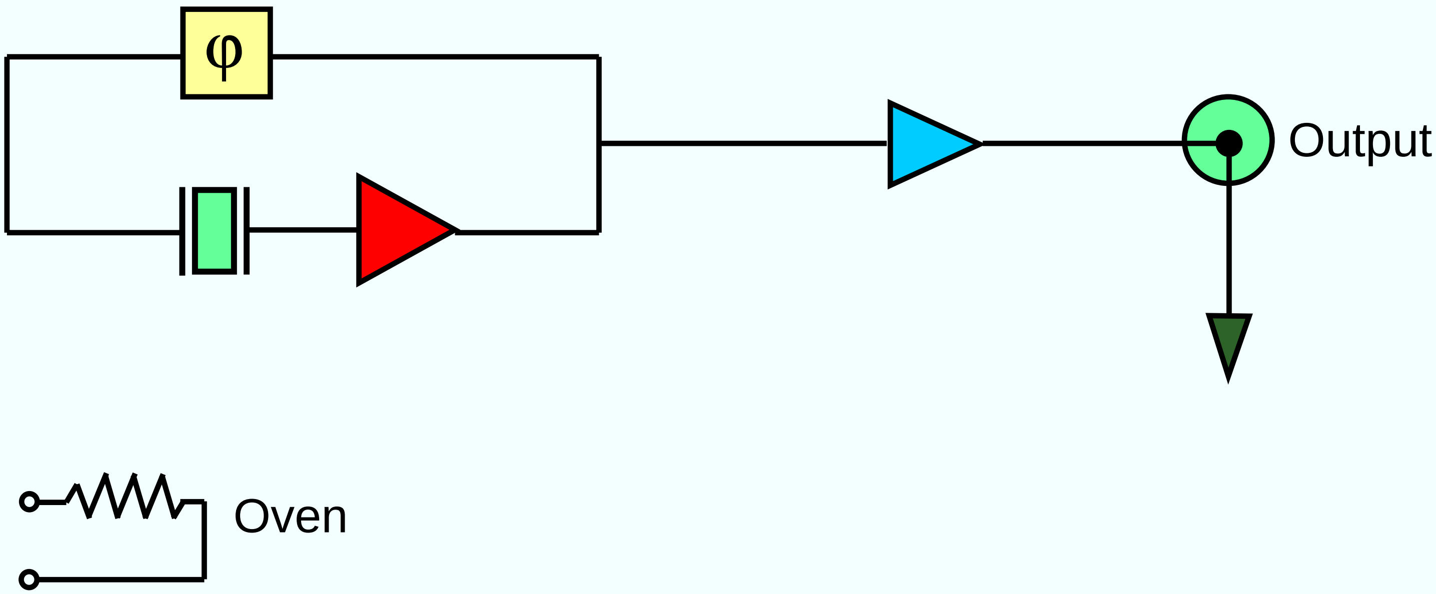


Modified Butler



Gate

OCXO Block Diagram



Each of the three main parts of an OCXO, i.e., the crystal, the sustaining circuit, and the oven, contribute to instabilities. The various instabilities are discussed in the rest of chapter 3 and in chapter 4.

Oscillator Instabilities - General Expression

$$\frac{\Delta f}{f_{oscillator}} \approx \frac{\Delta f}{f_{resonator}} + \frac{1}{2Q_L}\left[1+\left(\frac{2f_f Q_L}{f}\right)^2\right]^{-1/2} d\varphi(f_f)$$

where Q_L = loaded Q of the resonator, and $d\phi(f_f)$ is a small change in loop phase at offset frequency f_f away from carrier frequency f. Systematic phase changes and phase noise within the loop can originate in either the resonator or the sustaining circuits. Maximizing Q_L helps to reduce the effects of noise and environmentally induced changes in the sustaining electronics. In a properly designed oscillator, the short-term instabilities are determined by the resonator at offset frequencies smaller than the resonator's half-bandwidth, and by the sustaining circuit and the amount of power delivered from the loop for larger offsets.

Instabilities due to Sustaining Circuit

- **Load reactance change** - adding a load capacitance to a crystal changes the frequency by

$$\delta f \equiv \frac{\Delta f}{f} \cong \frac{C_1}{2\left(C_0 + C_L\right)}$$

$$\text{then, } \frac{\Delta\left(\delta f\right)}{\Delta C_L} \cong -\frac{C_1}{2\left(C_0 + C_L\right)^2}$$

- **Example**: If C_0 = 5 pF, C_1 = 14fF and C_L = 20pF, then a ΔC_L = 10 fF (= 5 X 10^{-4}) causes $\approx$1 X 10^{-7} frequency change, and a C_L aging of 10 ppm per day causes 2 X 10^{-9} per day of oscillator aging.

- **Drive level changes:** Typically 10^{-8} per ma^2 for a 10 MHz 3rd SC-cut.

- **DC bias** on the crystal also contributes to oscillator aging.

Oscillator Instabilities - Tuned Circuits

Many oscillators contain tuned circuits - to suppress unwanted modes, as matching circuits, and as filters. The effects of small changes in the tuned circuit's inductance and capacitance is given by:

$$\frac{\Delta f}{f}_{\text{oscillator}} \approx \frac{d\varphi(f_f)}{2Q_L} \approx \left(\frac{1}{1 + \frac{2f_f}{BW}} \right) \left(\frac{Q_c}{Q} \right) \left(\frac{dC_c}{C_c} + \frac{dL_c}{L_c} \right)$$

where BW is the bandwidth of the filter, f_f is the frequency offset of the center frequency of the filter from the carrier frequency, Q_L is the loaded Q of the resonator, and Q_c, L_c and C_c are the tuned circuit's Q, inductance and capacitance, respectively.

Oscillator Instabilities - Circuit Noise

Flicker PM noise in the sustaining circuit causes flicker FM contribution to the oscillator output frequency given by:

$$\mathcal{L}_{osc}(f_f) = \mathcal{L}_{ckt}(1\text{Hz}) \frac{f^2}{4 f_f^3 Q_L^2}$$

and

$$\sigma_y(\tau) = \frac{1}{Q_L} \sqrt{\ln 2\, \mathcal{L}_{ckt}(1\text{Hz})}$$

where f_f is the frequency offset from the carrier frequency f, Q_L is the loaded Q of the resonator in the circuit, $\mathcal{L}_{ckt}(1\text{Hz})$ is the flicker PM noise at $f_f = 1\text{Hz}$, and τ is any measurement time in the flicker floor range. For $Q_L = 10^6$ and $\mathcal{L}_{ckt}(1\text{Hz}) = -140\text{dBc/Hz}$, $\sigma_y(\tau) = 8.3 \times 10^{-14}$. ($\mathcal{L}_{ckt}(1\text{Hz}) = -155\text{dBc/Hz}$ has been achieved.)

Oscillator Instabilities - External Load

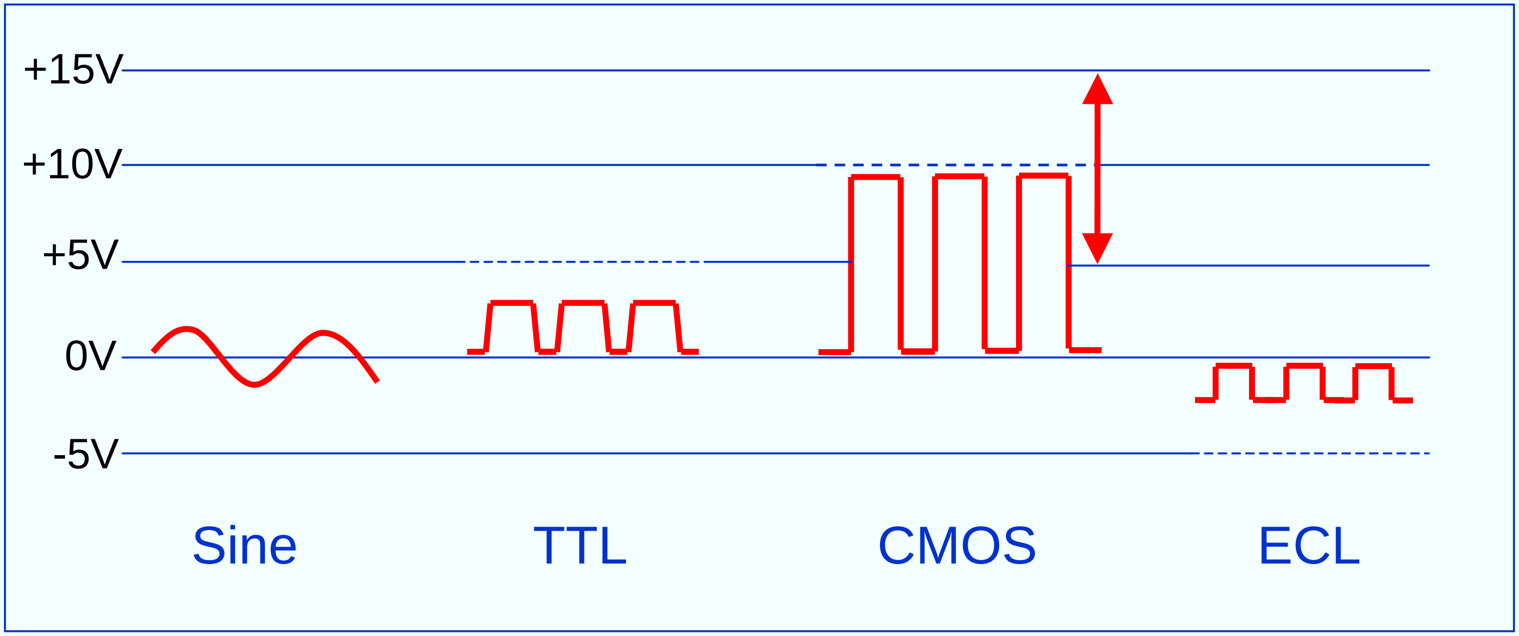
If the external load changes, there is a change in the amplitude or phase of the signal reflected back into the oscillator. The portion of that signal which reaches the oscillating loop changes the oscillation phase, and hence the frequency by

$$\frac{\Delta f}{f_{\text{oscillator}}} \approx \frac{d\phi(f_f)}{2Q} \approx \left(\frac{1}{2Q}\right)\left(\frac{\Gamma - 1}{\Gamma + 1}\right)(\sin\theta)\sqrt{\text{isolation}}$$

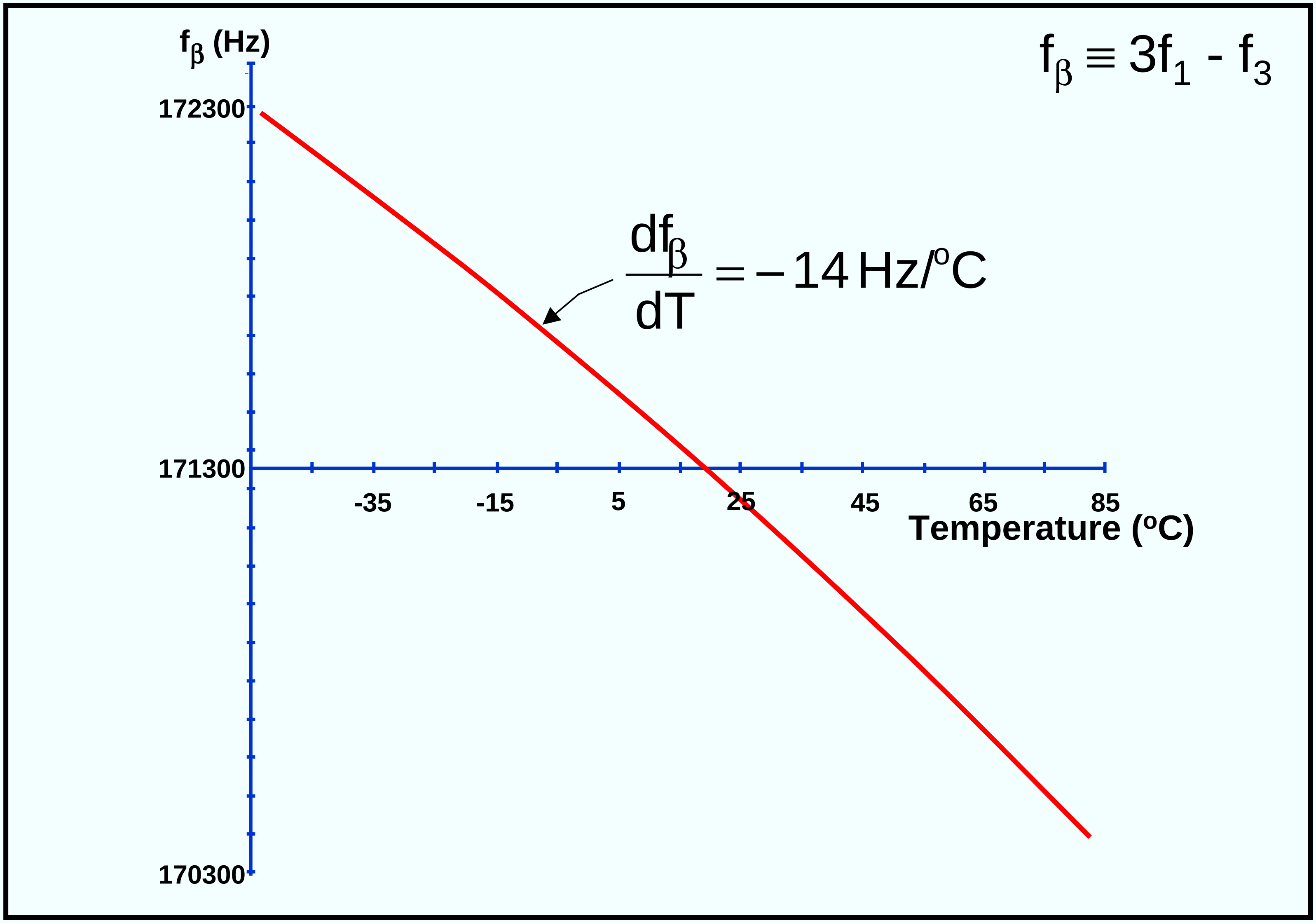
where Γ is the VSWR of the load, and θ is the phase angle of the reflected wave; e.g., if Q ~ 10^{6}, and isolation ~40 dB (i.e., ~10^{-4}), then the worst case (100% reflection) pulling is ~5 x 10^{-9}. A VSWR of 2 reduces the maximum pulling by only a factor of 3. The problem of load pulling becomes worse at higher frequencies, because both the Q and the isolation are lower.

Oscillator Outputs

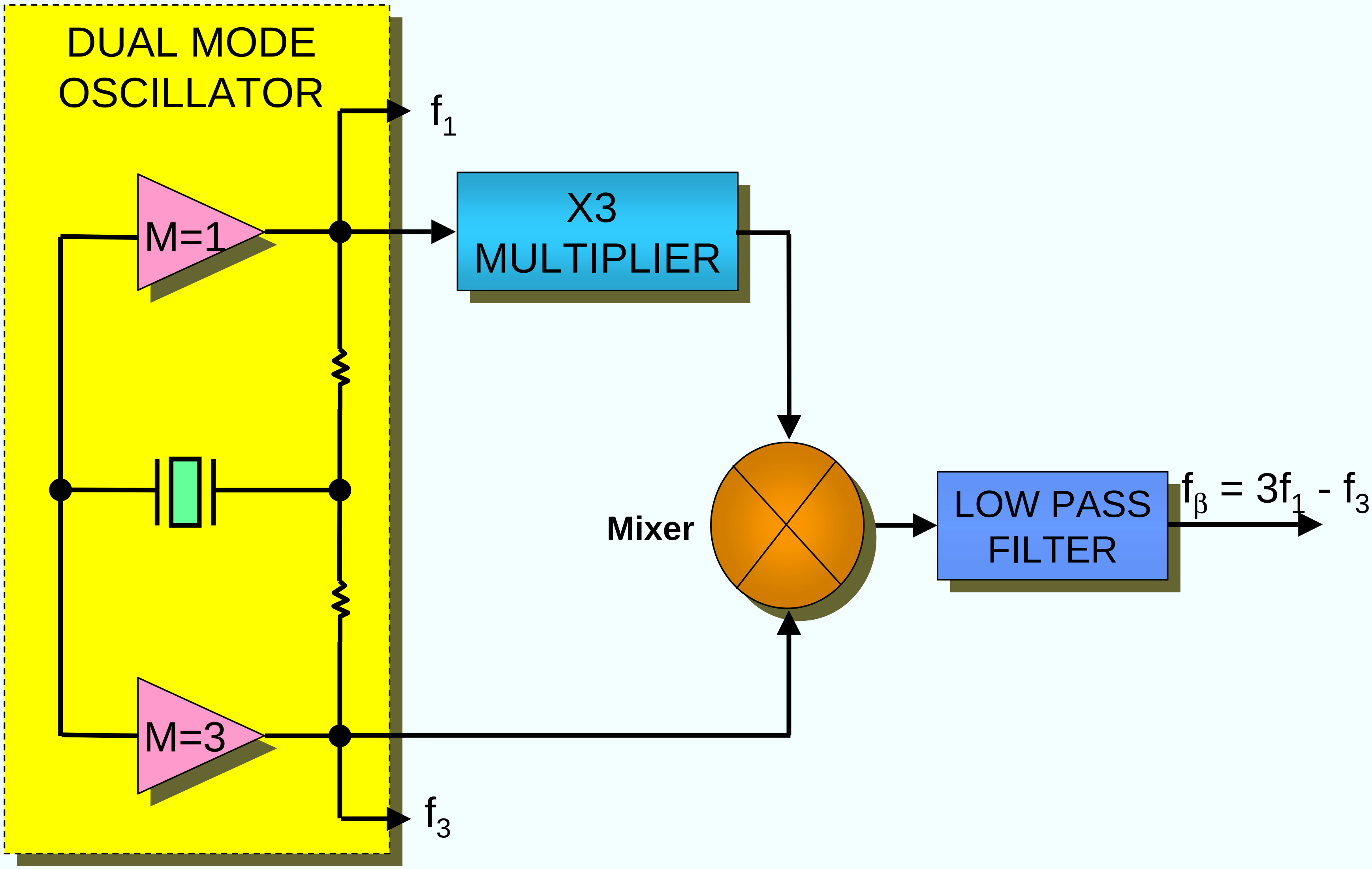
Most users require a sine wave, a TTL-compatible, a CMOS-compatible, or an ECL-compatible output. The latter three can be simply generated from a sine wave. The four output types are illustrated below, with the dashed lines representing the supply voltage inputs, and the bold solid lines, the outputs. (There is no "standard" input voltage for sine wave oscillators, and the input voltage for CMOS typically ranges from 3V to 15V.)



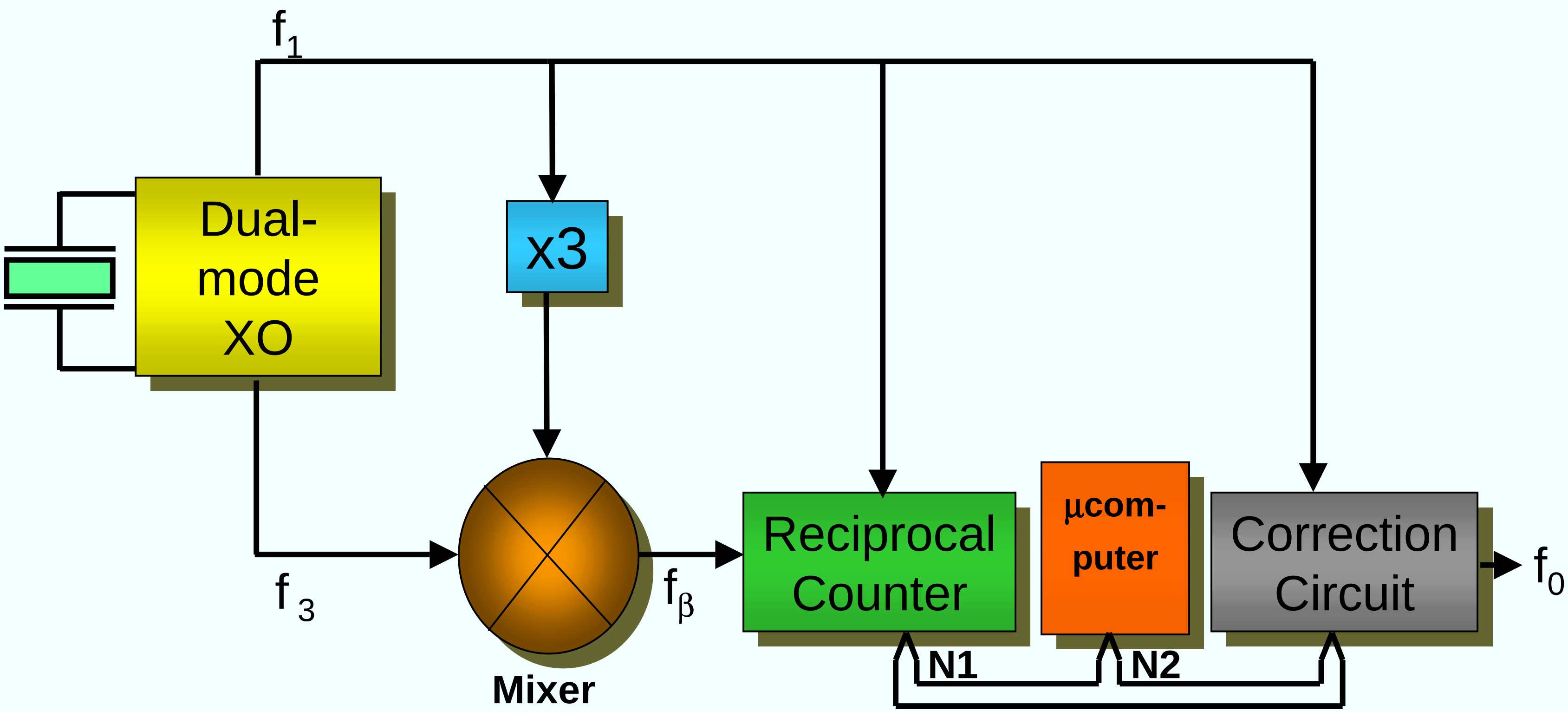
Resonator Self-Temperature Sensing



Thermometric Beat Frequency Generation

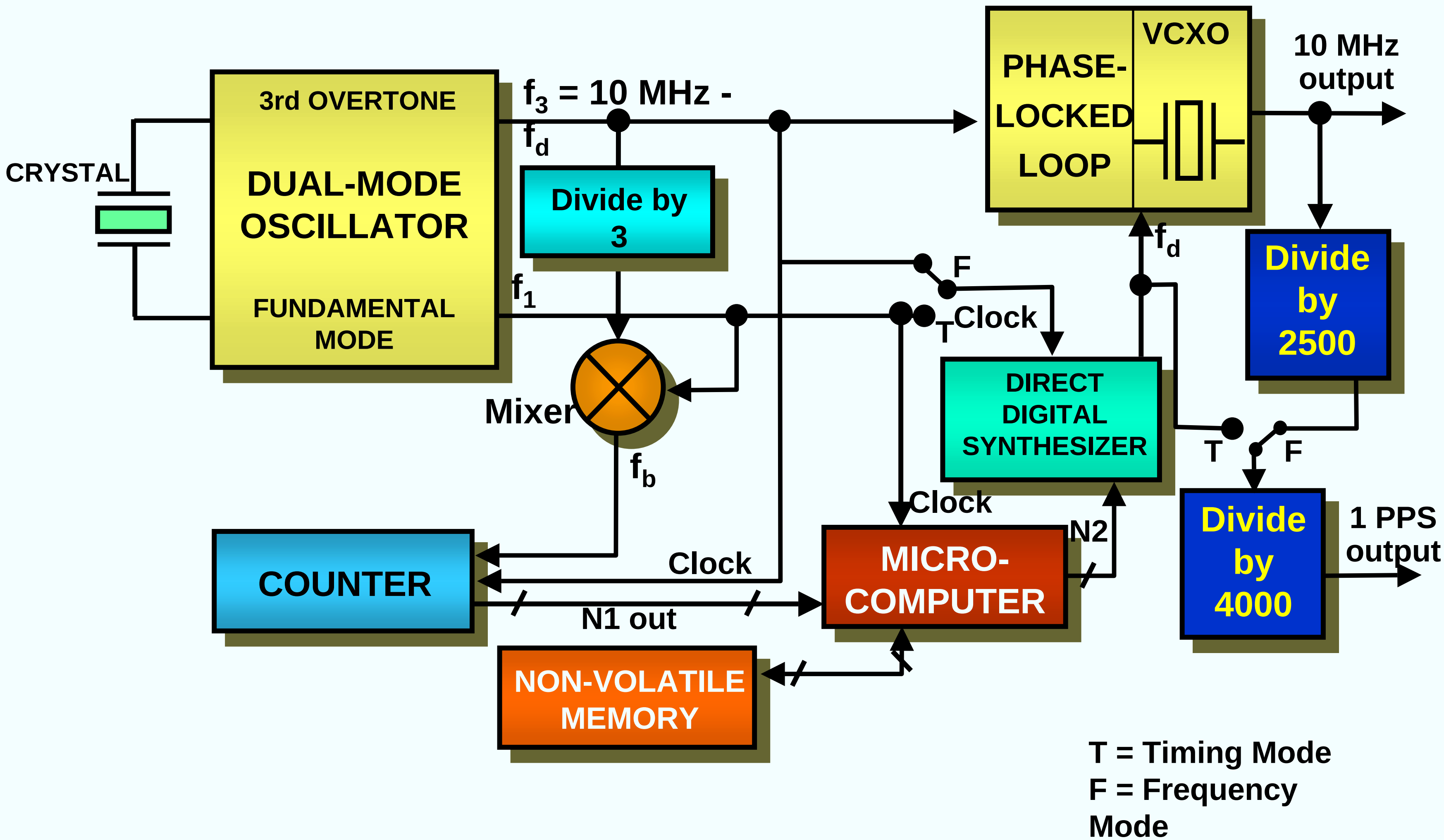


Microcomputer Compensated Crystal Oscillator (MCXO)

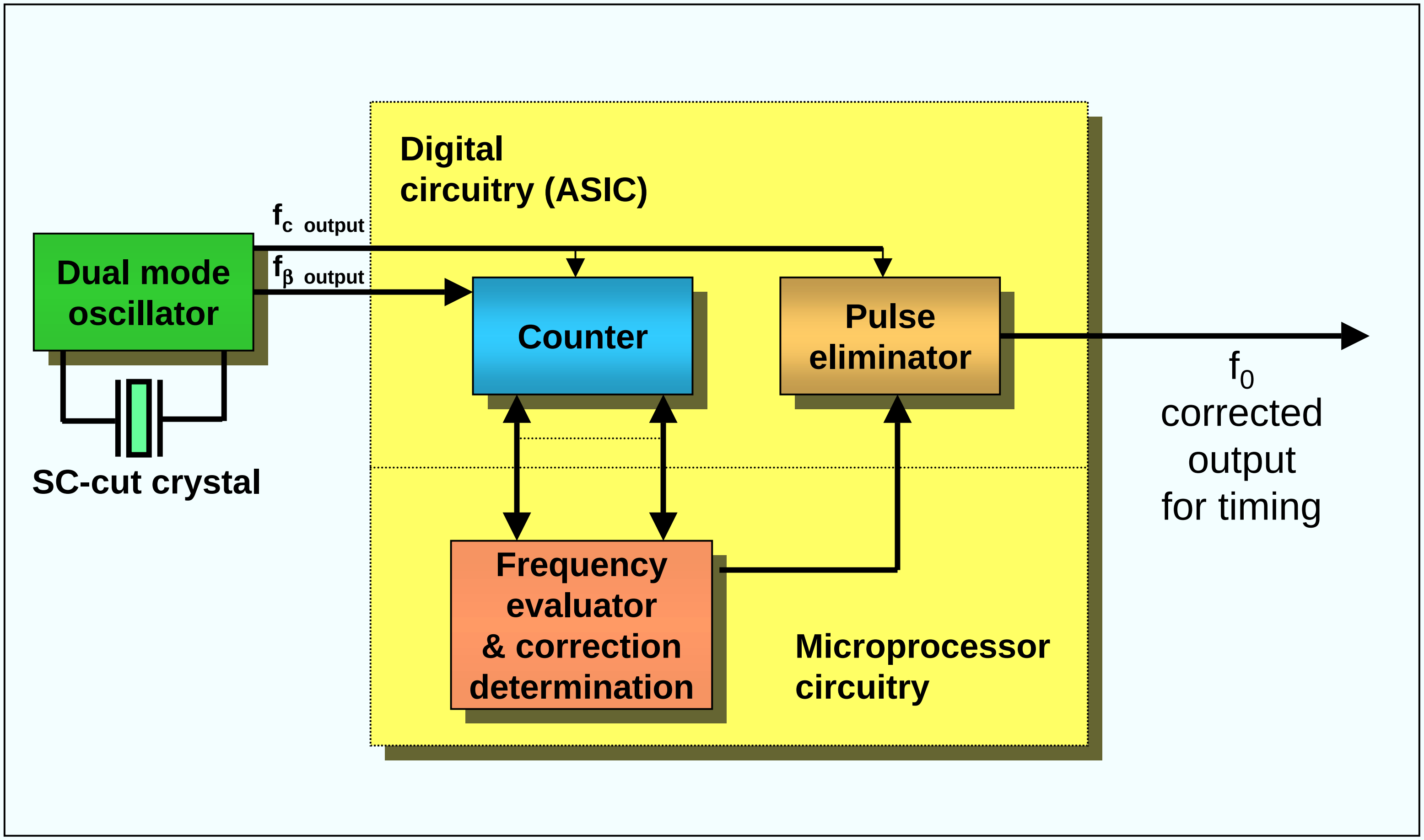


MCXO Frequency Summing Method

Block Diagram



MCXO - Pulse Deletion Method

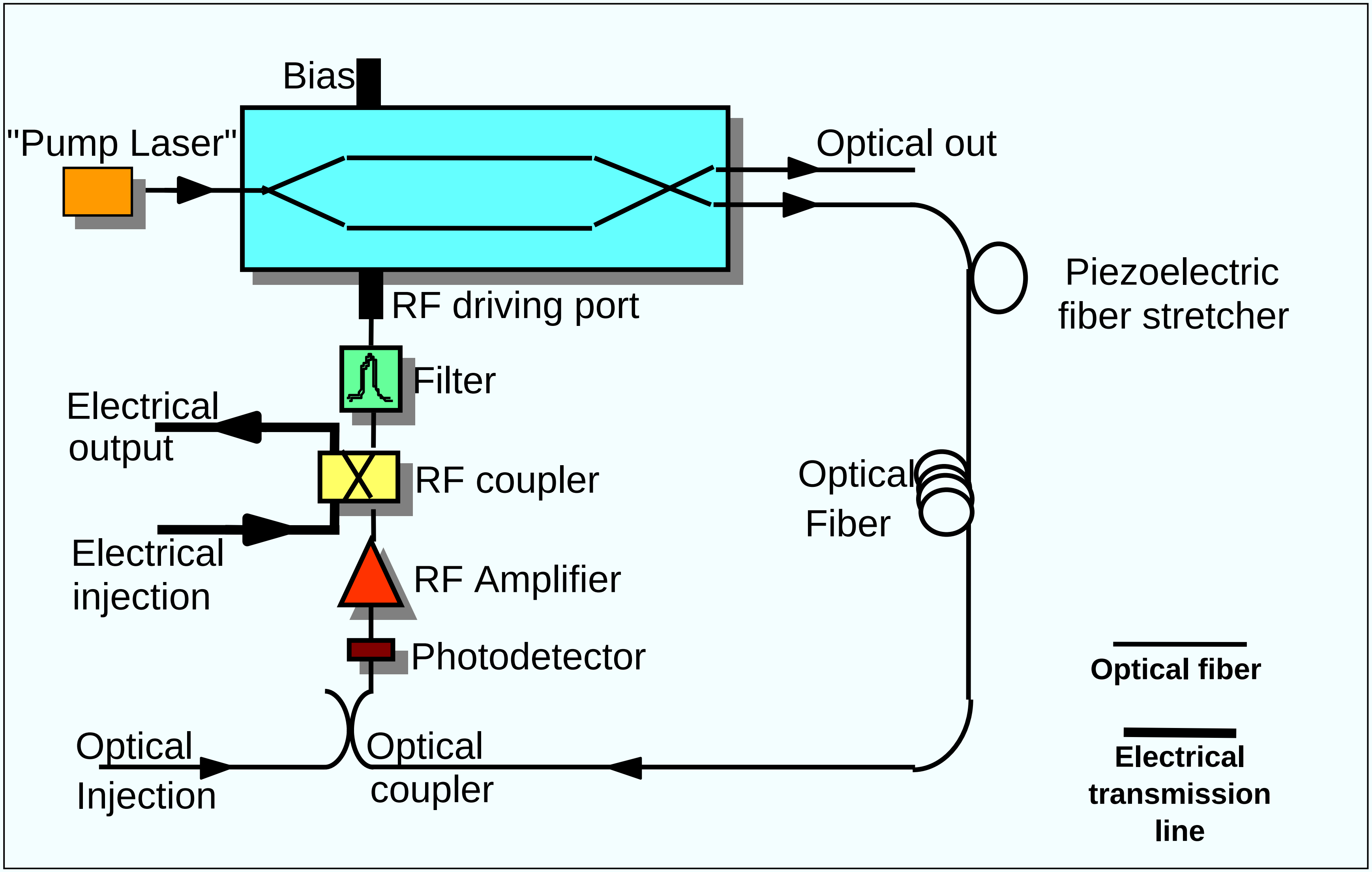


Microcomputer compensated crystal oscillator (MCXO) block diagram - pulse deletion method.

MCXO - TCXO Resonator Comparison

Parameter	MCXO	TCXO
Cut, overtone	SC-cut, 3rd	AT-cut, fund.
Angle-of-cut tolerance	Loose	Tight
Blank f and plating tolerance	Loose	Tight
Activity dip incidence	Low	Significant
Hysteresis (-55^{0}C to +85^{0}C)	10^{-9} to 10^{-8}	10^{-7} to 10^{-6}
Aging per year	10^{-8} to 10^{-7}	10^{-7} to 10^{-6}

Opto-Electronic Oscillator (OEO)



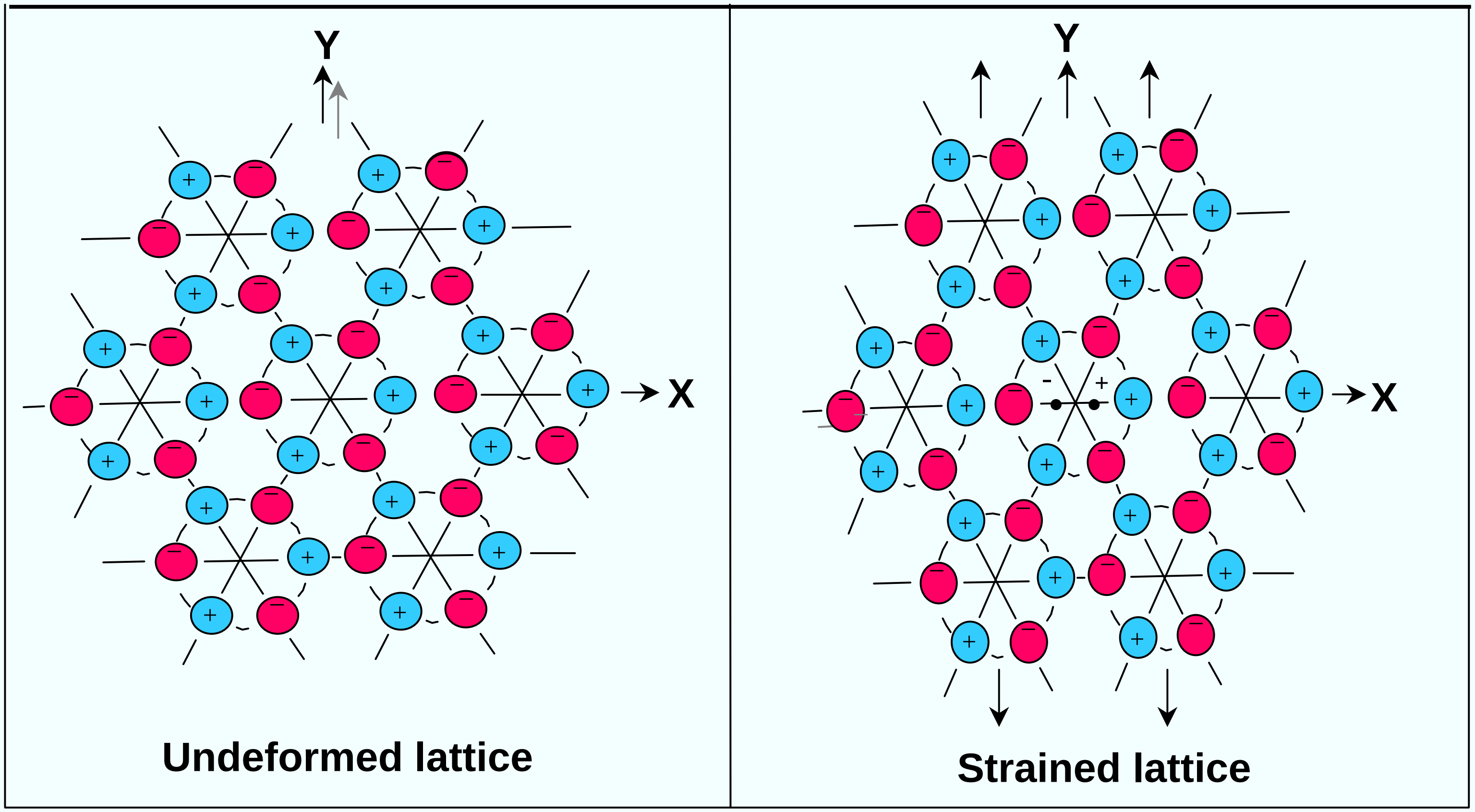
CHAPTER 3
Quartz Crystal Resonators

Why Quartz?

Quartz is the only material known that possesses the following combination of properties:

- Piezoelectric ("pressure-electric"; piezein = to press, in Greek)
- Zero temperature coefficient cuts exist
- Stress compensated cut exists
- Low loss (i.e., high Q)
- Easy to process; low solubility in everything, under "normal" conditions, except the fluoride etchants; hard but not brittle
- Abundant in nature; easy to grow in large quantities, at low cost, and with relatively high purity and perfection. Of the man-grown single crystals, quartz, at ~3,000 tons per year, is second only to silicon in quantity grown (3 to 4 times as much Si is grown annually, as of 1997).

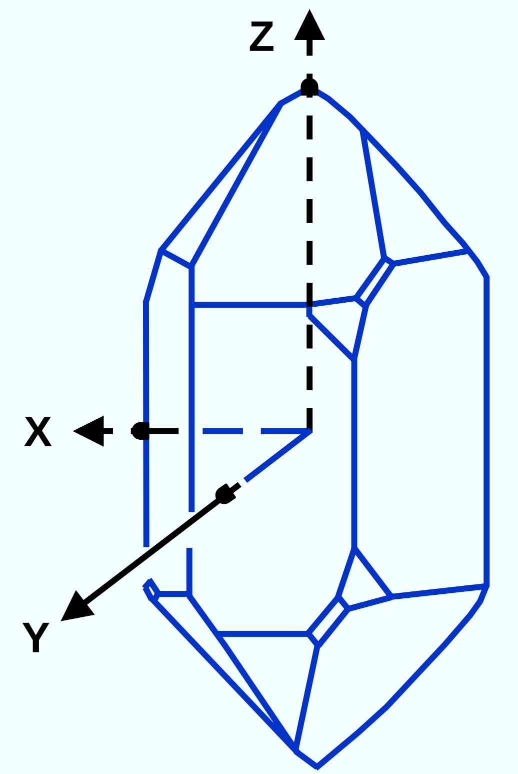
The Piezoelectric Effect



The piezoelectric effect provides a coupling between the mechanical properties of a piezoelectric crystal and an electrical circuit.

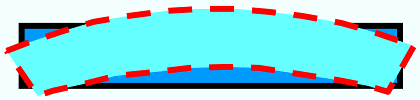
The Piezoelectric Effect in Quartz

STRAIN		FIELD along: X	FIELD along: Y	FIELD along: Z
EXTENSIONAL along:	X	√		
	Y	√		
	Z			
SHEAR about:	X	√		
	Y		√	
	Z		√	

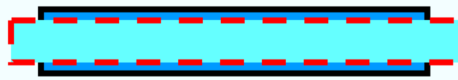


In quartz, the five strain components shown may be generated by an electric field. The modes shown on the next page may be excited by suitably placed and shaped electrodes. The shear strain about the Z-axis produced by the Y-component of the field is used in the rotated Y-cut family, including the AT, BT, and ST-cuts.

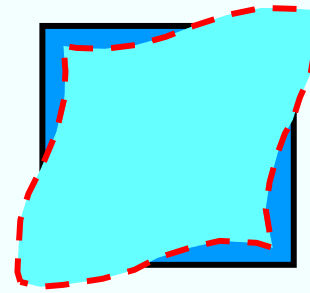
Modes of Motion



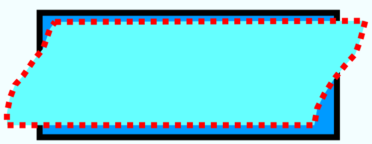
Flexure Mode



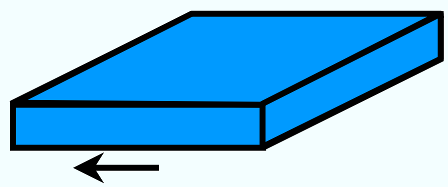
Extensional Mode



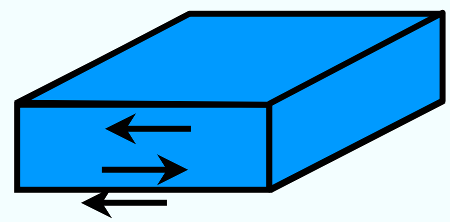
Face Shear Mode



Thickness Shear Mode

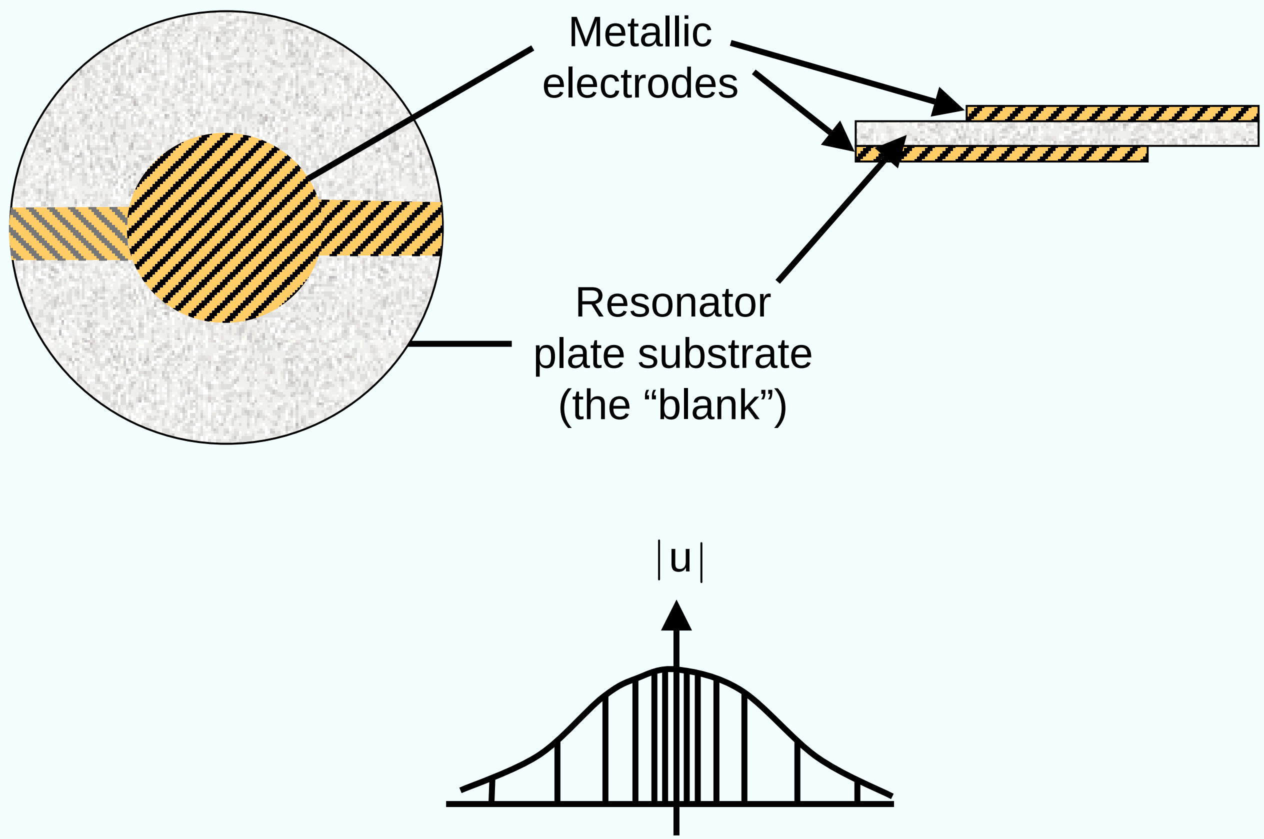


Fundamental Mode Thickness Shear



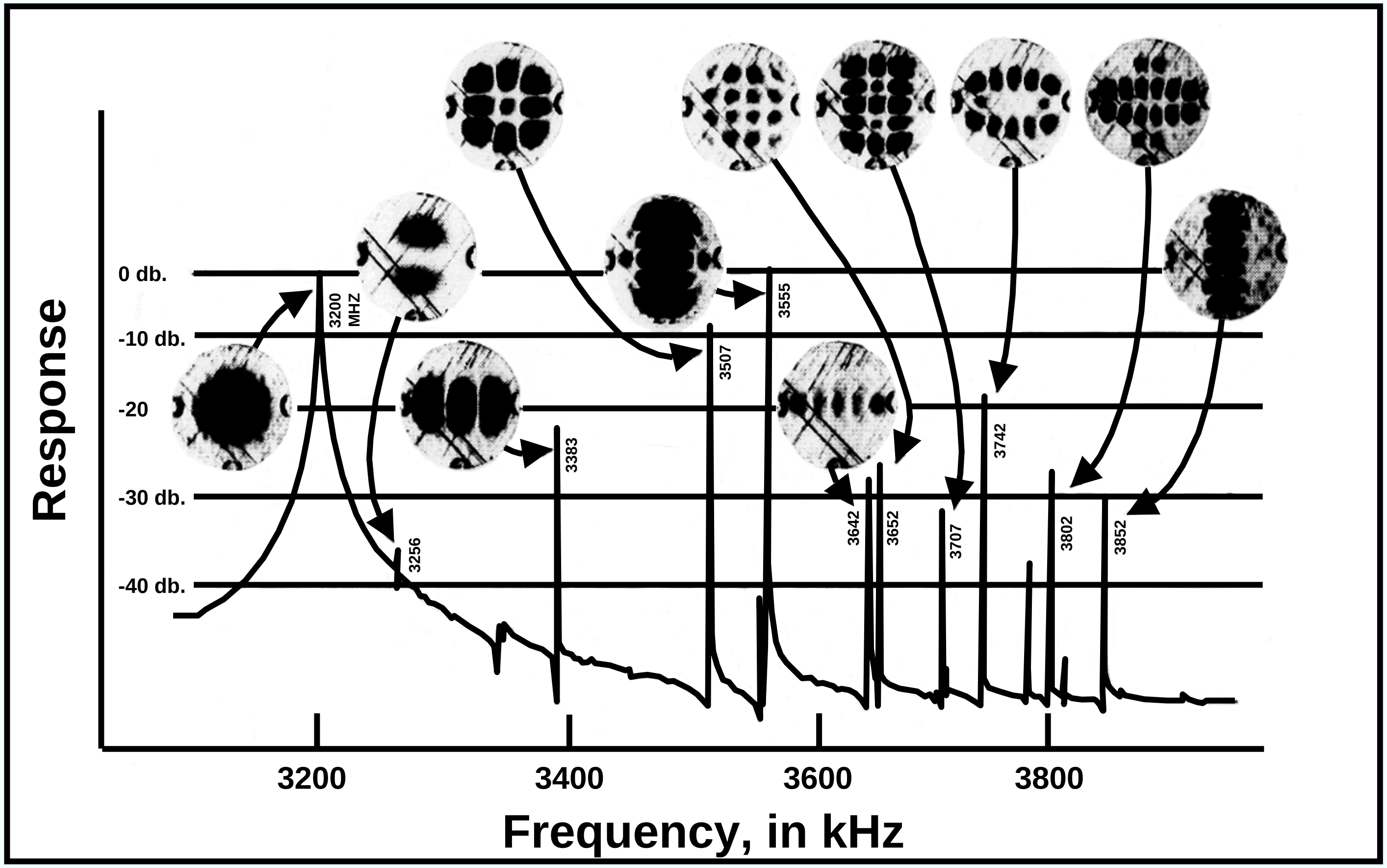
Third Overtone Thickness Shear

Resonator Vibration Amplitude Distribution



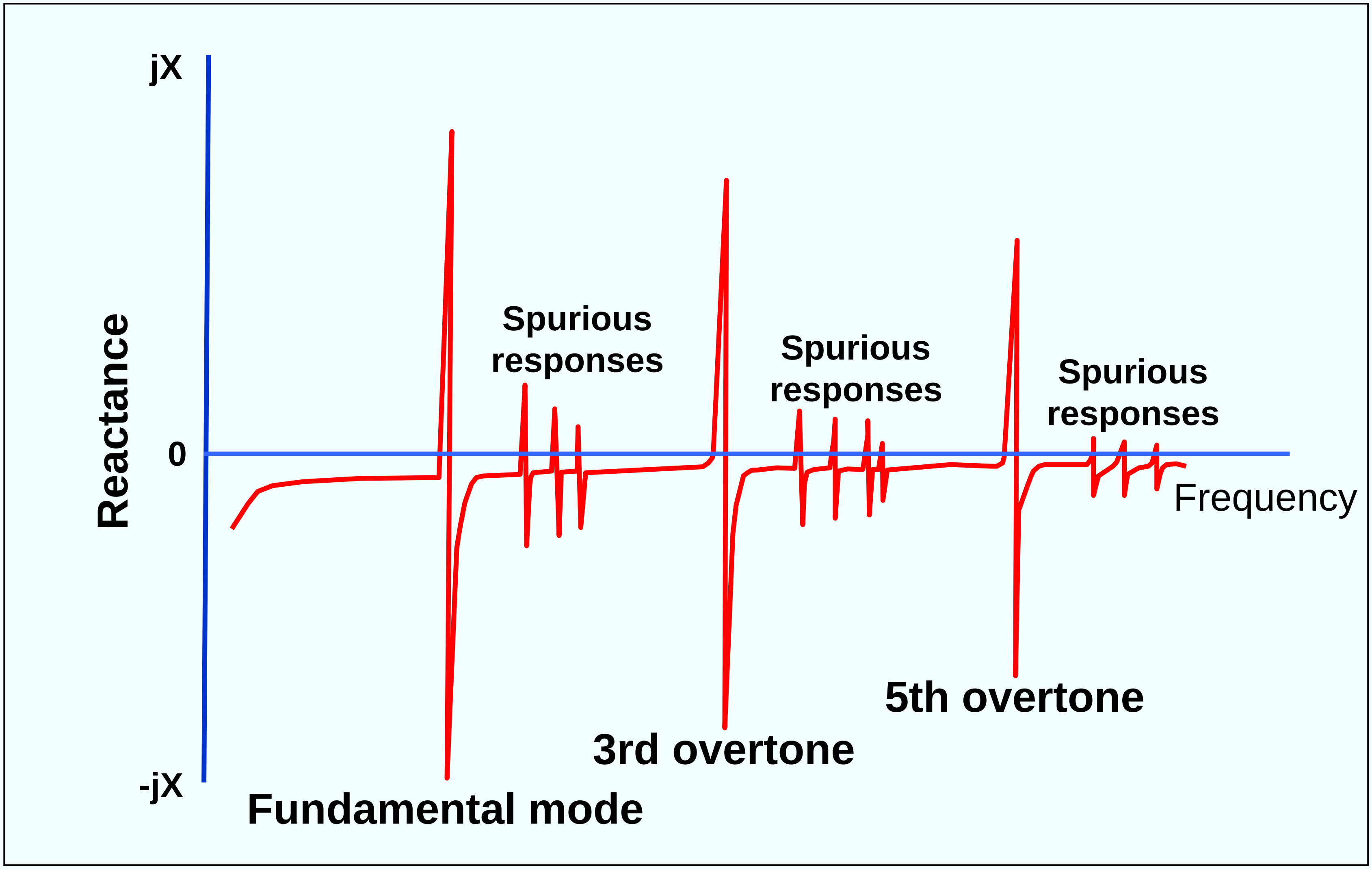
Conventional resonator geometry
and amplitude distribution, |u|

Resonant Vibrations of a Quartz Plate



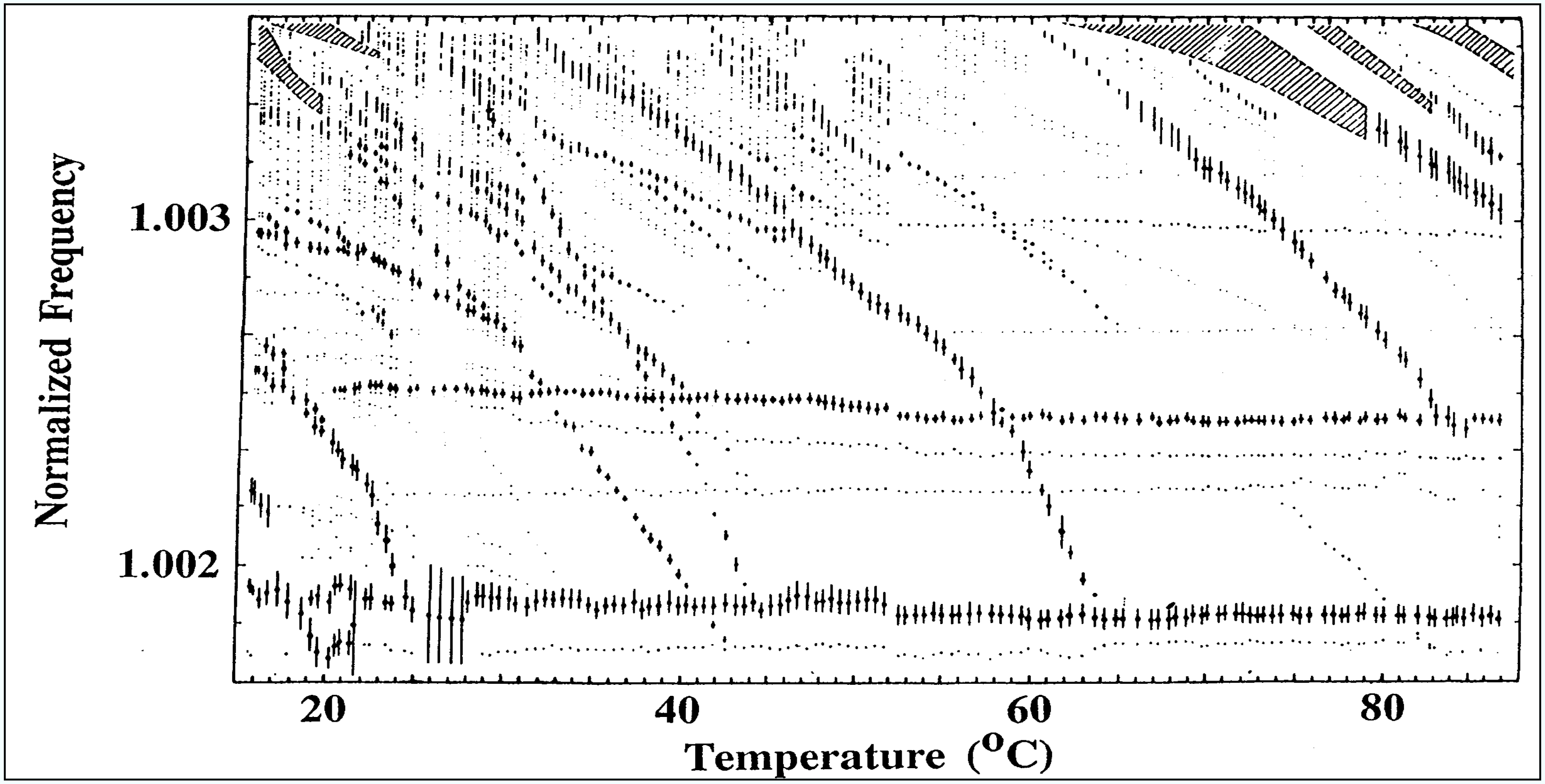
X-ray topographs (21•$\bar{0}$ plane) of various modes excited during a frequency scan of a fundamental mode, circular, AT-cut resonator. The first peak, at 3.2 MHz, is the main mode; all others are unwanted modes. Dark areas correspond to high amplitudes of displacement.

Overtone Response of a Quartz Crystal



Unwanted Modes vs. Temperature

(3 MHz rectangular AT-cut resonator, 22 X 27 X 0.552 mm)



Activity dips occur where the f vs. T curves of unwanted modes intersect the f vs. T curve of the wanted mode. Such activity dips are highly sensitive to drive level and load reactance.

Mathematical Description of a Quartz Resonator

- In piezoelectric materials, electrical current and voltage are coupled to elastic displacement and stress:

$$\{T\} = [C]\{S\} - [e]\{E\}$$
$$\{D\} = [e]\{S\} + [\in]\{E\}$$

where {T} = stress tensor, [C] = elastic stiffness matrix, {S} = strain tensor, [e] = piezoelectric matrix
{E} = electric field vector, {D} = electric displacement vector, and [$\in$] = is the dielectric matrix

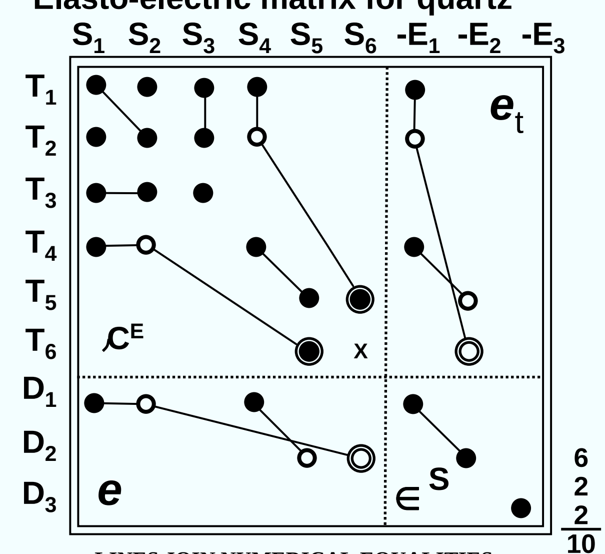
- For a linear piezoelectric material

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ D_1 \\ D_2 \\ D_3 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} & -e_{11} & -e_{21} & -e_{31} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} & -e_{12} & -e_{22} & -e_{32} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} & -e_{13} & -e_{23} & -e_{33} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} & -e_{14} & -e_{24} & -e_{34} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} & -e_{15} & -e_{25} & -e_{35} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} & -e_{16} & -e_{26} & -e_{36} \\ e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} & \in_{11} & \in_{12} & \in_{13} \\ e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} & \in_{21} & \in_{22} & \in_{23} \\ e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36} & \in_{31} & \in_{32} & \in_{33} \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ E_1 \\ E_2 \\ E_3 \end{pmatrix},$$

where

$T_1 = T_{11}$, $S_1 = S_{11}$,
$T_2 = T_{22}$, $S_2 = S_{22}$,
$T_3 = T_{33}$, $S_3 = S_{33}$,
$T_4 = T_{23}$, $S_4 = 2S_{23}$,
$T_5 = T_{13}$, $S_5 = 2S_{13}$,
$T_6 = T_{12}$, $S_6 = 2S_{12}$,

- **Elasto-electric matrix for quartz**



LINES JOIN NUMERICAL EQUALITIES
EXCEPT FOR COMPLETE RECIPROCITY
ACROSS PRINCIPAL DIAGONAL
○ INDICATES NEGATIVE OF ●
◉ ◎ INDICATES TWICE THE NUMERICAL EQUALITIES
X INDICATES 1/2 (c_{11} - c_{12})

Mathematical Description - Continued

- Number of independent non-zero constants depend on crystal symmetry. For quartz (trigonal, class 32), there are 10 independent linear constants - 6 elastic, 2 piezoelectric and 2 dielectric. "Constants" depend on temperature, stress, coordinate system, etc.

- To describe the behavior of a resonator, the differential equations for Newton's law of motion for a continuum, and for Maxwell's equation* must be solved, with the proper electrical and mechanical boundary conditions at the plate surfaces. $(F = ma \Rightarrow \frac{\partial T_{ij}}{\partial x_j} = \rho\, \ddot{u}_i; \quad \overline{\nabla} \cdot \overline{D} = 0 \Rightarrow \frac{\partial D_i}{\partial x_i} = 0,$

$$E_i = -\frac{\partial \varphi}{\partial x_i}; \; S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right); \text{ etc.)}$$

- Equations are very "messy" - they have never been solved in closed form for physically realizable three-dimensional resonators. Nearly all theoretical work has used approximations.

- Some of the most important resonator phenomena (e.g., acceleration sensitivity) are due to nonlinear effects. Quartz has numerous higher order constants, e.g., 14 third-order and 23 fourth-order elastic constants, as well as 16 third-order piezoelectric coefficients are known; nonlinear equations are extremely messy.

---

* Magnetic field effects are generally negligible; quartz is diamagnetic, however, magnetic fields can affect the mounting structure and electrodes.

Infinite Plate Thickness Shear Resonator

$$f_n = \frac{n}{2h}\sqrt{\frac{c_{ij}}{\rho}}, \quad n = 1, 3, 5...$$

Where f_n = resonant frequency of n-th harmonic
h = plate thickness
ρ = density
c_{ij} = elastic modulus associated with the elastic wave being propagated

$$T_f = \frac{d(\log f_n)}{dT} = \frac{1}{f_n}\frac{df_n}{dT} = \frac{-1}{h}\frac{dh}{dT} - \frac{1}{2\rho}\frac{d\rho}{dT} + \frac{1}{2c_{ij}}\frac{dc_{ij}}{dT}$$

where T_f is the linear temperature coefficient of frequency. The temperature coefficient of c_{ij} is negative for most materials (i.e., “springs” become “softer” as T increases). The coefficients for quartz can be +, - or zero (see next page).

Quartz is Highly Anisotropic

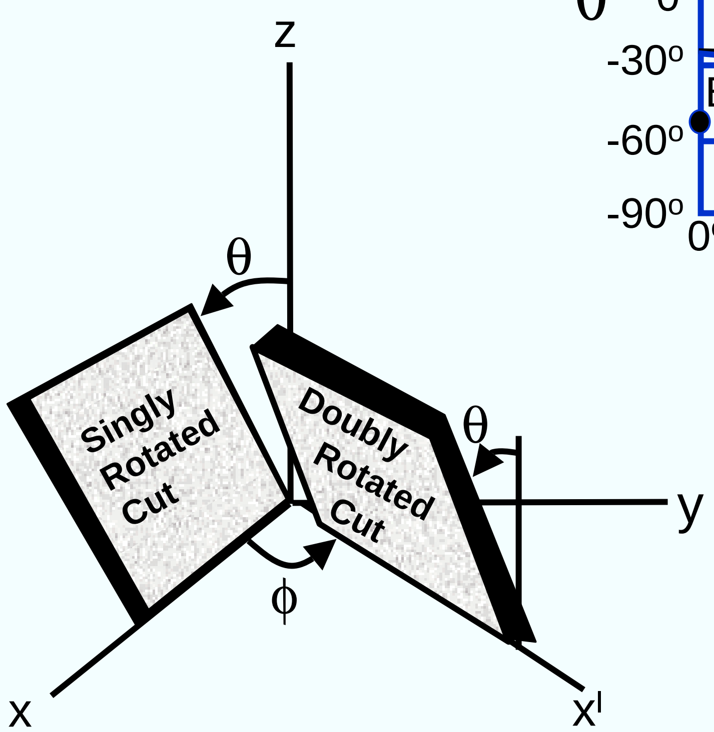
- The properties of quartz vary greatly with crystallographic direction. For example, when a quartz sphere is etched deeply in HF, the sphere takes on a triangular shape when viewed along the Z-axis, and a lenticular shape when viewed along the Y-axis. The etching rate is more than 100 times faster along the fastest etching rate direction (the Z-direction) than along the slowest direction (the slow-X-direction).

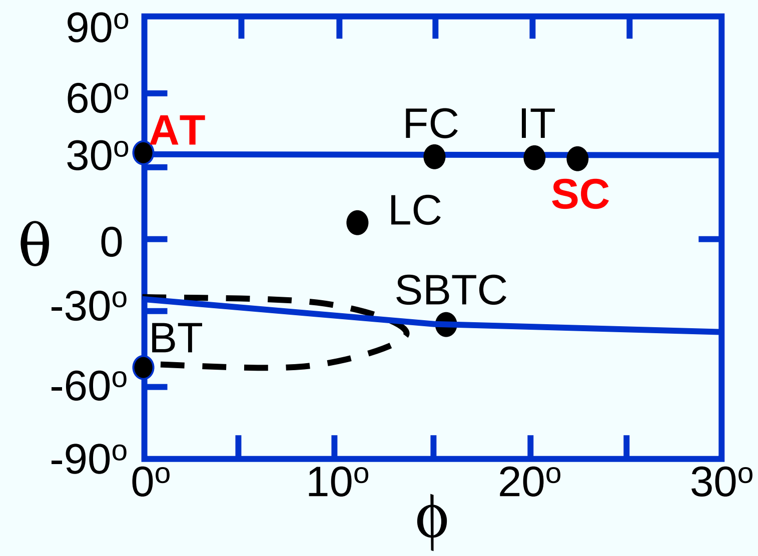
- The thermal expansion coefficient is 7.8 x 10^{-6}/°C along the Z-direction, and 14.3 x 10^{-6}/°C perpendicular to the Z-direction; the temperature coefficient of density is, therefore, -36.4 x 10^{-6}/°C.

- The temperature coefficients of the elastic constants range from -3300 x 10^{-6}/°C (for C_{12}) to +164 x 10^{-6}/°C (for C_{66}).

- For the proper angles of cut, the sum of the first two terms in T_f on the previous page is cancelled by the third term, i.e., temperature compensated cuts exist in quartz. (See next page.)

Zero Temperature Coefficient Quartz Cuts





The AT, FC, IT, SC, BT, and SBTC-cuts are some of the cuts on the locus of zero temperature coefficient cuts. The LC is a "linear coefficient" cut that has been used in a quartz thermometer.

Y-cut: ≈ +90 ppm/^{0}C
(thickness-shear mode)

X-cut: ≈ -20 ppm/^{0}C
(extensional mode)

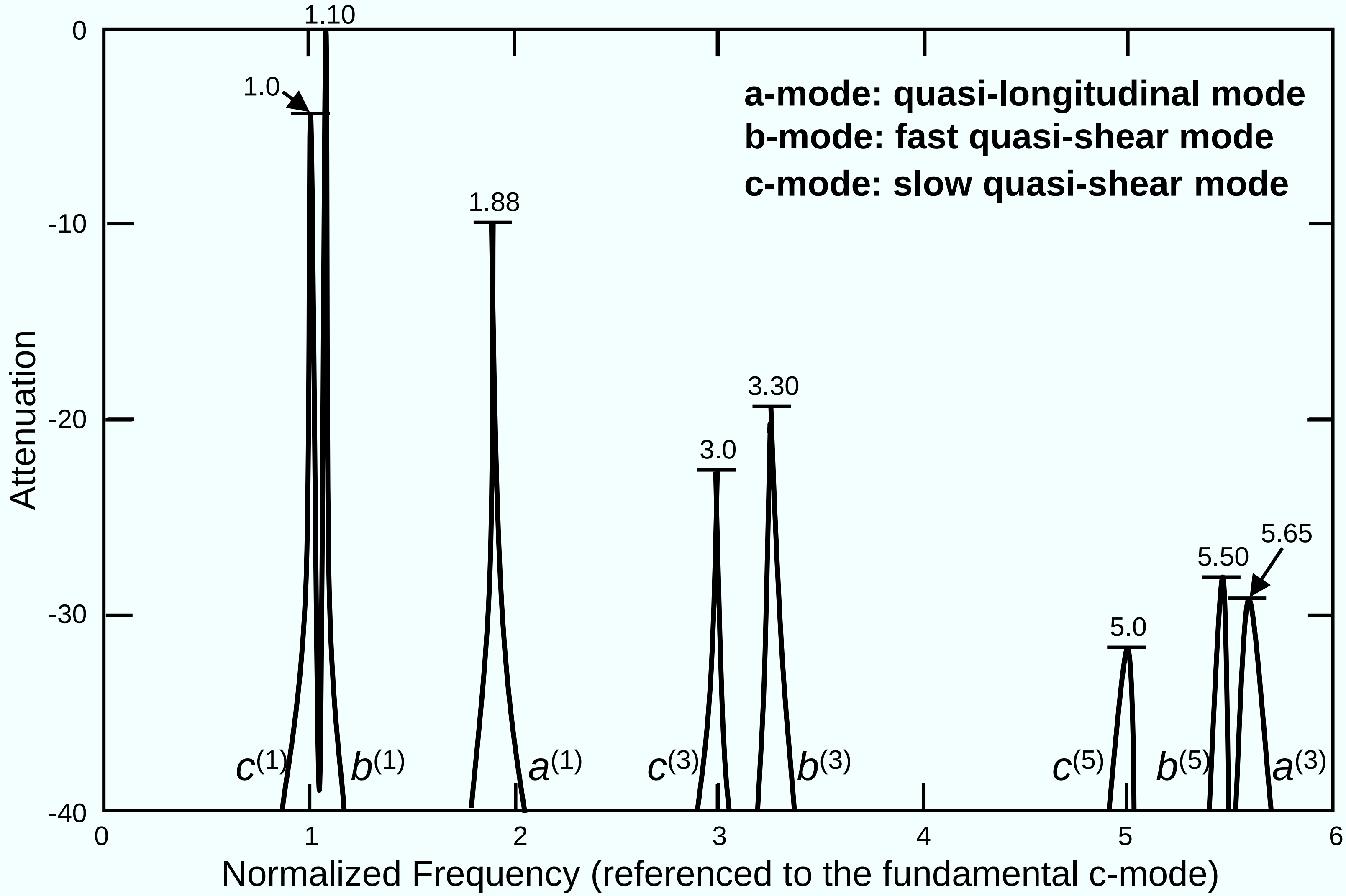
Comparison of SC and AT-cuts

- **Advantages of the SC-cut**
  - Thermal transient compensated (allows faster warmup OCXO)
  - Static and dynamic f vs. T allow higher stability OCXO and MCXO
  - Better f vs. T repeatability allows higher stability OCXO and MCXO
  - Far fewer activity dips
  - Lower drive level sensitivity
  - Planar stress compensated; lower Δf due to edge forces and bending
  - Lower sensitivity to radiation
  - Higher capacitance ratio (less Δf for oscillator reactance changes)
  - Higher Q for fundamental mode resonators of similar geometry
  - Less sensitive to plate geometry - can use wide range of contours

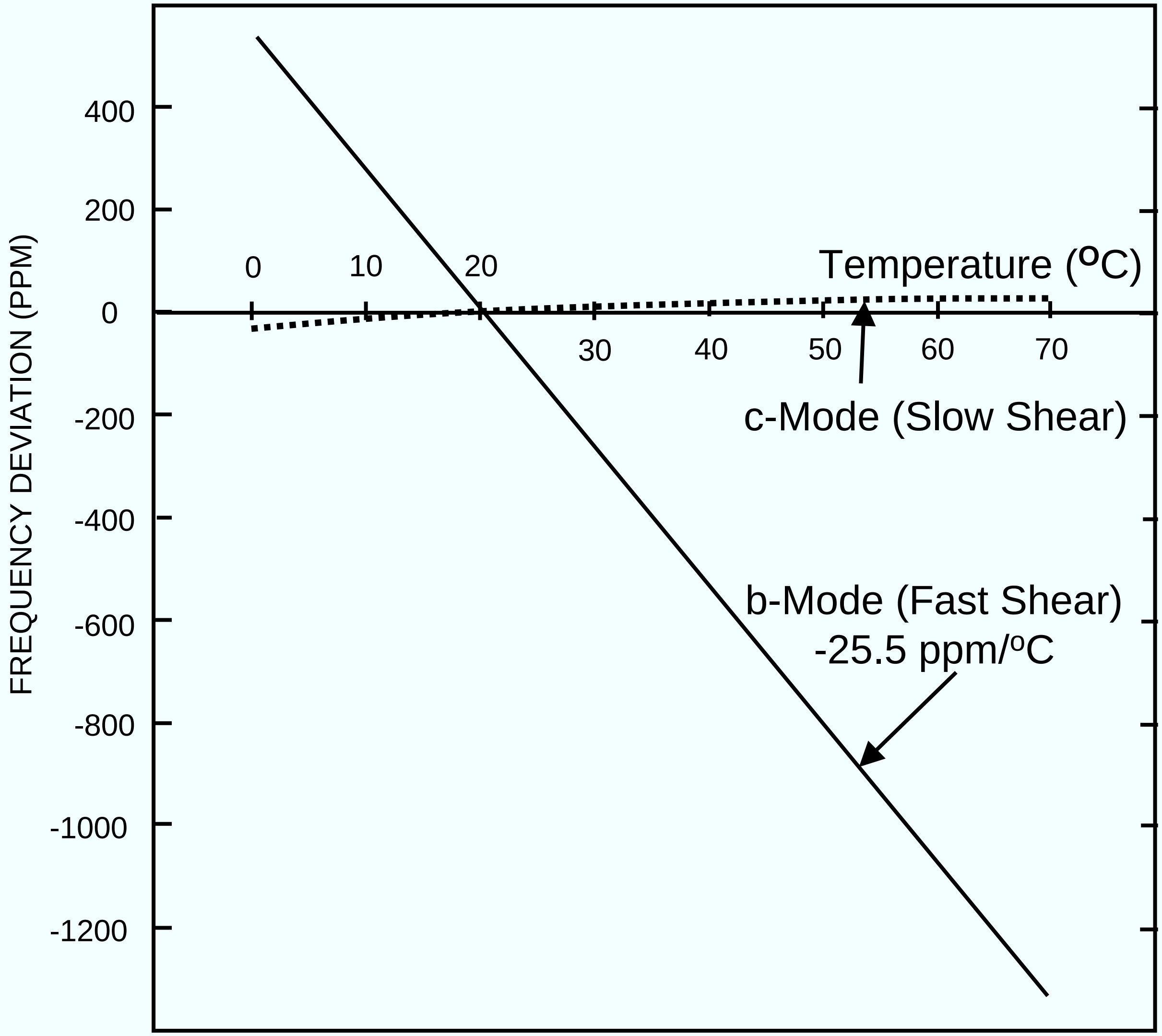
- **Disadvantage of the SC-cut :** More difficult to manufacture for OCXO (but is easier to manufacture for MCXO than is an AT-cut for precision TCXO)

- **Other Significant Differences**
  - B-mode is excited in the SC-cut, although not necessarily in LFR's
  - The SC-cut is sensitive to electric fields (which can be used for compensation)

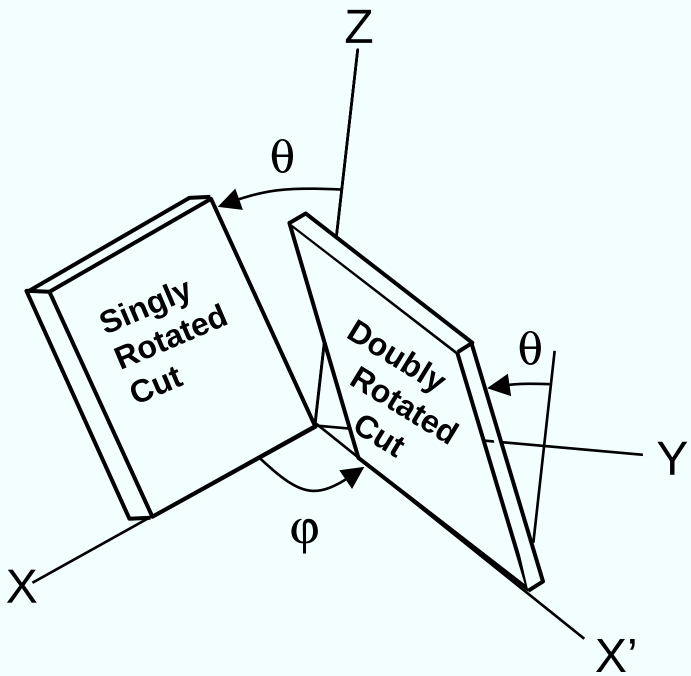
Mode Spectrograph of an SC-cut

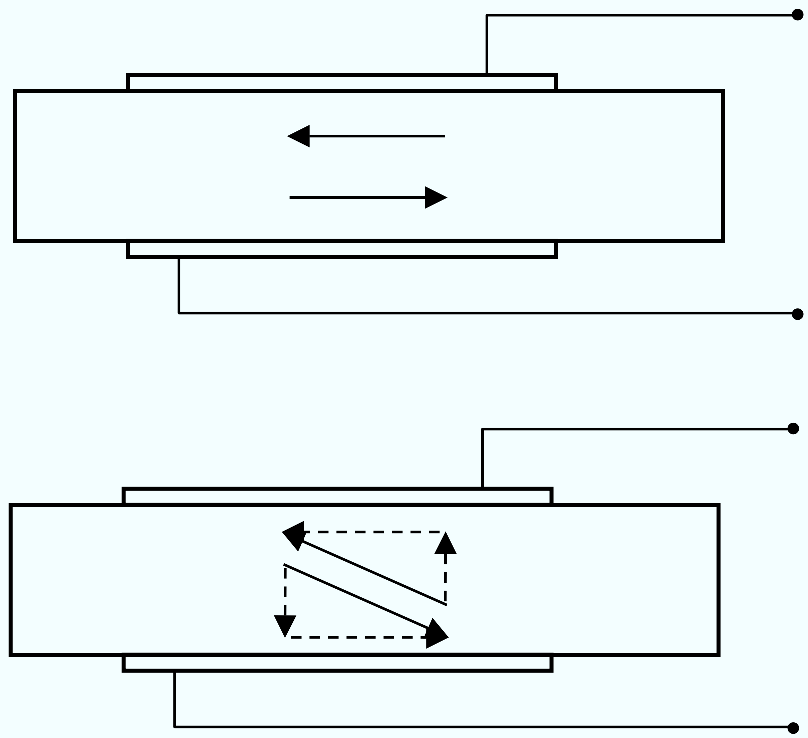


SC- cut f vs. T for b-mode and c-mode

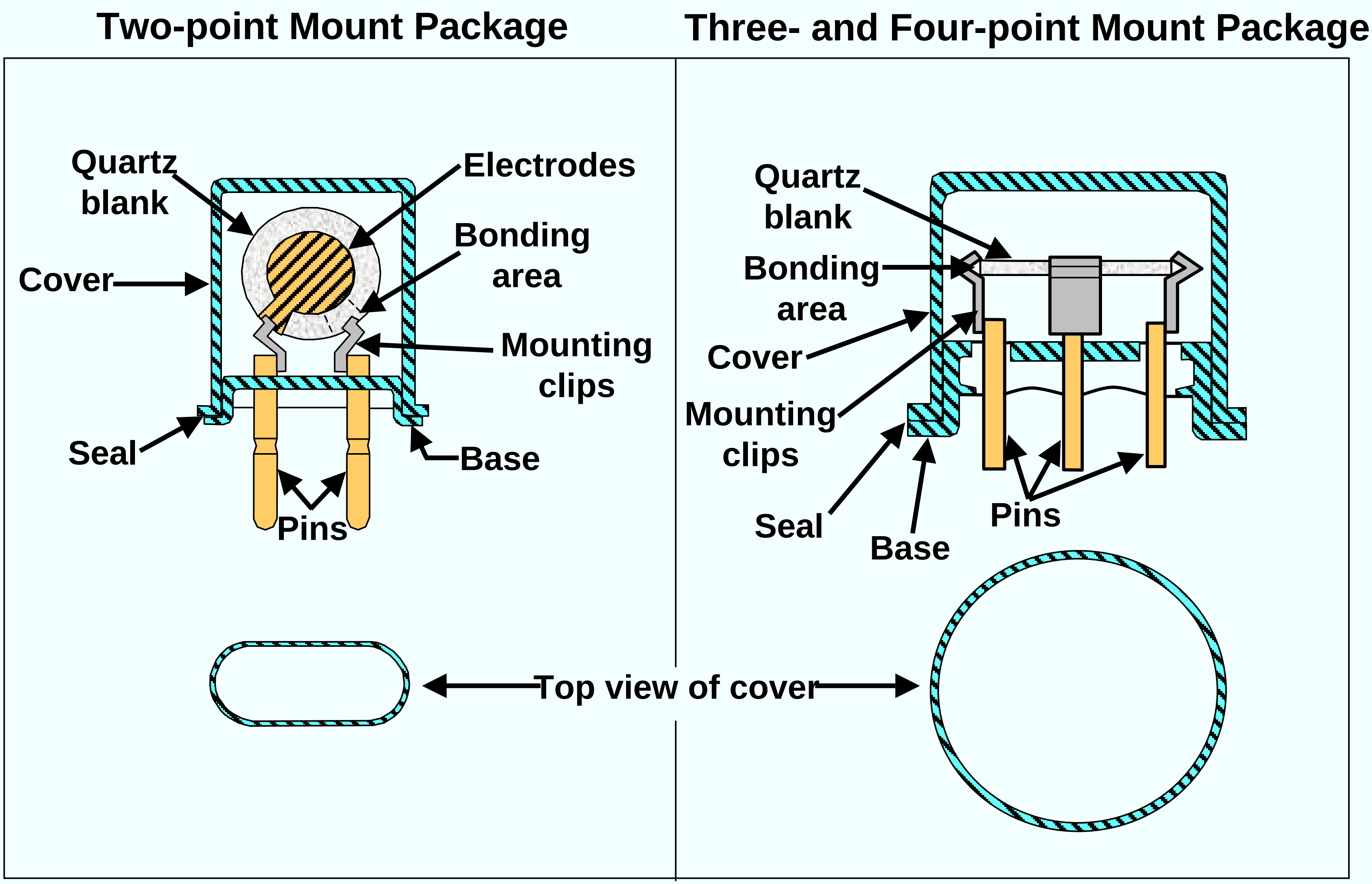


Singly Rotated and Doubly Rotated Cuts' Vibrational Displacements

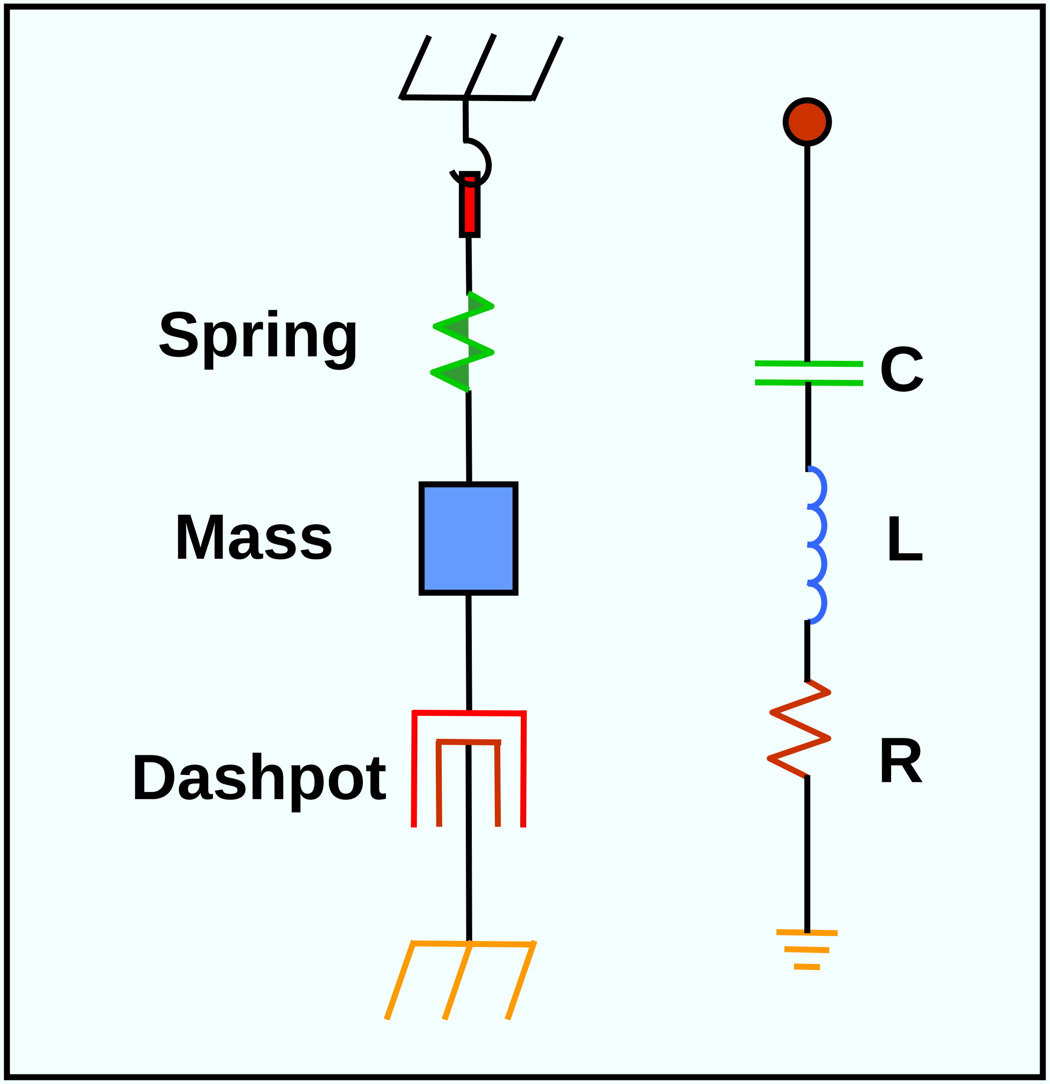




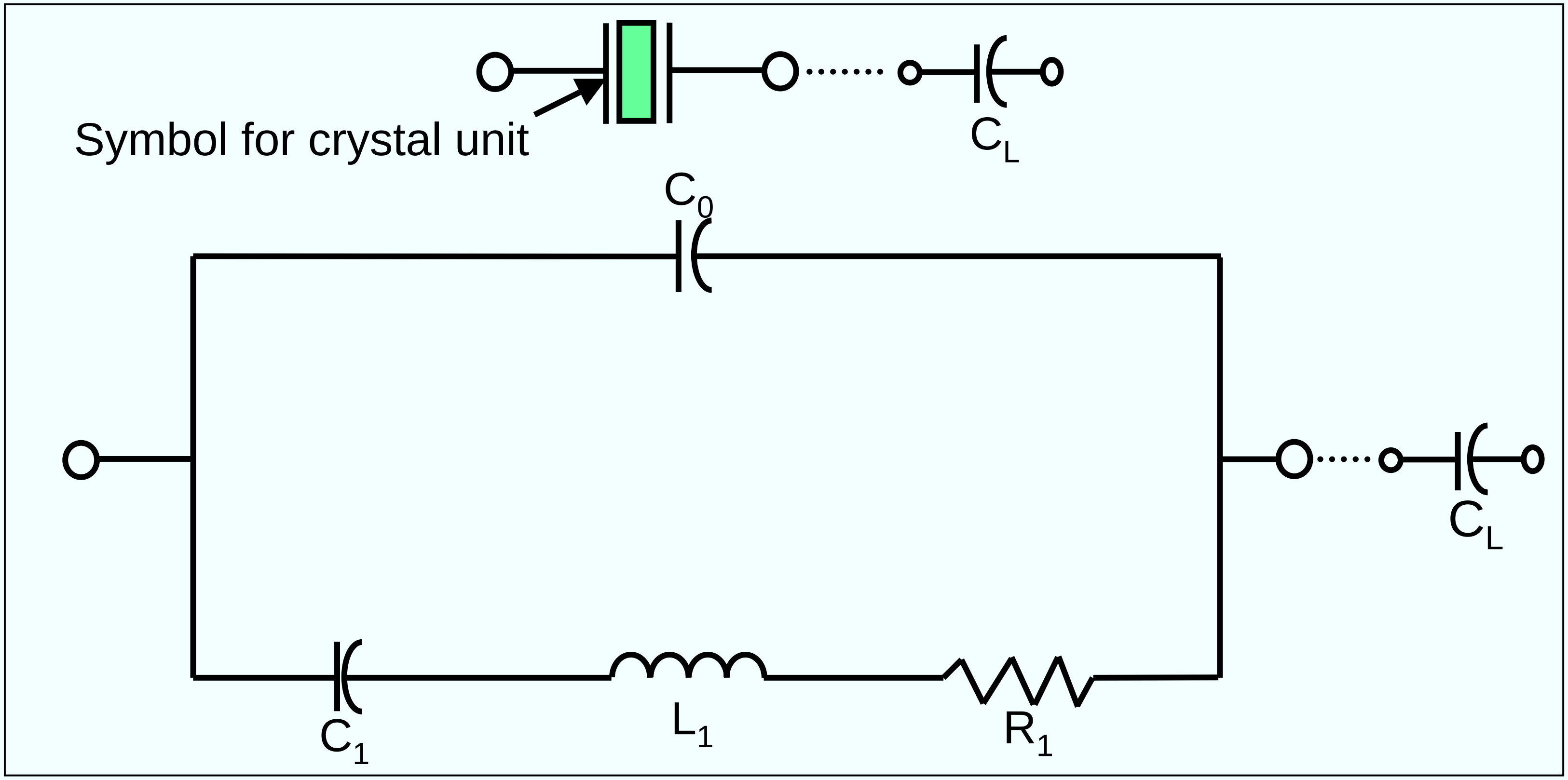
Resonator Packaging



Equivalent Circuits



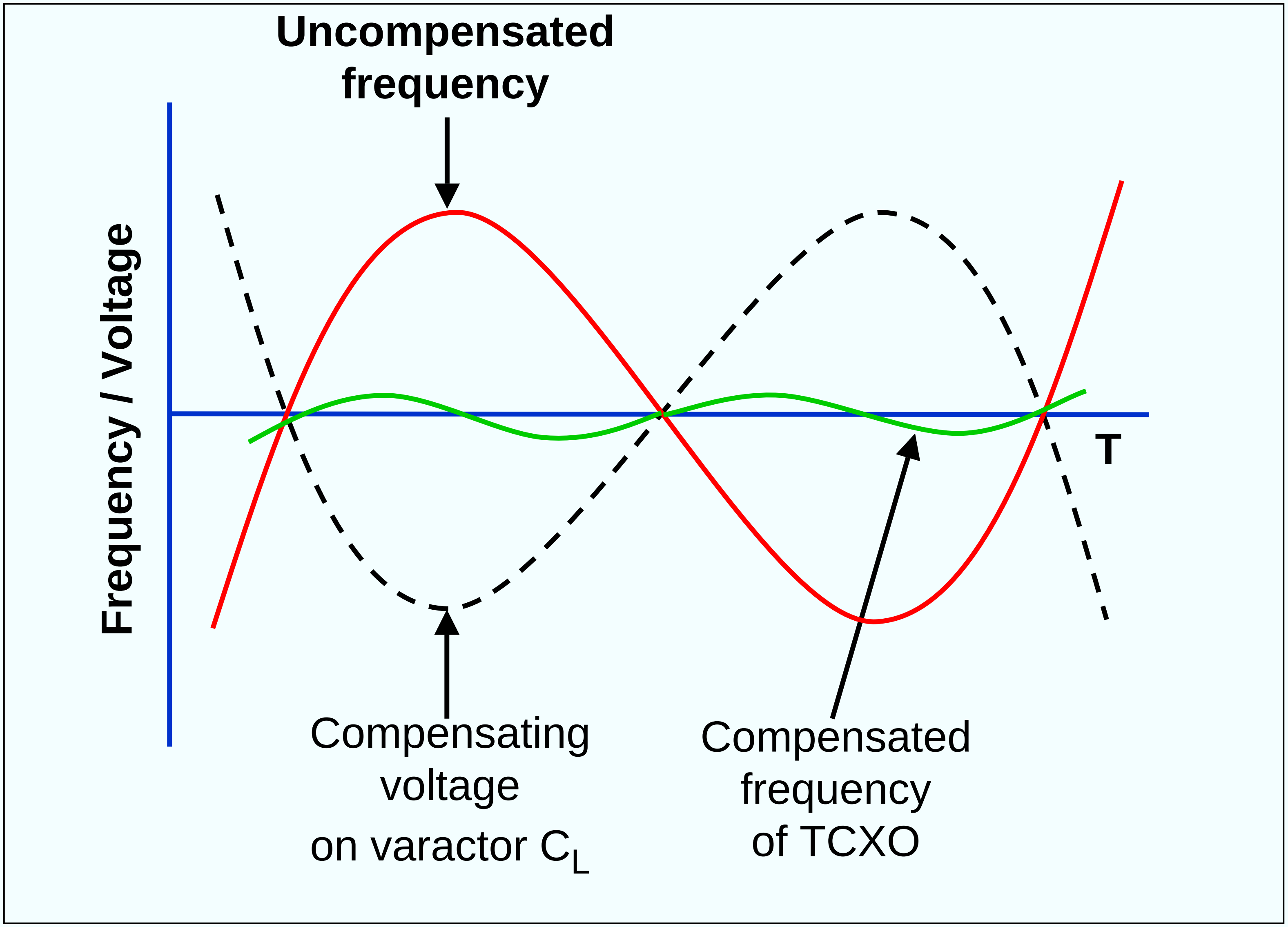
Equivalent Circuit of a Resonator



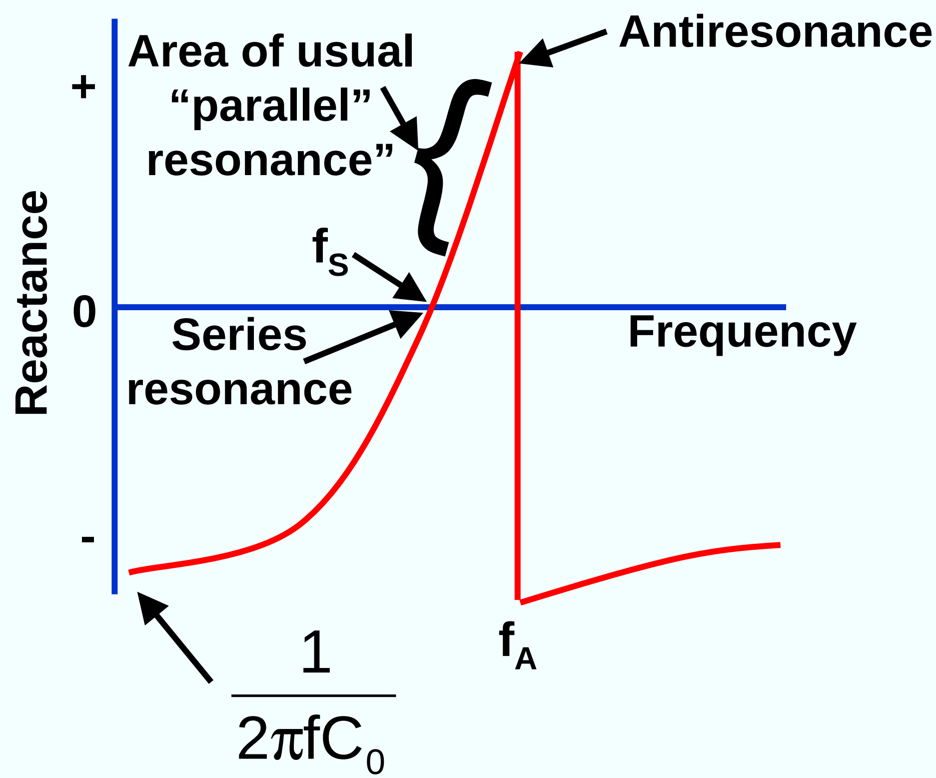
$$\frac{\Delta f}{f_S} \approx \frac{C_1}{2\left(C_0 + C_L\right)} \rightarrow$$

1. Voltage control (VCXO)
2. Temperature compensation (TCXO)

Crystal Oscillator f vs. T Compensation



Resonator Frequency vs. Reactance



Equivalent Circuit Parameter Relationships

$$C_0 \cong \varepsilon \frac{A}{t} \qquad r \equiv \frac{C_0}{C_1}$$

$$f_S = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1}} \qquad f_a - f_s \cong \frac{f_s}{2r}$$

$$Q = \frac{1}{2\pi f_S R_1 C_1} \qquad \varphi = \frac{\omega L_1 - \frac{1}{\omega C_1}}{R_1}$$

$$\tau_1 = R_1 C_1 \cong 10^{-14}\,\mathrm{s} \qquad \frac{d\varphi}{df} \cong \frac{360}{\pi}\frac{Q}{f_s}$$

$$C_{1n} \approx \frac{r' C_{11}}{n^3} \qquad L_{1n} \approx \frac{n^3 L_{11}}{r'^3} \qquad R_{1n} \approx \frac{n^3 R_{11}}{r'} \qquad 2r = \left(\frac{\pi n}{2k}\right)^2$$

n: Overtone number
C_0: Static capacitance
C_1: Motional capacitance
C_{1n}: C_1 of n-th overtone
L_1: Motional inductance
L_{1n}: L_1 of n-th overtone
R_1: Motional resistance
R_{1n}: R_1 of n-th overtone
ε: Dielectric permittivity of quartz $\approx 40 \times 10^{-13}$ pF/mm (average)
A: Electrode area
t: Plate thickness
r: Capacitance ratio
r': f_1/f_n
f_s: Series resonance frequency $\approx f_R$
f_a: Antiresonance frequency
Q; Quality factor
τ_1: Motional time constant
ω: Angular frequency = $2\pi f$
φ: Phase angle of the impedance
k; Piezoelectric coupling factor =8.8% for AT-cut, 4.99% for SC

What is Q and Why is it Important?

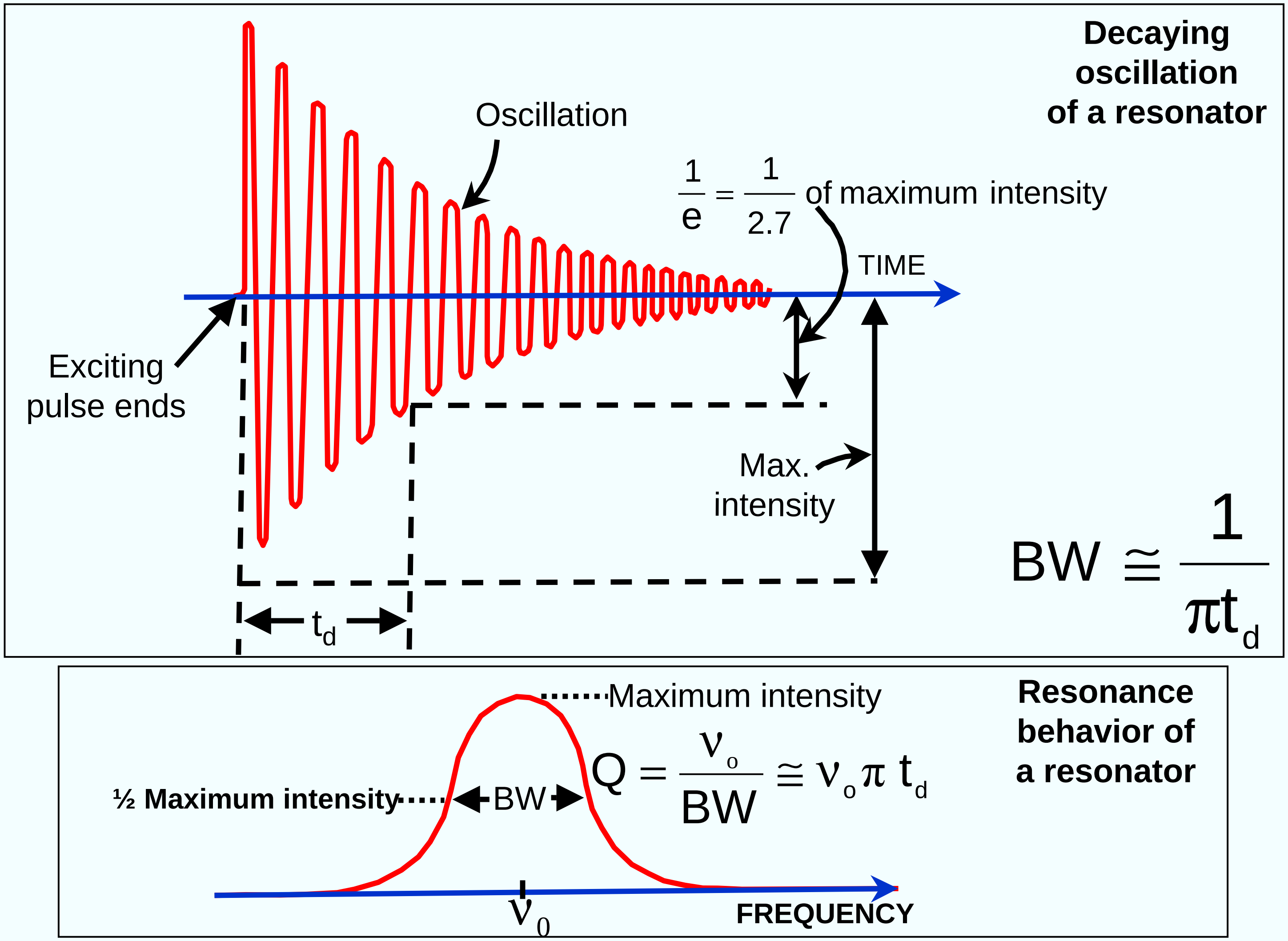
$$Q \equiv 2\pi \frac{\text{Energy stored during a cycle}}{\text{Energy lost during the cycle}}$$

Q is proportional to the decay-time, and is inversely proportional to the linewidth of resonance (see next page).

- The higher the Q, the higher the frequency stability and accuracy **capability** of a resonator (i.e., high Q is a necessary but not a sufficient condition). If, e.g., $Q = 10^6$, then 10^{-10} accuracy requires ability to determine center of resonance curve to 0.01% of the linewidth, and stability (for some averaging time) of 10^{-12} requires ability to stay near peak of resonance curve to 10^{-6} of linewidth.

- Phase noise close to the carrier has an especially strong dependence on Q ($\mathcal{L}(f) \propto 1/Q^4$).

Decay Time, Linewidth, and Q



Factors that Determine Resonator Q

The **maximum Q** of a resonator can be expressed as:

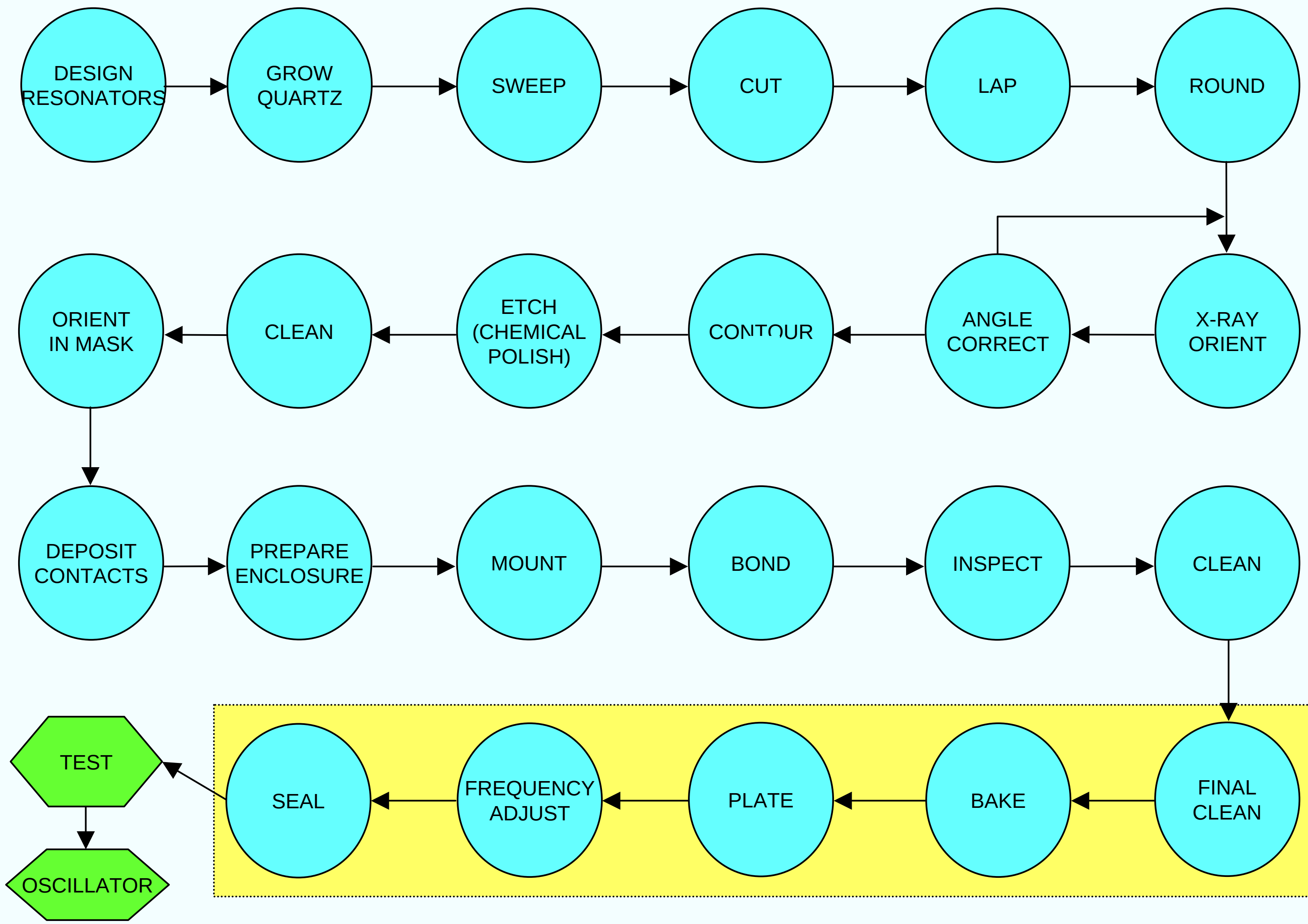
$$Q_{max} = \frac{1}{2\pi f\tau},$$

where f is the frequency in Hz, and τ is an empirically determined “motional time constant” in seconds, which varies with the angles of cut and the mode of vibration. For example, $\tau = 1 \times 10^{-14}$s for the AT-cut's c-mode (Q_{max} = 3.2 million at 5 MHz), $\tau = 9.9 \times 10^{-15}$s for the SC-cut's c-mode, and $\tau = 4.9 \times 10^{-15}$s for the BT-cut's b-mode.

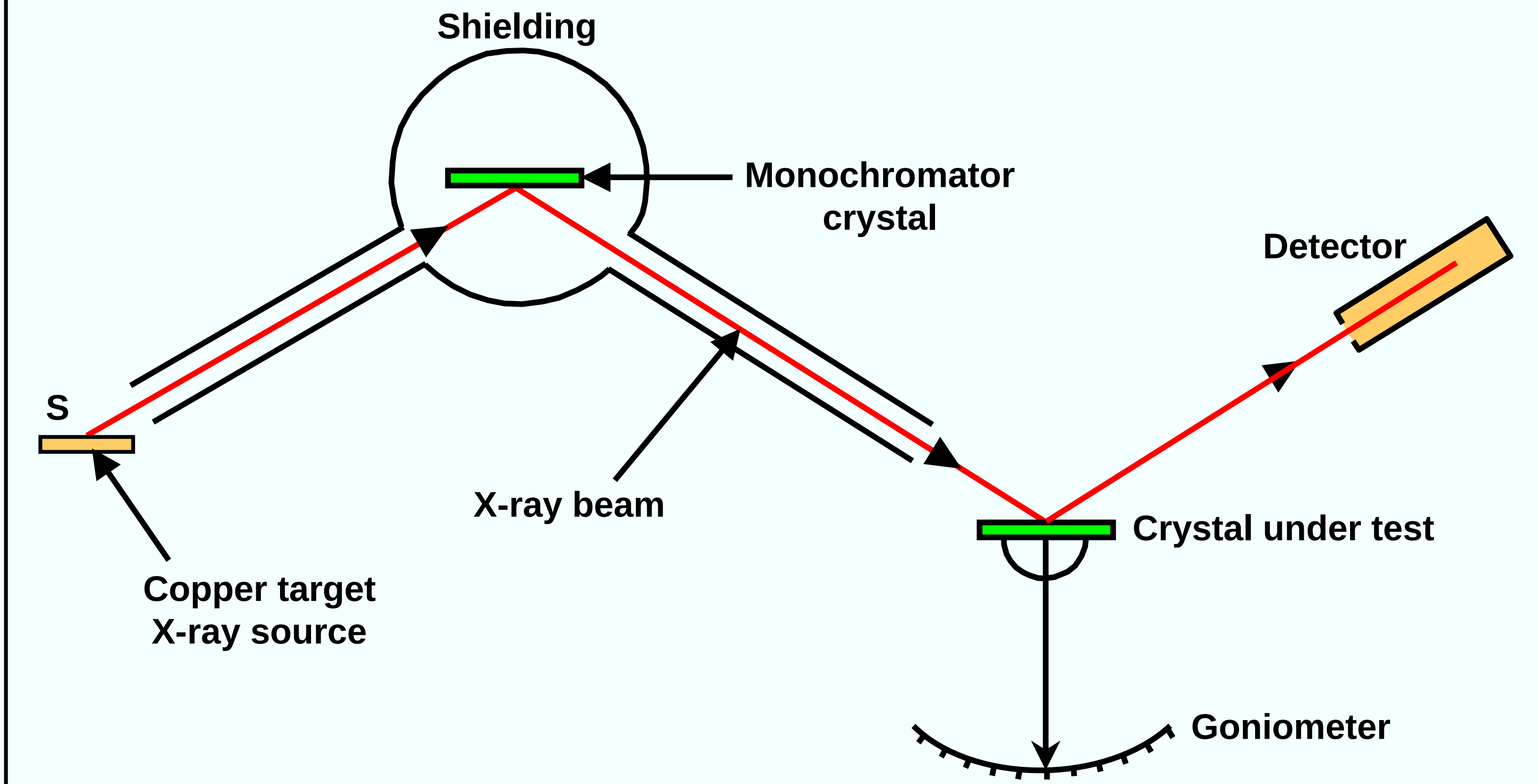
Other factors which affect the Q of a resonator include:

- Overtone
- Surface finish
- Material impurities and defects
- Mounting stresses
- Bonding stresses
- Temperature
- Electrode geometry and type
- Blank geometry (contour, dimensional ratios)
- Drive level
- Gases inside the enclosure (pressure, type of gas)
- Interfering modes
- Ionizing radiation

Resonator Fabrication Steps



X-ray Orientation of Crystal Plates



Double-crystal x-ray diffraction system

Contamination Control

Contamination control is essential during the fabrication of resonators because contamination can adversely affect:

- Stability (see chapter 4)
  - aging
  - hysteresis
  - retrace
  - noise
  - nonlinearities and resistance anomalies (high starting resistance, second-level of drive, intermodulation in filters)
  - frequency jumps?
- Manufacturing yields
- Reliability

Crystal Enclosure Contamination

The enclosure and sealing process can have important influences on resonator stability.

- A monolayer of adsorbed contamination contains ~ 10^{15} molecules/cm^2 (on a smooth surface)
- An enclosure at 10^{-7} torr contains ~10^9 gaseous molecules/cm^3

Therefore:

In a 1 cm^3 enclosure that has a monolayer of contamination on its inside surfaces, there are ~10^6 times more adsorbed molecules than gaseous molecules when the enclosure is sealed at 10^{-7} torr. The desorption and adsorption of such adsorbed molecules leads to aging, hysteresis, noise, etc.

What is an "f-squared"?

It is standard practice to express the thickness removed by lapping, etching and polishing, and the mass added by the electrodes, in terms of frequency change, Δf, in units of "f^2", where the Δf is in kHz and f is in MHz. For example, etching a 10MHz AT-cut plate $1f^2$ means that a thickness is removed that produces Δf= 100 kHz; and etching a 30 MHz plate $1f^2$ means that the Δf= 900 kHz. In both cases, $\Delta f=1f^2$ produces the same thickness change.

To understand this, consider that for a thickness-shear resonator (AT-cut, SC-cut, etc.)

$$f = \frac{N}{t}$$

where f is the fundamental mode frequency, t is the thickness of the resonator plate and N is the frequency constant (1661 MHz•μm for an AT-cut, and 1797 MHz•μm for a SC-cut's c-mode). Therefore,

$$\frac{\Delta f}{f} = -\frac{\Delta t}{t}$$

and,

$$\Delta t = -N\frac{\Delta f}{f^2}$$

So, for example, $\Delta f = 1f^2$ corresponds to the same thickness removal for all frequencies. For an AT-cut, Δt=1.661 μm of quartz (=0.83 μm per side) per f^2. An important advantage of using units of f^2 is that frequency changes can be measured much more accurately than thickness changes. The reason for expressing Δf in kHz and f in MHz is that by doing so, the numbers of f^2 are typically in the range of 0.1 to 10, rather than some very small numbers.

Milestones in Quartz Technology

1880	Piezoelectric effect discovered by Jacques and Pierre Curie
1905	First hydrothermal growth of quartz in a laboratory - by G. Spezia
1917	First application of piezoelectric effect, in sonar
1918	First use of piezoelectric crystal in an oscillator
1926	First quartz crystal controlled broadcast station
1927	First temperature compensated quartz cut discovered
1927	First quartz crystal clock built
1934	First practical temp. compensated cut, the AT-cut, developed
1949	Contoured, high-Q, high stability AT-cuts developed
1956	First commercially grown cultured quartz available
1956	First TCXO described
1972	Miniature quartz tuning fork developed; quartz watches available
1974	The SC-cut (and TS/TTC-cut) predicted; verified in 1976
1982	First MCXO with dual c-mode self-temperature sensing

Quartz Resonators for Wristwatches

Requirements:

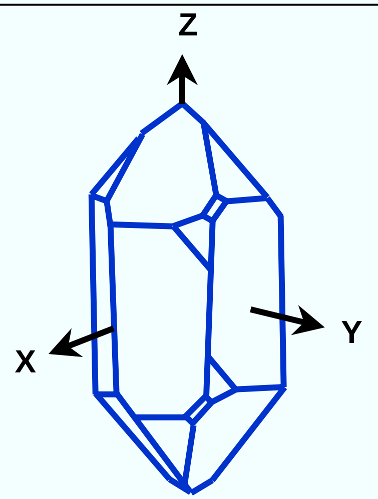
- Small size
- Low power dissipation (including the oscillator)
- Low cost
- High stability (temperature, aging, shock, attitude)

These requirements can be met with 32,768 Hz quartz tuning forks

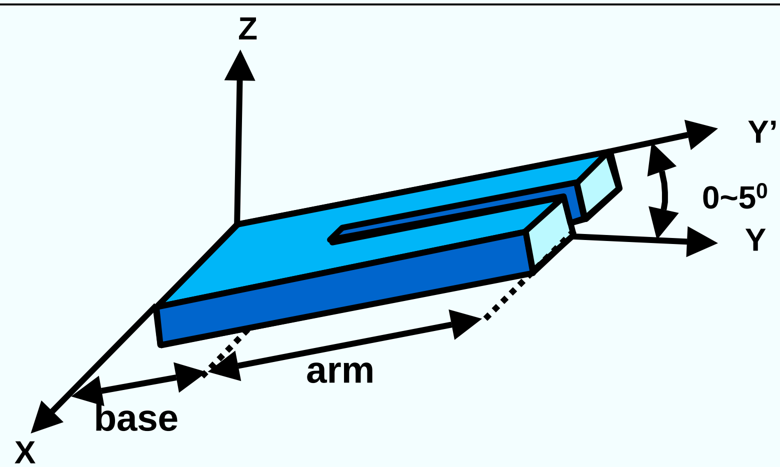
Why 32,768 Hz?

$32{,}768 = 2^{15}$ • In an analog watch, a stepping motor receives one impulse per second which advances the second hand by 6º, i.e., 1/60th of a circle, every second. • Dividing 32,768 Hz by two 15 times results in 1 Hz. • The 32,768 Hz is a compromise among size, power requirement (i.e., battery life) and stability.	32,768 16,384 8,192 4,096 2,048 1,024 512 256 128 64 32 16 8 4 2 1

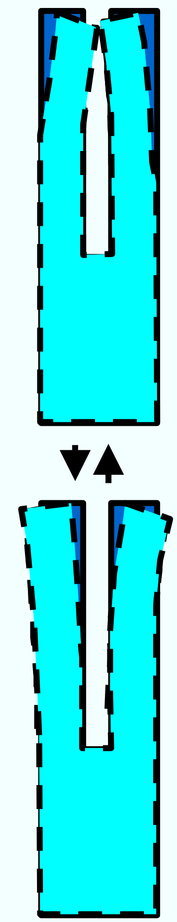
Quartz Tuning Fork



a) natural faces and crystallographic axes of quartz

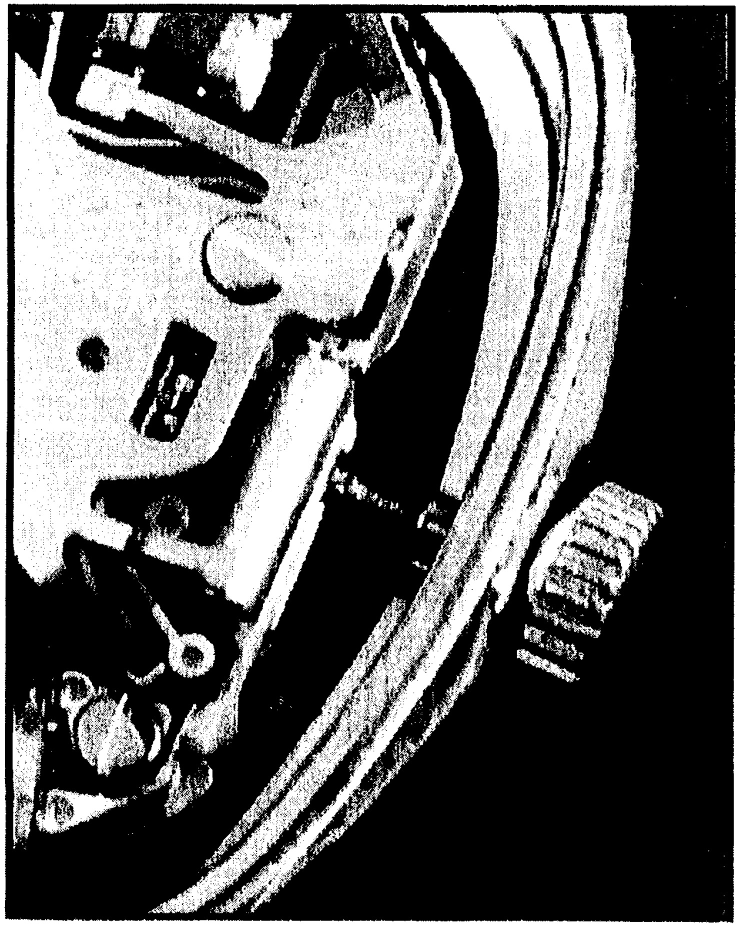


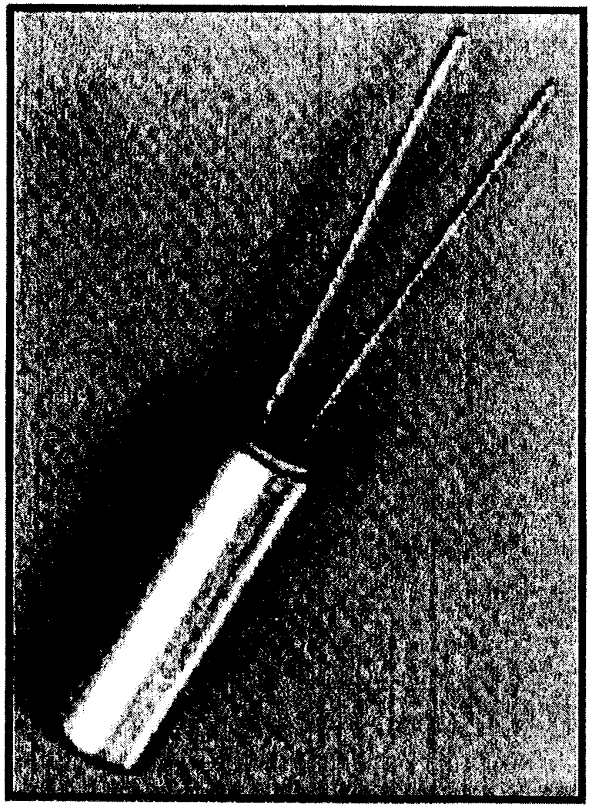
b) crystallographic orientation of tuning fork



c) vibration mode of tuning fork

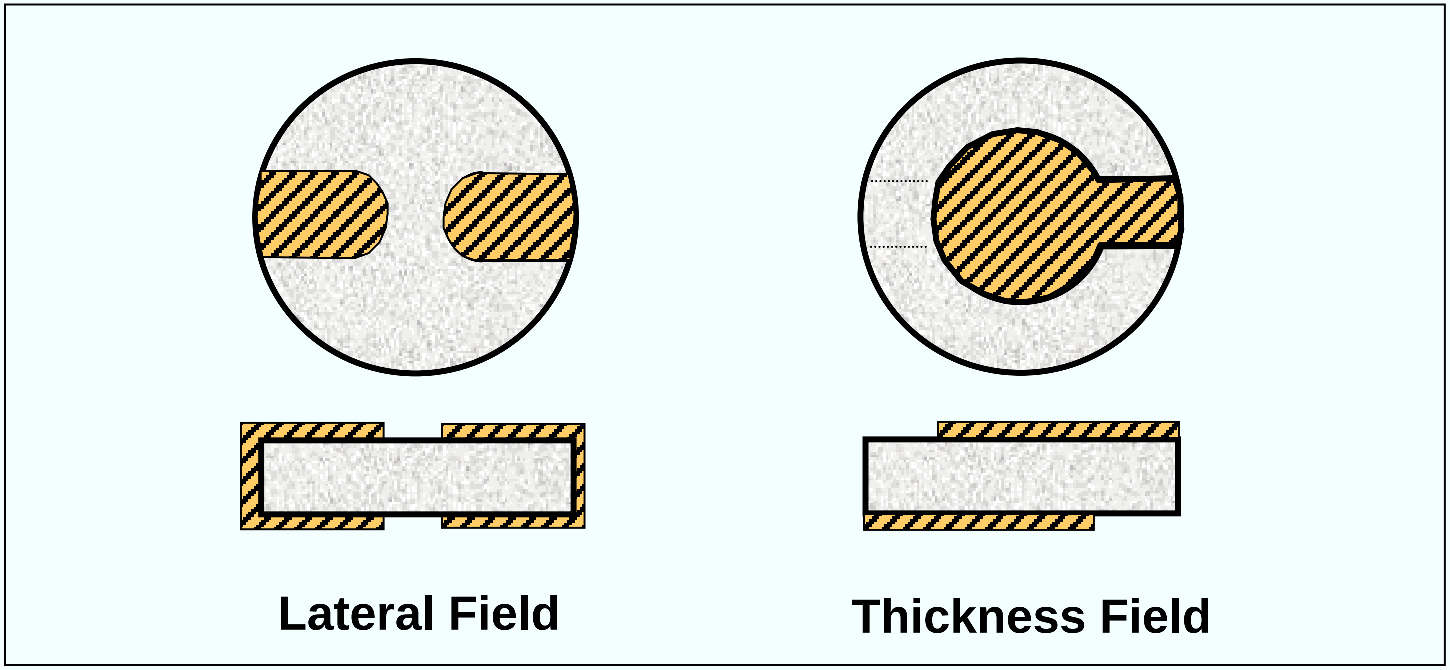
Watch Crystal





2 mm diameter x 6 mm tall cylinder
(0.08" diameter x 0.24" tall)

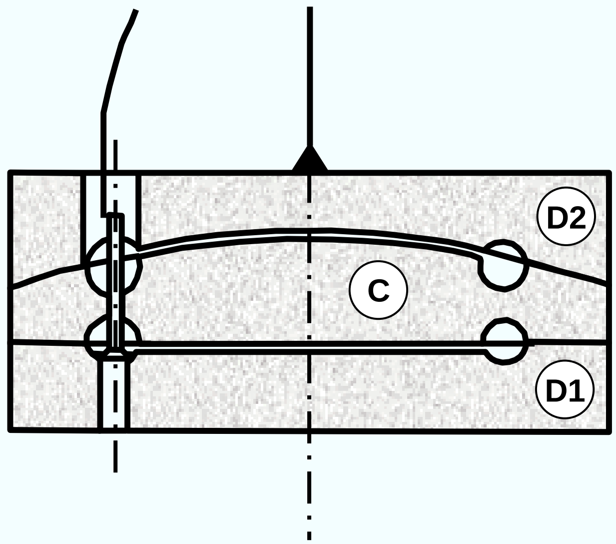
Lateral Field Resonator



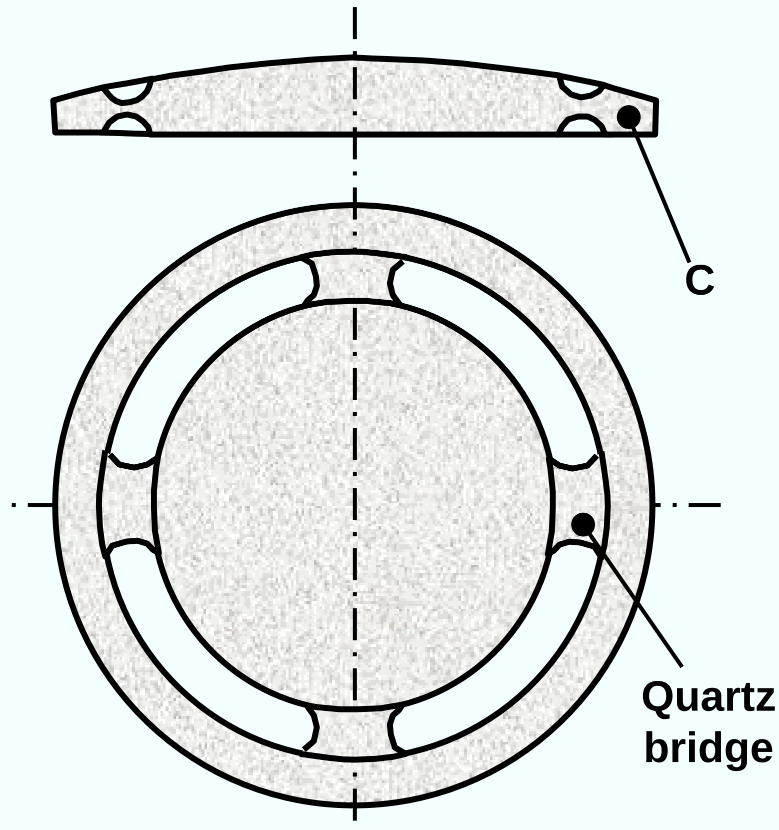
In lateral field resonators (LFR): 1. the electrodes are absent from the regions of greatest motion, and 2. varying the orientation of the gap between the electrodes varies certain important resonator properties. Advantages of LFR are:

- Ability to eliminate undesired modes, e.g., the b-mode in SC-cuts
- Potentially higher Q (less damping due to electrodes and mode traps)
- Potentially higher stability (less electrode and mode trap effects, smaller C_1)

Electrodeless (BVA) Resonator



Side view of BVA_2
resonator construction



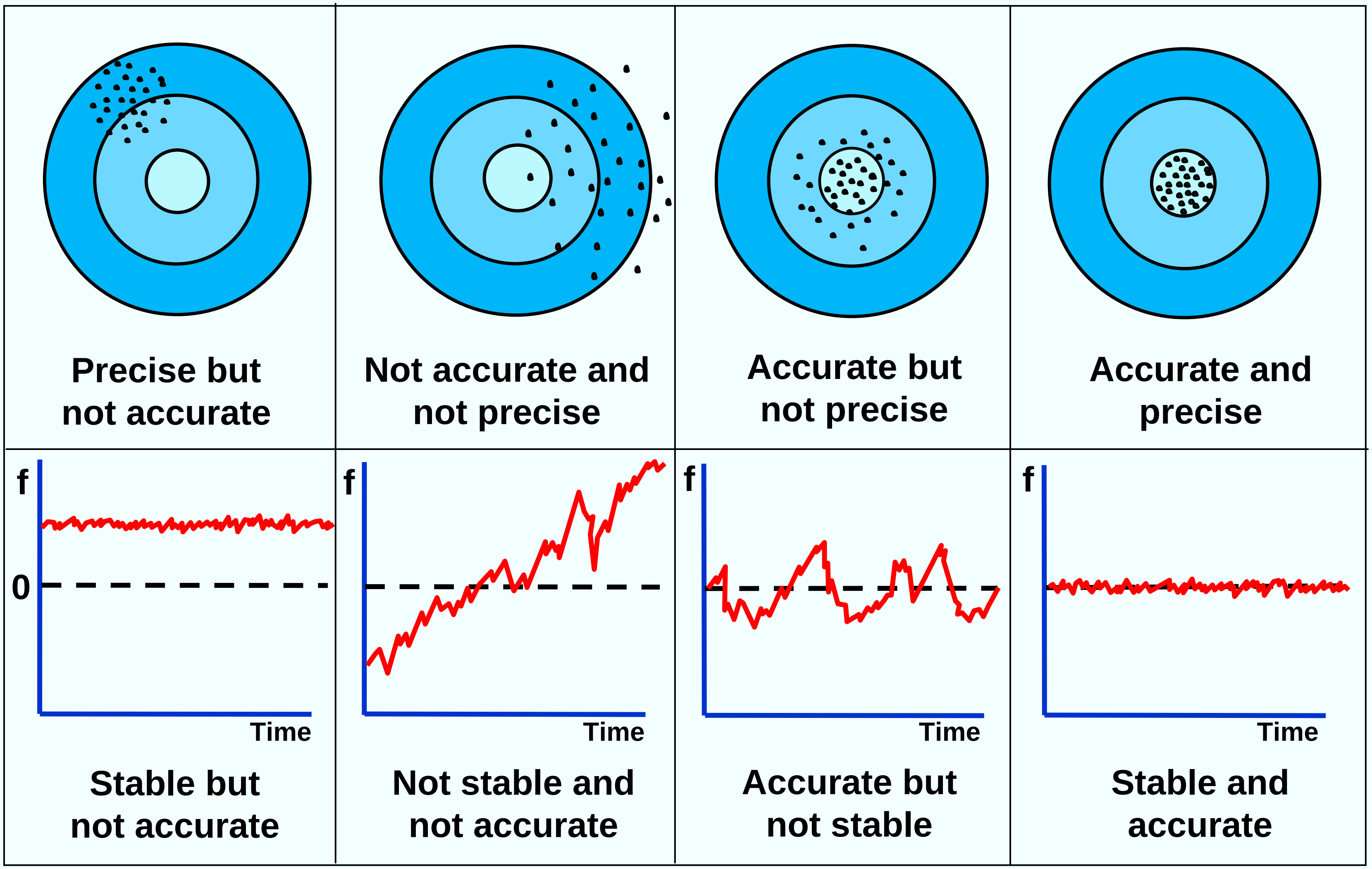
Side and top views of
center plate C

CHAPTER 4
Oscillator Stability

The Units of Stability in Perspective

- What is one part in 10^{10} ? (As in 1×10^{-10}/day aging.)
  - ~1/2 cm out of the circumference of the earth.
  - ~1/4 second per human lifetime (of ~80 years).
- What is -170 dB? (As in -170 dBc/Hz phase noise.)
  - -170 dB = 1 part in $10^{17} \approx$ thickness of a sheet of paper out of total distance traveled by all the cars in the world in a day.

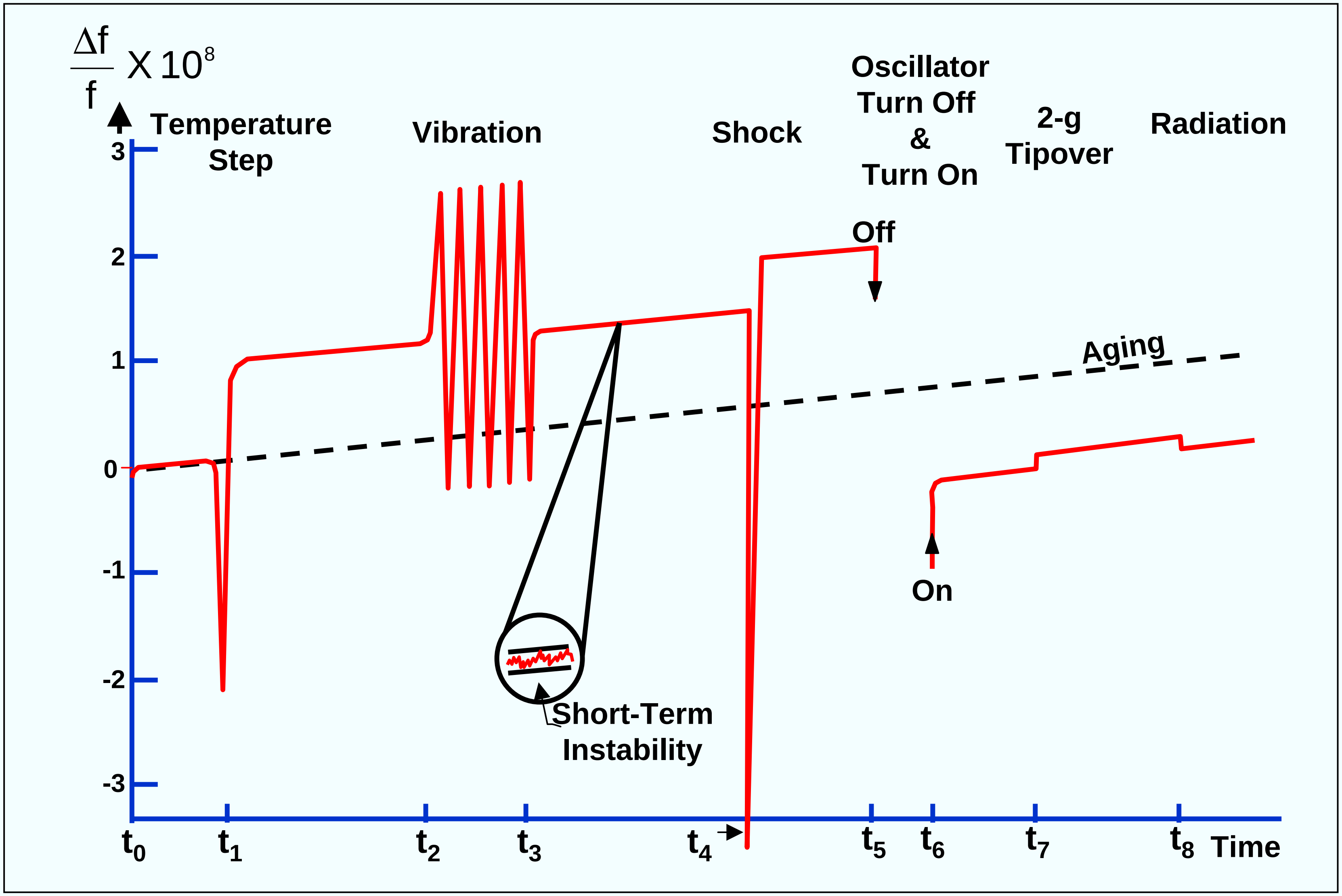
Accuracy, Precision, and Stability



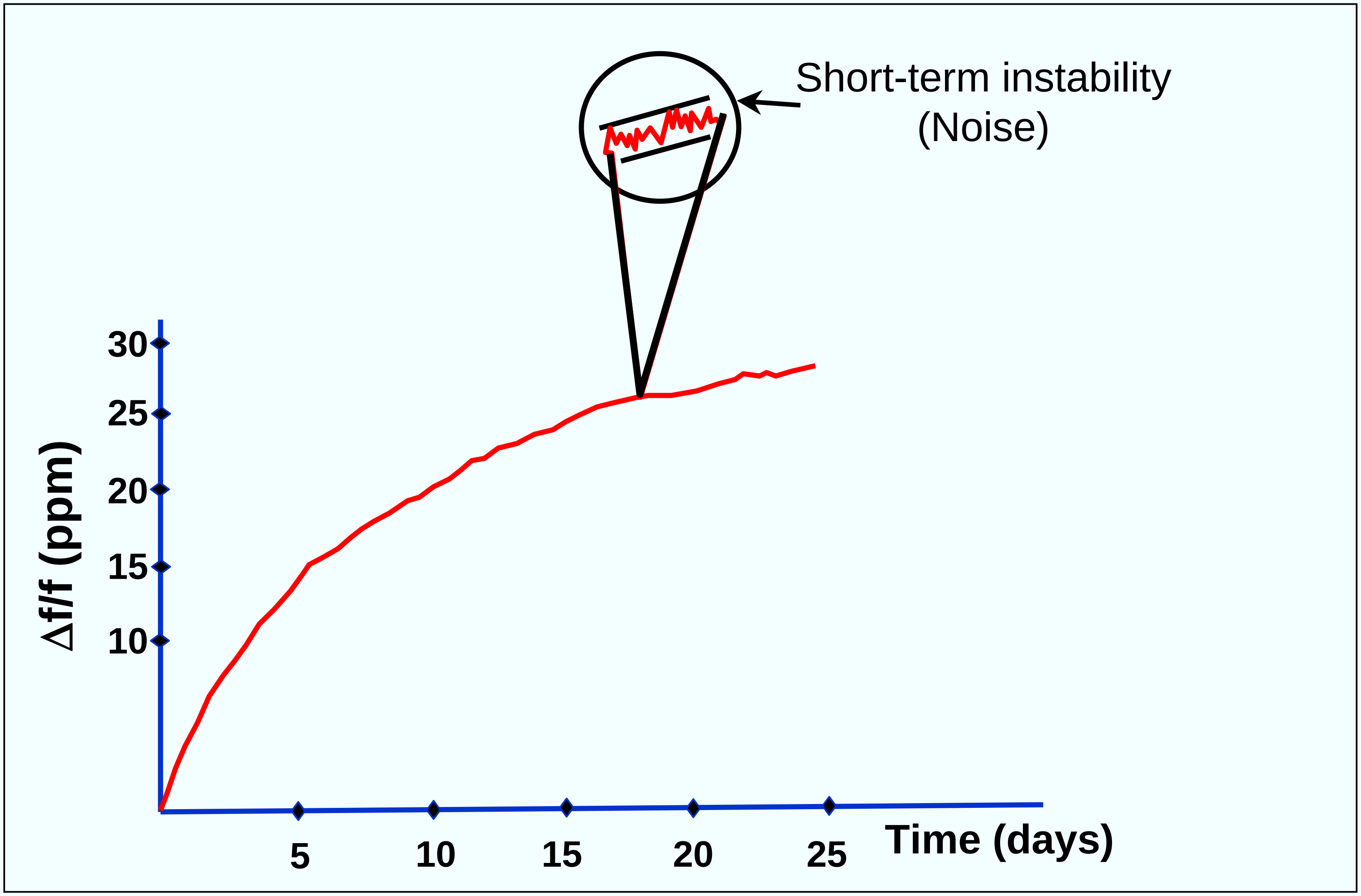
Influences on Oscillator Frequency

- **Time**
  - Short term (noise)
  - Intermediate term (e.g., due to oven fluctuations)
  - Long term (aging)
- **Temperature**
  - Static frequency vs. temperature
  - Dynamic frequency vs. temperature (warmup, thermal shock)
  - Thermal history ("hysteresis," "retrace")
- **Acceleration**
  - Gravity (2g tipover)
  - Vibration
  - Acoustic noise
  - Shock
- **Ionizing radiation**
  - Steady state
  - Pulsed
  - Photons (X-rays, γ-rays)
  - Particles (neutrons, protons, electrons)
- **Other**
  - Power supply voltage
  - Atmospheric pressure (altitude)
  - Humidity
  - Load impedance
  - Magnetic field

Idealized Frequency-Time-Influence Behavior



Aging and Short-Term Stability



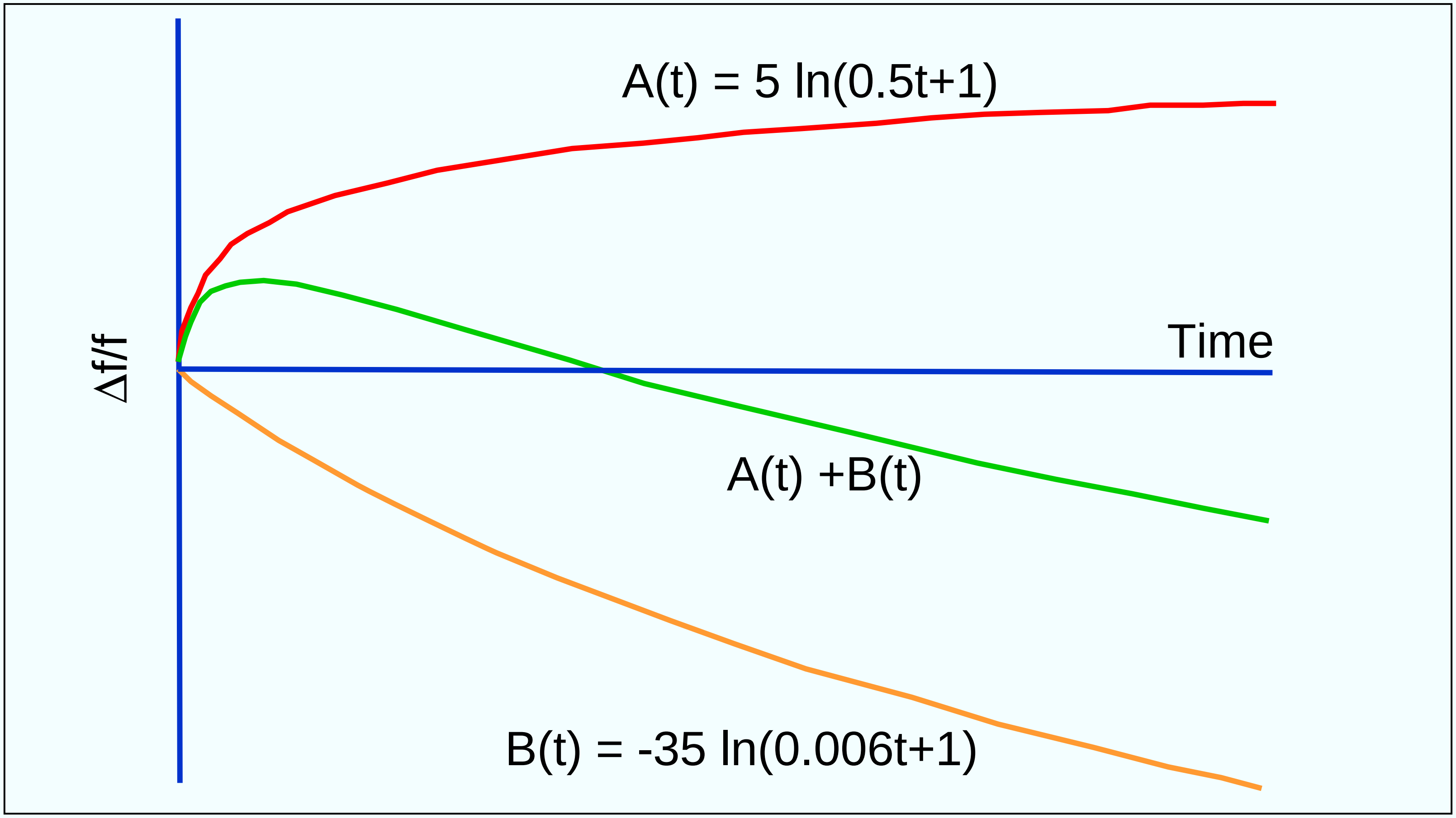
Aging Mechanisms

- **Mass transfer due to contamination**
  Since $f \propto 1/t$, $\Delta f/f = -\Delta t/t$; e.g., $f_{5MHz} \approx 10^6$ molecular layers, therefore, 1 quartz-equivalent monolayer $\Rightarrow \Delta f/f \approx 1$ ppm

- **Stress relief** in the resonator's: mounting and bonding structure, electrodes, and in the quartz (?)

- **Other effects**
  - Quartz outgassing
  - Diffusion effects
  - Chemical reaction effects
  - Pressure changes in resonator enclosure (leaks and outgassing)
  - Oscillator circuit aging (load reactance and drive level changes)
  - Electric field changes (doubly rotated crystals only)
  - Oven-control circuitry aging

Typical Aging Behaviors



Stresses on a Quartz Resonator Plate

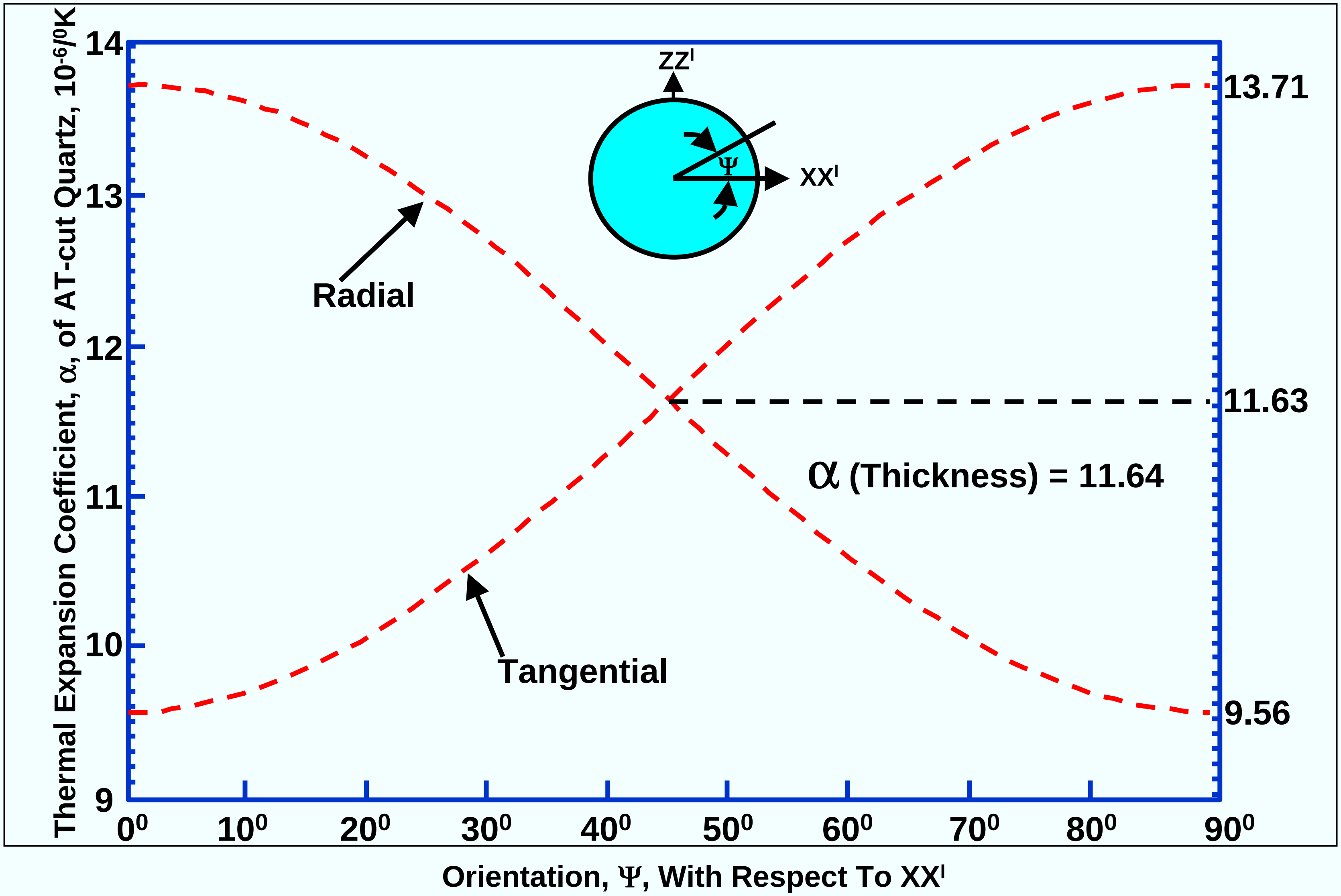
Causes:

- Thermal expansion coefficient differences
- Bonding materials changing dimensions upon solidifying/curing
- Residual stresses due to clip forming and welding operations, sealing
- Intrinsic stresses in electrodes
- Nonuniform growth, impurities & other defects during quartz growing
- Surface damage due to cutting, lapping and (mechanical) polishing

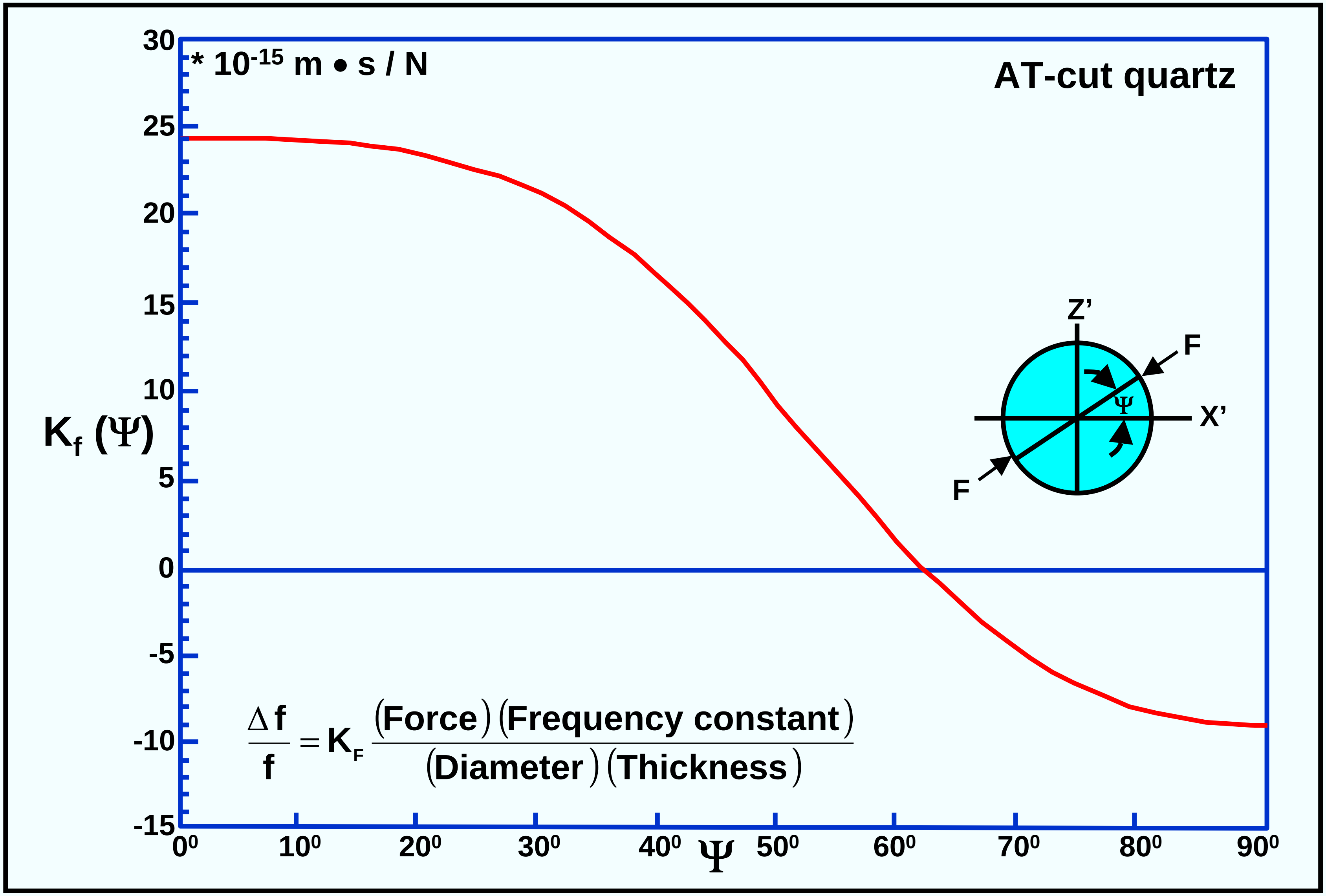
Effects:

- In-plane diametric forces
- Tangential (torsional) forces, especially in 3 and 4-point mounts
- Bending (flexural) forces, e.g., due to clip misalignment and electrode stresses
- Localized stresses in the quartz lattice due to dislocations, inclusions, other impurities, and surface damage

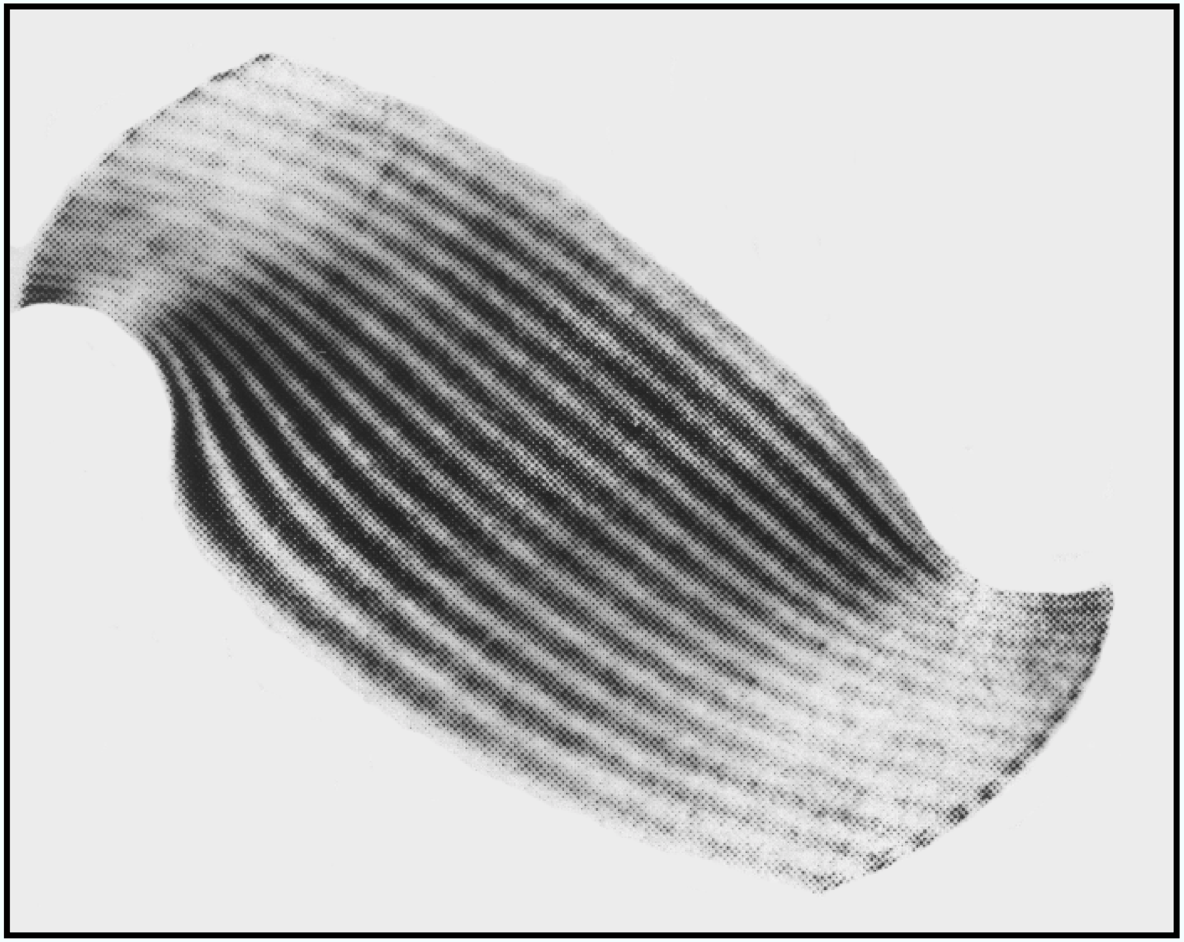
Thermal Expansion Coefficients of Quartz



Force-Frequency Coefficient

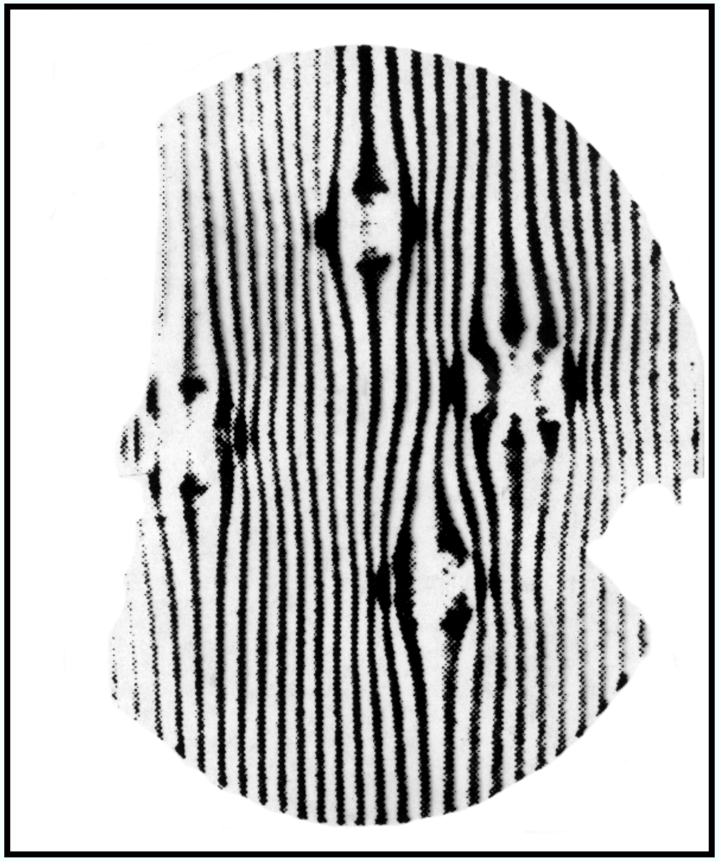


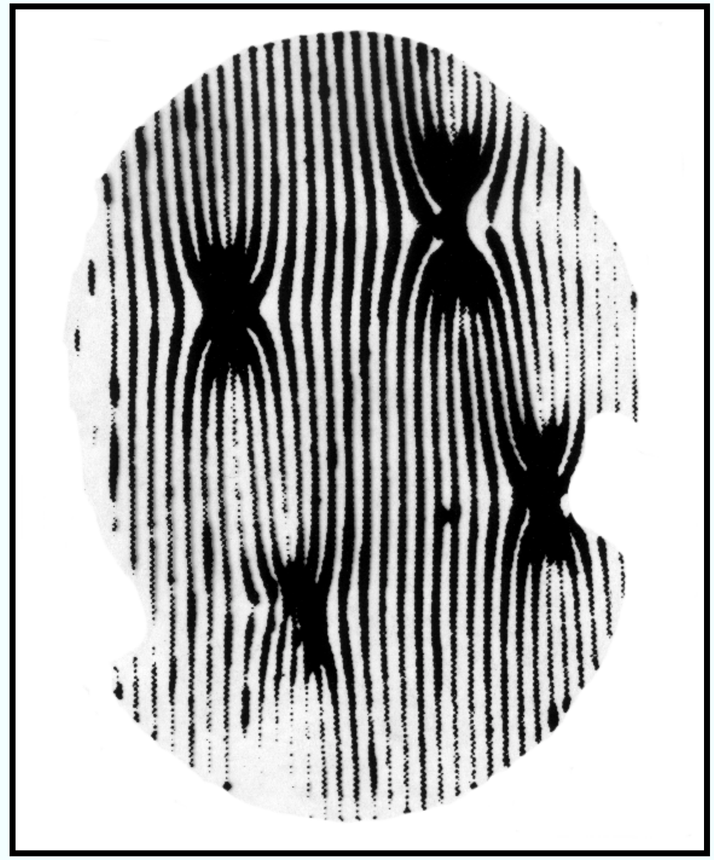
Strains Due To Mounting Clips



X-ray topograph of an AT-cut, two-point mounted resonator. The topograph shows the lattice deformation due to the stresses caused by the mounting clips.

Strains Due To Bonding Cements





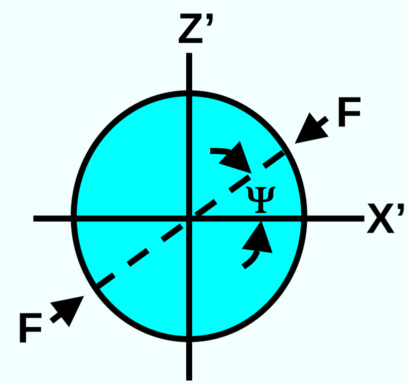
(a) (b)

X-ray topographs showing lattice distortions caused by bonding cements; (a) Bakelite cement - expanded upon curing, (b) DuPont 5504 cement - shrank upon curing

Mounting Force Induced Frequency Change

The force-frequency coefficient, K_F (ψ), is defined by

$$\frac{\Delta f}{f} = K_F \frac{(\text{Force})(\text{Frequency} - \text{constant})}{(\text{Diameter})(\text{Thickness})}$$



Maximum K_F (AT-cut) = 24.5 x 10^{-15} m-s/N at $\psi = 0^o$

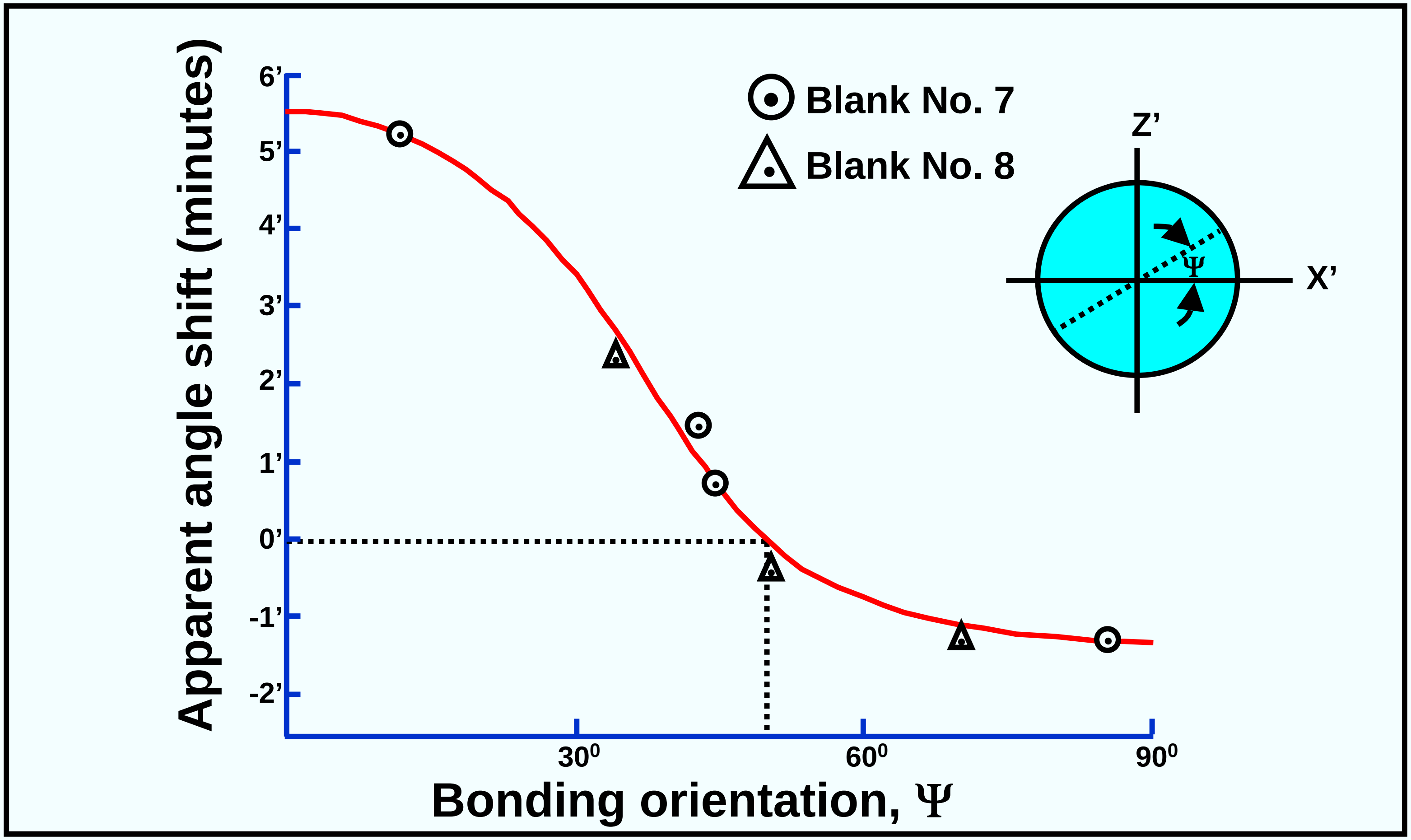
Maximum K_F (SC-cut) = 14.7 x 10^{-15} m-s/N at $\psi = 44^o$

As an example, consider a 5 MHz 3rd overtone, 14 mm diameter resonator. Assuming the presence of diametrical forces only, (1 gram = 9.81 x 10^{-3} newtons),

$$\left(\frac{\Delta f}{f}\right)_{Max} = \begin{cases} 2.9 \times 10^{-8} \text{ per gram for an AT-cut resonator} \\ 1.7 \times 10^{-8} \text{ per gram for an SC-cut resonator} \end{cases}$$

$$\left(\frac{\Delta f}{f}\right)_{Min} =$$ 0 at $\psi = 61^o$ for an AT-cut resonator, and at $\psi = 82^o$ for an SC-cut.

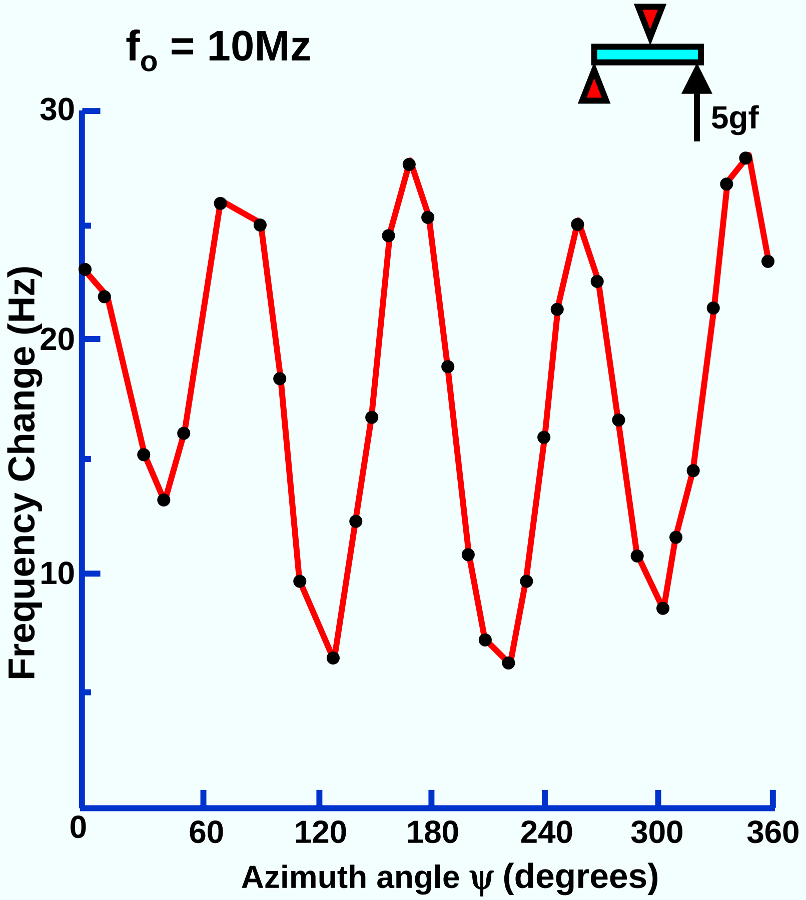
Bonding Strains Induced Frequency Changes



When 22 MHz fundamental mode AT-cut resonators were reprocessed so as to vary the bonding orientations, the frequency vs. temperature characteristics of the resonators changed as if the angles of cut had been changed. The resonator blanks were 6.4 mm in diameter plano-plano, and were bonded to low-stress mounting clips by nickel electrobonding.

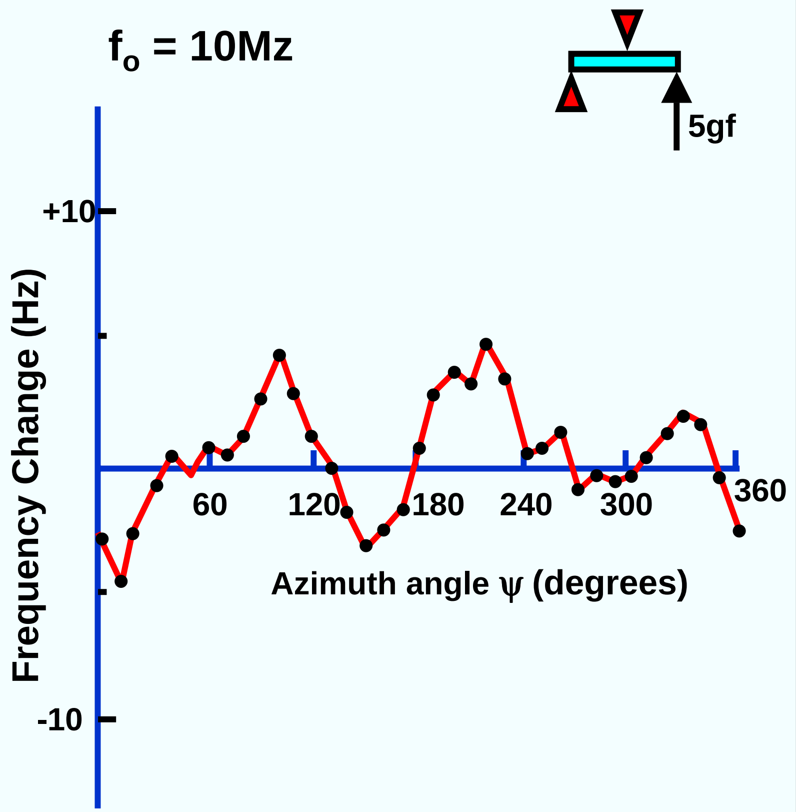
Bending Force vs. Frequency Change

AT-cut resonator



Frequency change for symmetrical bending, AT-cut crystal.

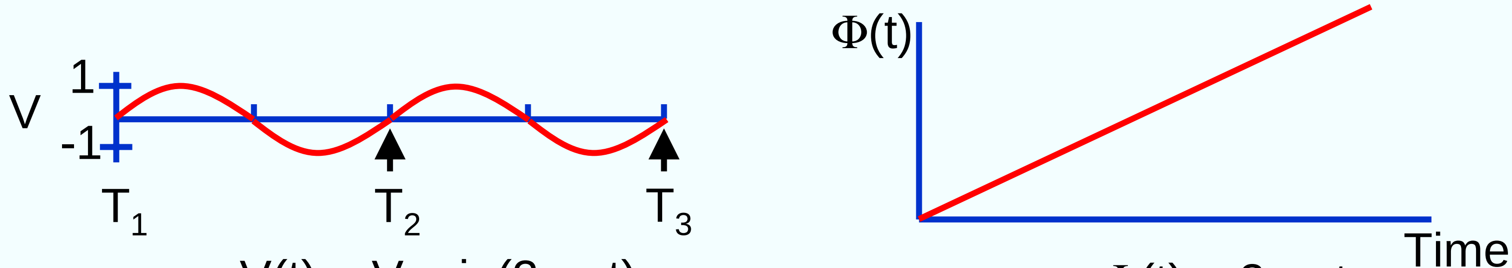
SC-cut resonator



Frequency change for symmetrical bending, SC-cut crystal.

Short Term Instability (Noise)

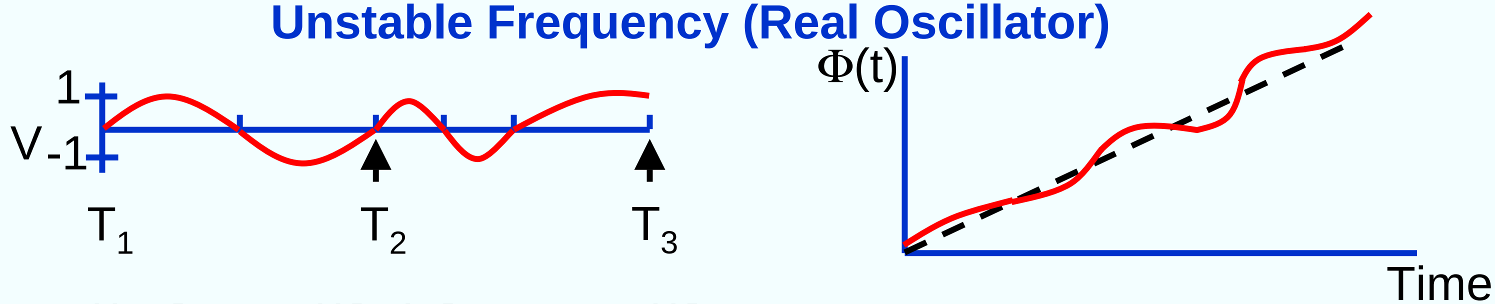
Stable Frequency (Ideal Oscillator)



$V(t) = V_0 \sin(2\pi\nu_0 t)$

$\Phi(t) = 2\pi\nu_0 t$

Unstable Frequency (Real Oscillator)



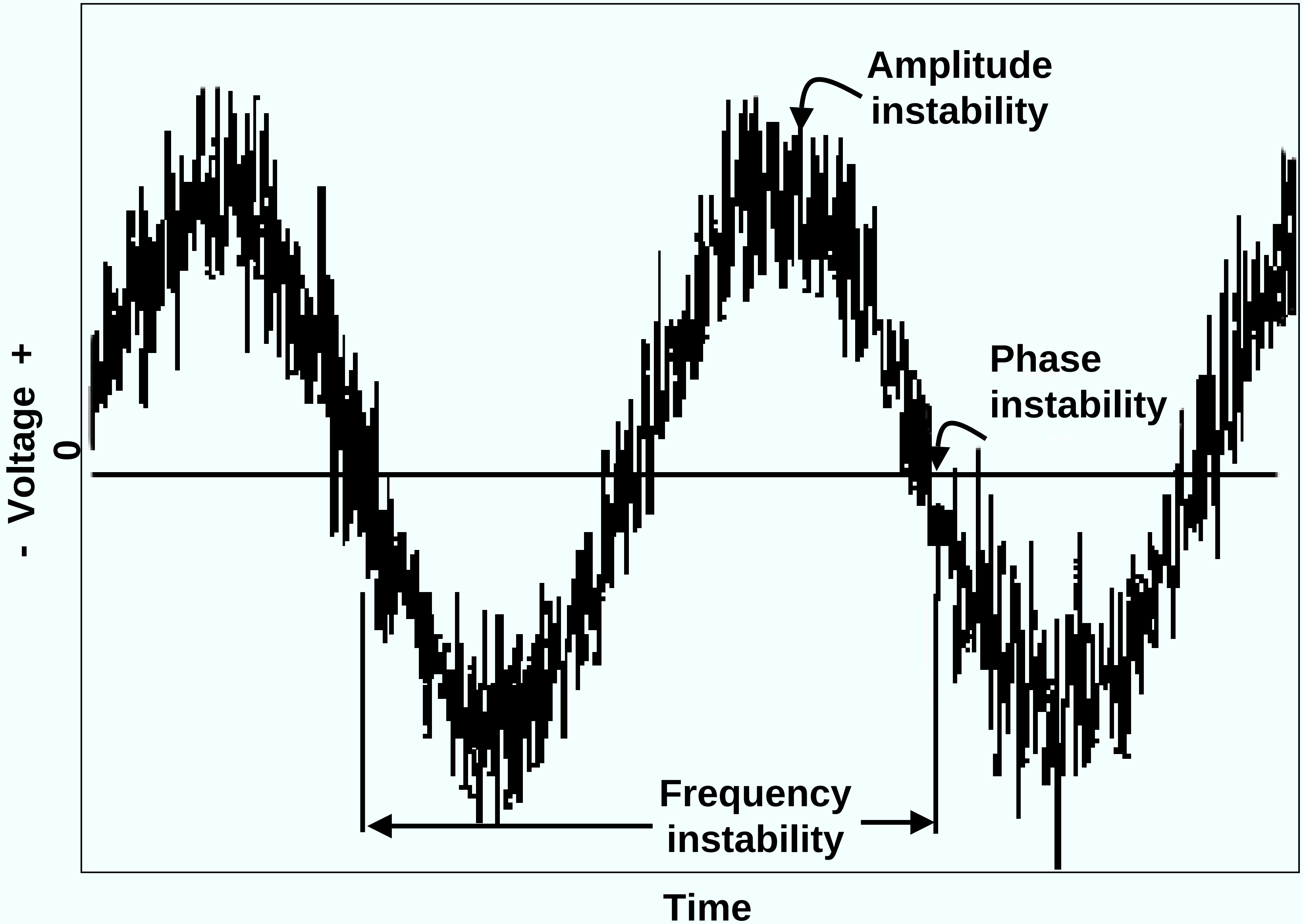
$V(t) = [V_0 + \varepsilon(t)] \sin[2\pi\nu_0 t + \phi(t)]$

$\Phi(t) = 2\pi\nu_0 t + \phi(t)$

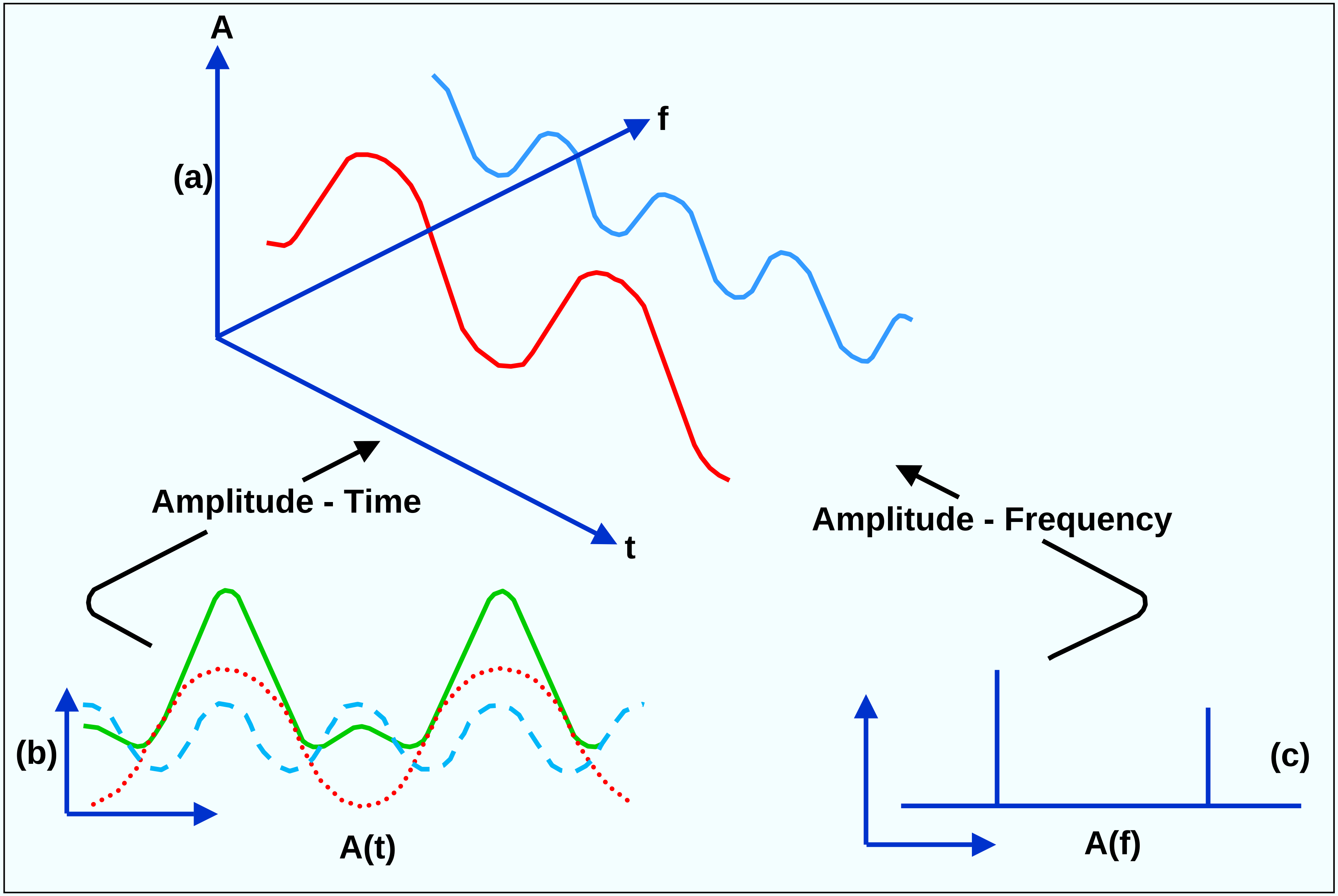
Instantaneous frequency, $\nu(t) = \dfrac{1}{2\pi}\dfrac{d\Phi(t)}{dt} = \nu_0 + \dfrac{1}{2\pi}\dfrac{d\phi(t)}{dt}$

$V(t)$ = Oscillator output voltage, V_0 = Nominal peak voltage amplitude
$\varepsilon(t)$ = Amplitude noise, ν_0 = Nominal (or "carrier") frequency
$\Phi(t)$ = Instantaneous phase, and $\phi(t)$ = Deviation of phase from nominal (i.e., the ideal)

Instantaneous Output Voltage of an Oscillator



Time Domain - Frequency Domain



Causes of Short Term Instabilities

- Johnson noise (thermally induced charge fluctuations, i.e., "thermal emf" in resistive elements)
- Phonon scattering by defects & quantum fluctuations (related to Q)
- Noise due to oscillator circuitry (active and passive components)
- Temperature fluctuations- thermal transient effects
  - activity dips at oven set-point
- Random vibration
- Fluctuations in the number of adsorbed molecules
- Stress relief, fluctuations at interfaces (quartz, electrode, mount, bond)
- Shot noise in atomic frequency standards
- ? ? ?

Short-Term Stability Measures

Measure	Symbol
Two-sample deviation, also called "Allan deviation" Spectral density of phase deviations Spectral density of fractional frequency deviations Phase noise * Most frequently found on oscillator specification sheets	$\sigma_y(\tau)$* $S_\phi(f)$ $S_y(f)$ $L(f)$*

$f^2 S_\phi(f) = \nu^2 S_y(f)$; $L(f) \equiv ½ [S_\phi(f)]$ (per IEEE Std. 1139),

and

$$\sigma_y^2(\tau) = \frac{2}{(\pi\nu\tau)^2} \int_0^\infty S_\phi(f) \sin^4(\pi f \tau) df$$

Where τ = averaging time, ν = carrier frequency, and f = offset or Fourier frequency, or "frequency from the carrier".

Allan Deviation

Also called **two-sample deviation,** or square-root of the "**Allan variance,**" it is the standard method of describing the short term stability of oscillators in the time domain. It is denoted by $\sigma_y(\tau)$,

where $$\sigma_y^2(\tau) = \frac{1}{2} < (y_{k+1} - y_k)^2 > .$$

The fractional frequencies, $y = \frac{\Delta f}{f}$ are measured over a time interval, τ; $(y_{k+1} - y_k)$ are the differences between pairs of successive measurements of y, and, ideally, < > denotes a time average of an infinite number of $(y_{k+1} - y_k)^2$. A good estimate can be obtained by a limited number, m, of measurements (m≥100). $\sigma_y(\tau)$ generally denotes $\sqrt{\sigma_y^2(\tau, m)}$, i.e.,

$$\sigma_y^2(\tau) = \sigma_y^2(\tau, m) = \frac{1}{m}\sum_{j=1}^{m} \tfrac{1}{2}\left(y_{k+1} - y_k\right)_j^2$$

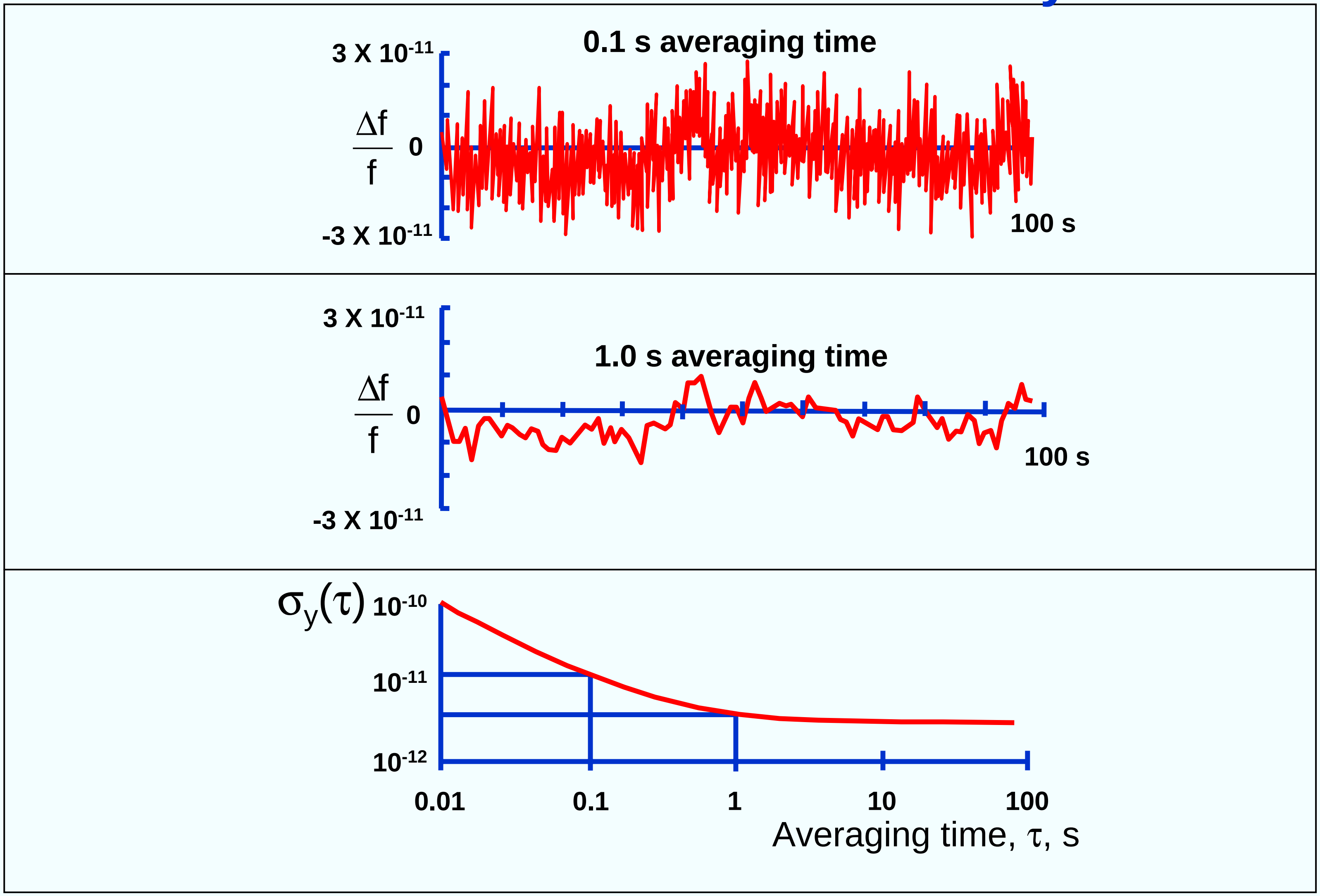
Why $\sigma_y(\tau)$?

- **Classical variance:** $$\sigma_{y_i}^2 = \frac{1}{m-1}\sum\left(y_i - \overline{y}\right)^2,$$

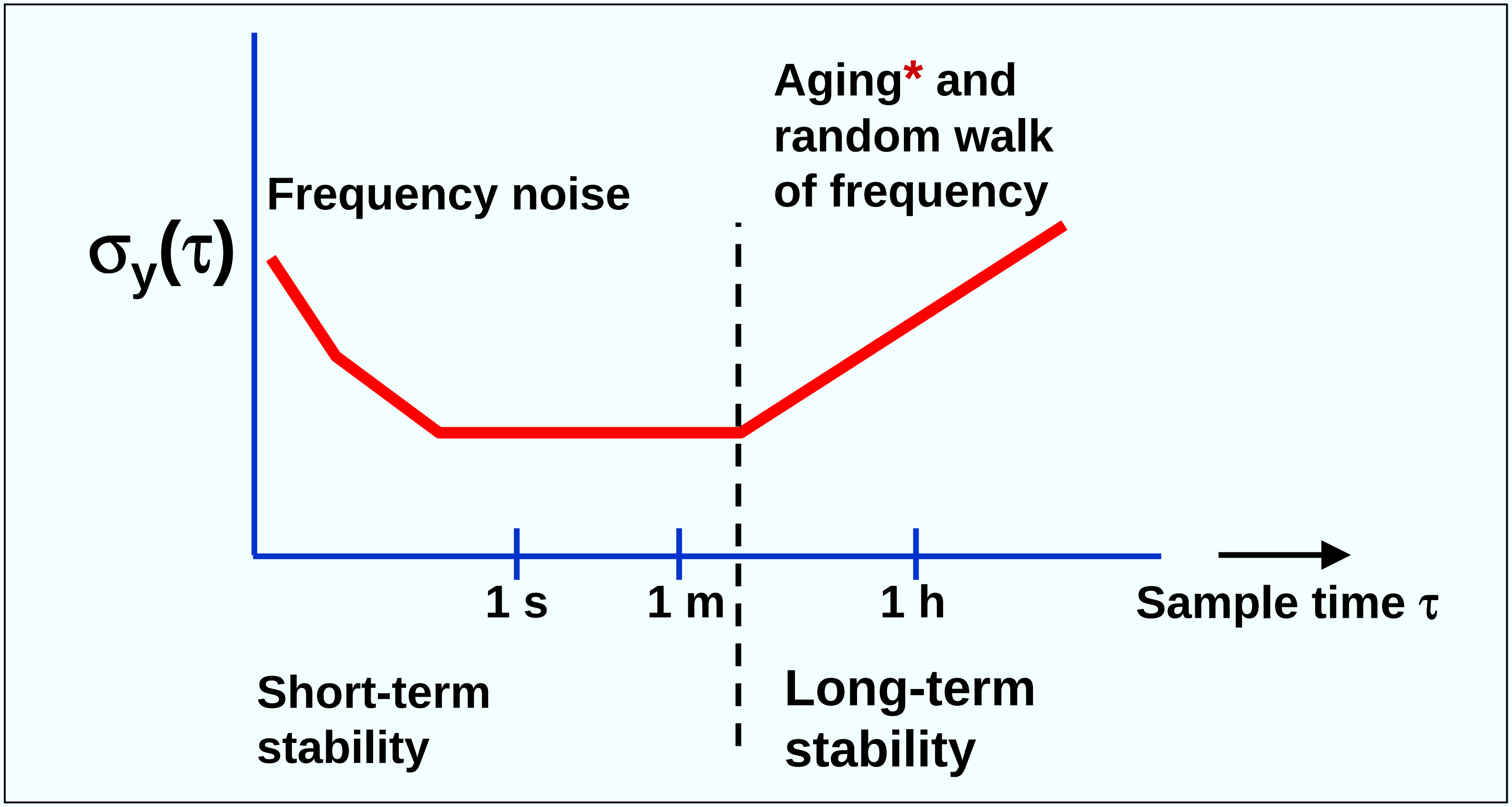
  diverges for some commonly observed noise processes, such as random walk, i.e., the variance increases with increasing number of data points.

- **Allan variance:**
  - Converges for all noise processes observed in precision oscillators.
  - Has straightforward relationship to power law spectral density types.
  - Is easy to compute.
  - Is faster and more accurate in estimating noise processes than the Fast Fourier Transform.

Frequency Noise and $\sigma_y(\tau)$

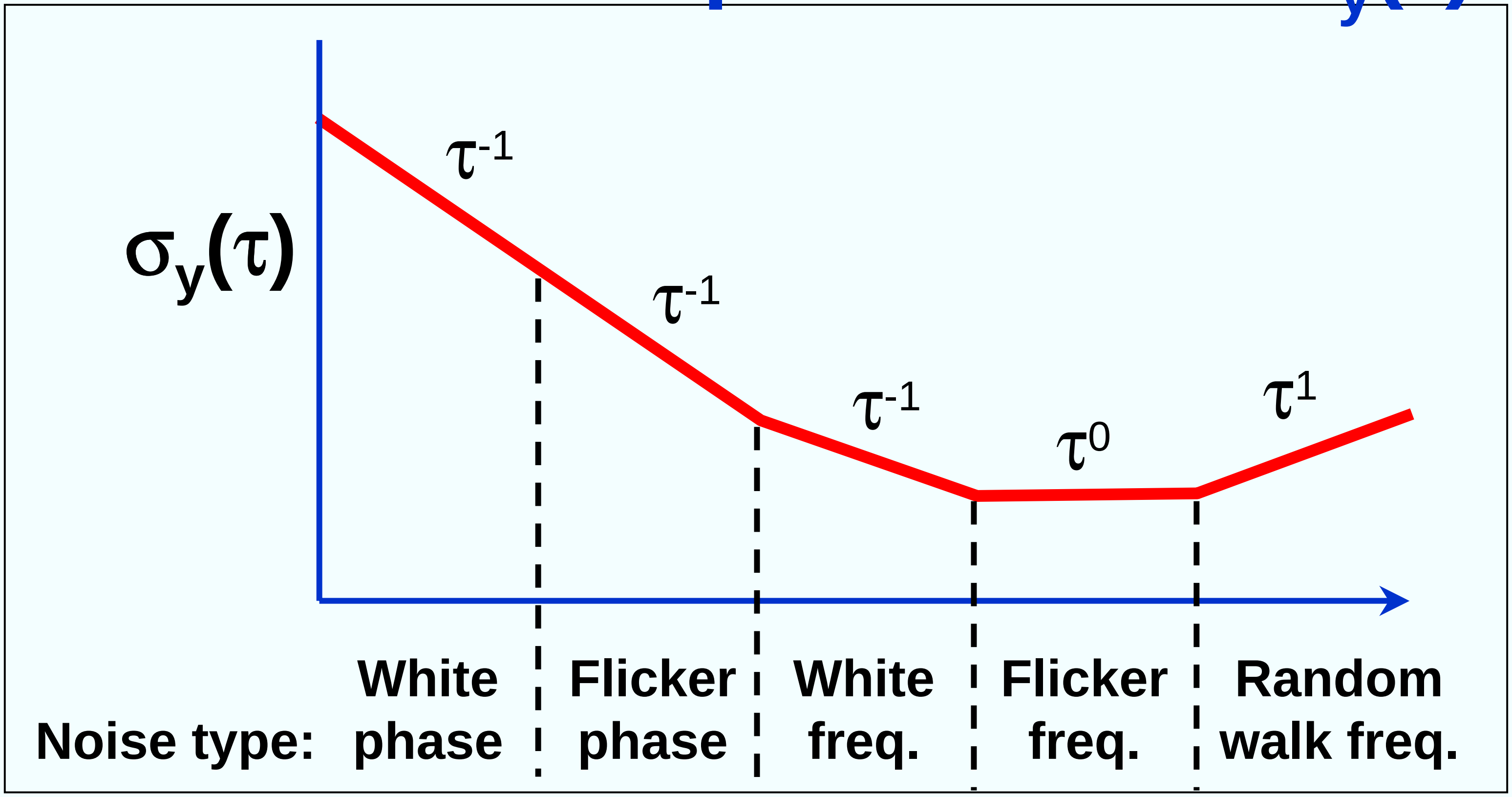


Time Domain Stability



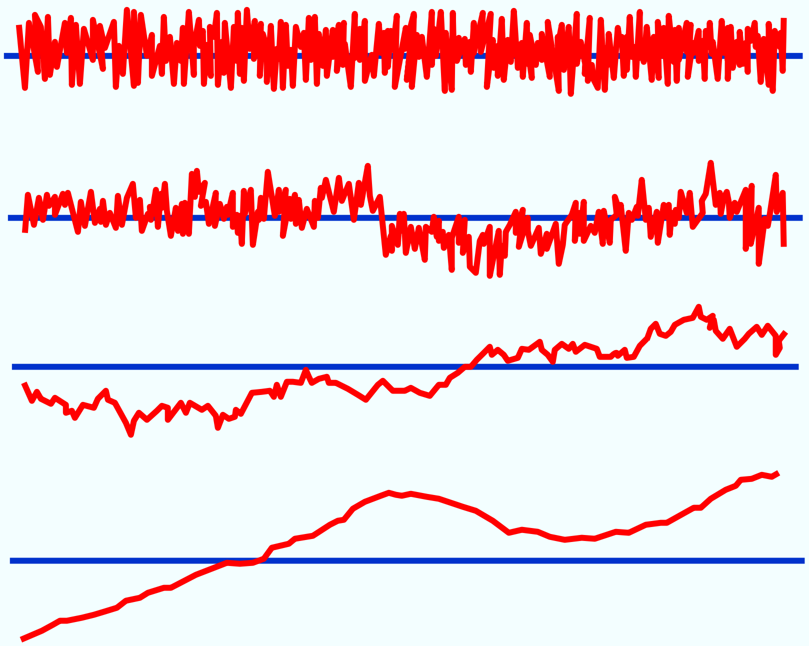
*For $\sigma_y(\tau)$ to be a proper measure of random frequency fluctuations, aging must be properly subtracted from the data at long τ's.

Power Law Dependence of $\sigma_y(\tau)$



Below the flicker of frequency noise (i.e., the “flicker floor”) region, crystal oscillators typically show τ^{-1} (white phase noise) dependence. Atomic standards show $\tau^{-1/2}$ (white frequency noise) dependence down to about the servo-loop time constant, and τ^{-1} dependence at less than that time constant. Typical τ’s at the start of flicker floors are: 1 second for a crystal oscillator, 10^3s for a Rb standard and 10^5s for a Cs standard.

Pictures of Noise

Plot of z(t) vs. t	$S_z(f) = h_\alpha f^\alpha$	Noise name
	$\alpha = 0$	White
	$\alpha = -1$	Flicker
	$\alpha = -2$	Random walk
	$\alpha = -3$	

Plots show fluctuations of a quantity z(t), which can be,e.g., the output of a counter (Δf vs. t) or of a phase detector (ϕ[t] vs. t). The plots show simulated time-domain behaviors corresponding to the most common (power-law) spectral densities; h_α is an amplitude coefficient. Note: since $S_{\Delta f} = f^2 S_\phi$, e.g. white frequency noise and random walk of phase are equivalent.

Spectral Densities

$$V(t) = \left[V_0 + \varepsilon\left(t\right)\right]\sin\left[2\pi\nu_0 t + \phi(t)\right]$$

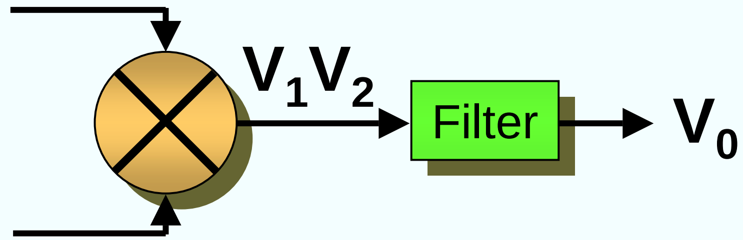
In the frequency domain, due to the phase deviation, $\phi(t)$, some of the power is at frequencies other than ν_0. The stabilities are characterized by "spectral densities." The spectral density, $S_V(f)$, the mean-square voltage $\langle V^2(t) \rangle$ in a unit bandwidth centered at f, is not a good measure of frequency stability because both $\varepsilon(t)$ and $\phi(t)$ contribute to it, and because it is not uniquely related to frequency fluctuations (although $\varepsilon(t)$ is often negligible in precision frequency sources.)

The spectral densities of phase and fractional-frequency fluctuations, $S_\phi(f)$ and $S_y(f)$, respectively, are used to measure the stabilities in the frequency domain. The spectral density $S_g(f)$ of a quantity g(t) is the mean square value of g(t) in a unit bandwidth centered at f. Moreover, the RMS value of g^2 in bandwidth BW is given by $g^2_{RMS}(t) = \int_{BW} S_g(f)\,df$.

Mixer Functions

$$V_1 = A_1 \sin(\omega_1 t + \varphi_1)$$

$$V_2 = A_2 \sin(\omega_2 t + \varphi_2)$$



Trigonometric identities: sin(x)sin(y) = ½cos(x-y) - ½cos(x+y)

cos(x±π/2) = sin(x)

Let $\omega_1 = \omega_2; \Phi_1 \equiv \omega_1 t + \varphi_1$, and $\Phi_2 \equiv \omega_2 t + \varphi_2$. Then the mixer can become :

- **Phase detector:** When $\Phi_1 = \Phi_2 + \pi/2$ and $A_1 = A_2 = 1$, then

$$V_0 = \frac{1}{2}\sin(\varphi_1 - \varphi_2) = \frac{1}{2}(\varphi_1 - \varphi_2) \text{ for small } \varphi\text{'s}$$

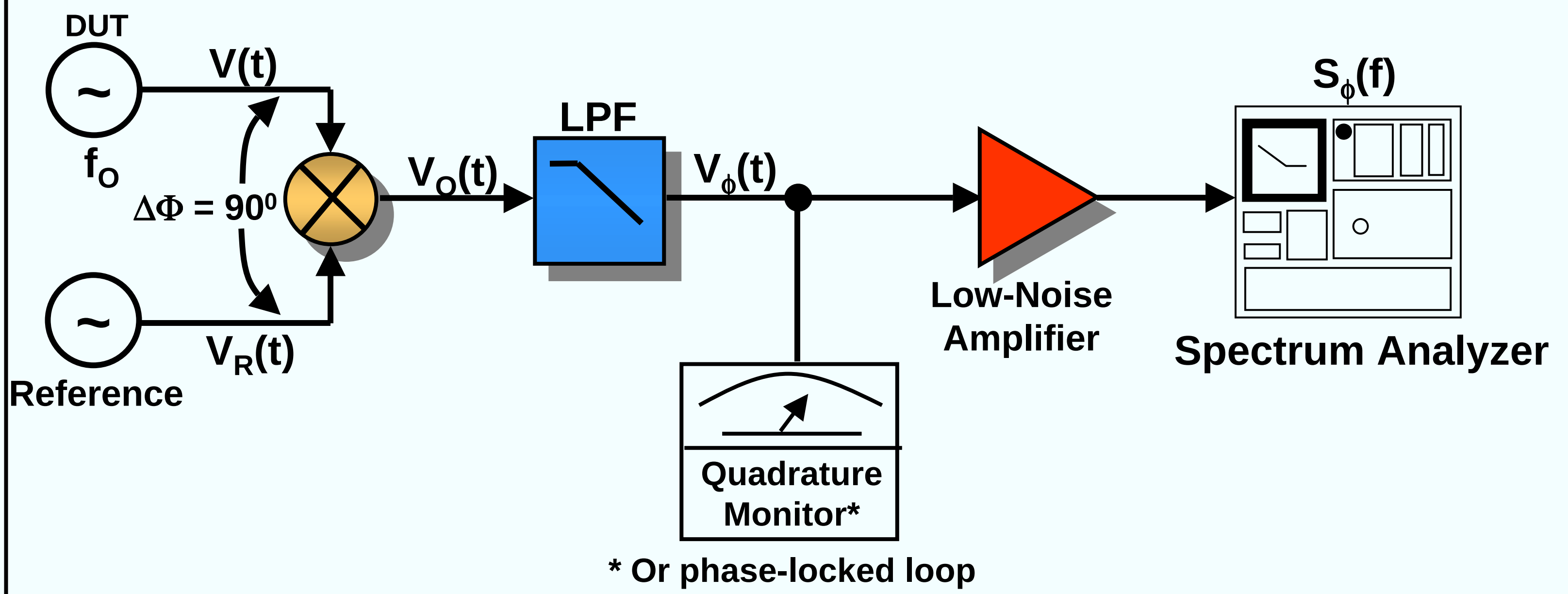
- **AM detector:** When $A_2 = 1$ and the filter is a low – pass filter, then

$$V_0 = \frac{1}{2}A_1\cos(\varphi_1 - \varphi_2); \text{ if } \varphi_1 \approx \varphi_2, \text{ then } V_0 \approx \frac{1}{2}A_1$$

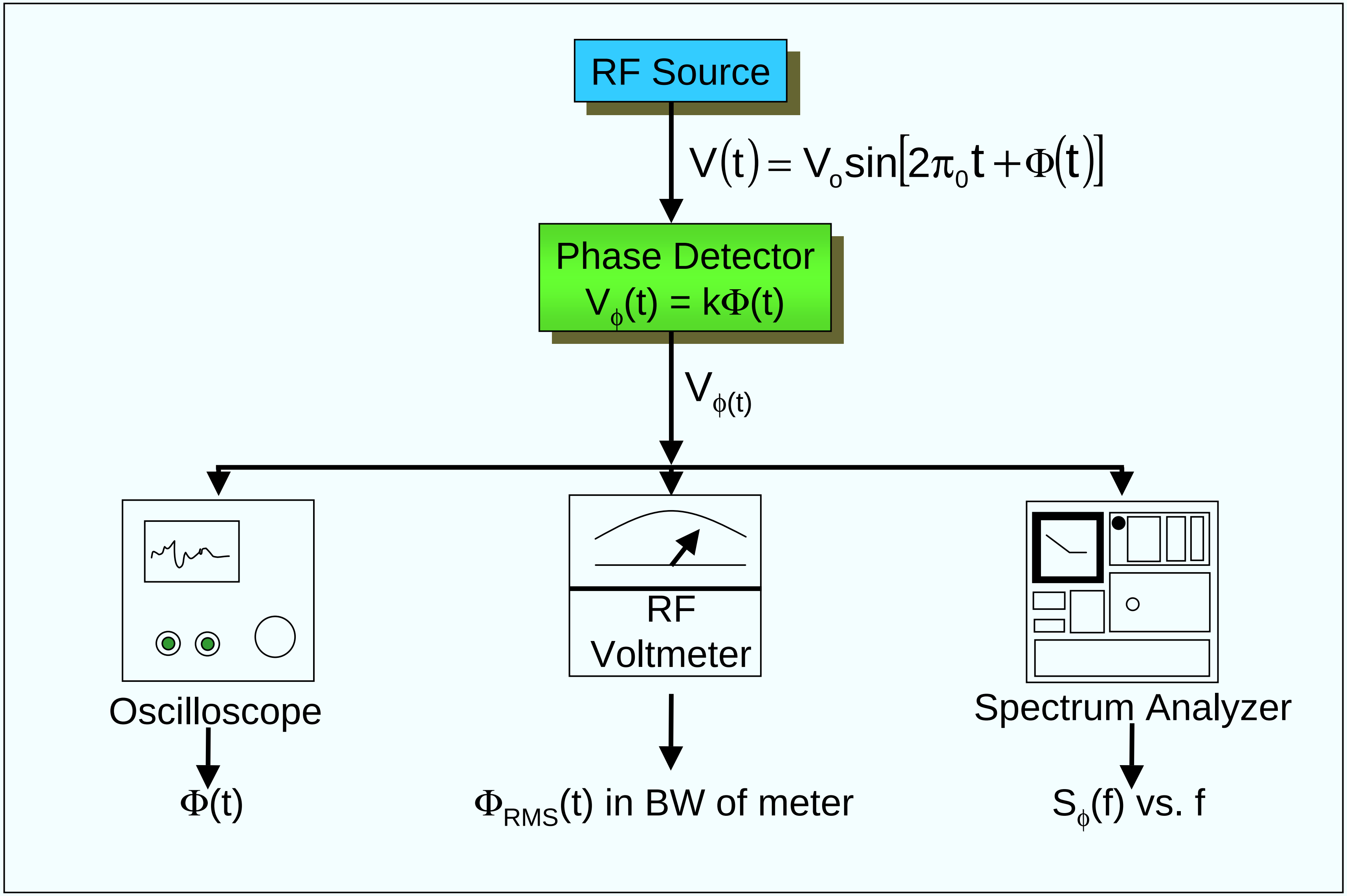
- **Frequency multiplier:** When $V_1 = V_2$ and the filter is bandpass at $2\omega_1$

then, $V_0 = \frac{1}{2}A_1^2\cos(2\omega_1 t + 2\varphi_1) \Rightarrow$ doubles the frequency and phase error.

Phase Detector



Phase Noise Measurement



Frequency - Phase - Time Relationships

$$\nu(t) = \nu_0 + \frac{1}{2\pi}\frac{d\phi(t)}{dt} = \text{"instantaneous" frequency;}\quad \phi(t) = \phi_0 + \int_0^t 2\pi\left[\nu(t') - \nu_0\right]dt'$$

$$y(t) \equiv \frac{\nu(t) - \nu_0}{\nu_0} = \frac{\dot{\phi}(t)}{2\pi\nu_0} = \text{normalized frequency;}\quad \phi^2_{RMS} = \int S_\phi(f)dt$$

$$S_\phi(f) = \frac{\phi^2_{RMS}}{BW} = \left(\frac{\nu_0}{f}\right)^2 S_y(f); \qquad \mathcal{L}(f) \equiv 1/2\, S_\phi(f),\ \text{per IEEE Standard 1139} - 1988$$

$$\sigma_y^2(\tau) = 1/2 < \left(\bar{y}_{k+1} - \bar{y}_k\right)^2 > = \frac{2}{(\pi\nu_0\tau)^2}\int_0^\infty S_\phi(f)\sin^4(\pi f\tau)\,df$$

The five common power-law noise processes in precision oscillators are:

$$S_y(f) = \underset{\text{(White PM)}}{h_2 f^2} + \underset{\text{(Flicker PM)}}{h_1 f} + \underset{\text{(White FM)}}{h_0} + \underset{\text{(Flicker FM)}}{h_{-1} f^{-1}} + \underset{\text{(Random-walk FM)}}{h_{-2} f^{-2}}$$

$$\text{Time deviation} = x(t) = \int_o^t y(t')dt' = \frac{\phi(t)}{2\pi\nu}$$

$S_\phi(f)$ to SSB Power Ratio Relationship

Consider the "simple" case of sinusoidal phase modulation at frequency f_m. Then, $\phi(t) = \phi_o(t)\sin(2\pi f_m t)$, and $V(t) = V_o\cos[2\pi f_c t + \phi(t)] = V_o\cos[2\pi f_c t + \phi_0(t)\sin(\pi f_m t)]$, where $\phi_o(t)$= peak phase excursion, and f_c=carrier frequency. Cosine of a sine function suggests a Bessel function expansion of V(t) into its components at various frequencies via the identities:

$$\cos(X+Y) = \cos X \cos Y - \sin X \sin Y$$

$$\cos X \cos Y = 1/2[\cos(X+Y) + \cos(X-Y)]$$

$$-\sin X \sin Y = [\cos(X+Y) - \cos(X-Y)]$$

$$\cos(B\sin X) = J_0(B) + 2\sum J_{2n}(B)\cos(2nX)$$

$$\sin(B\sin X) = 2\sum_{n=0}^{\infty} J_{2n+1}(B)\sin[(2n+1)X]$$

After some messy algebra, $S_V(f)$ and $S_\phi(f)$ are as shown on the next page. Then,

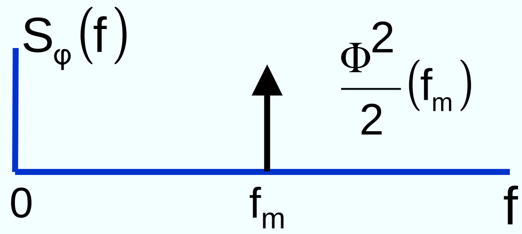
$$\text{SSB Power Ratio at } f_m = \frac{V_0{}^2 J_1{}^2[\Phi(f_m)]}{V_0{}^2 J_0{}^2[\Phi(f_m)] + 2\sum_{i=1}^{\infty} J_i{}^2[\Phi(f_m)]}$$

$$\text{if } \Phi(f_m) << 1, \text{ then } J_0 = 1,\ J_1 = 1/2\Phi(f_m),\ J_n = 0 \text{ for } n > 1, \text{ and}$$

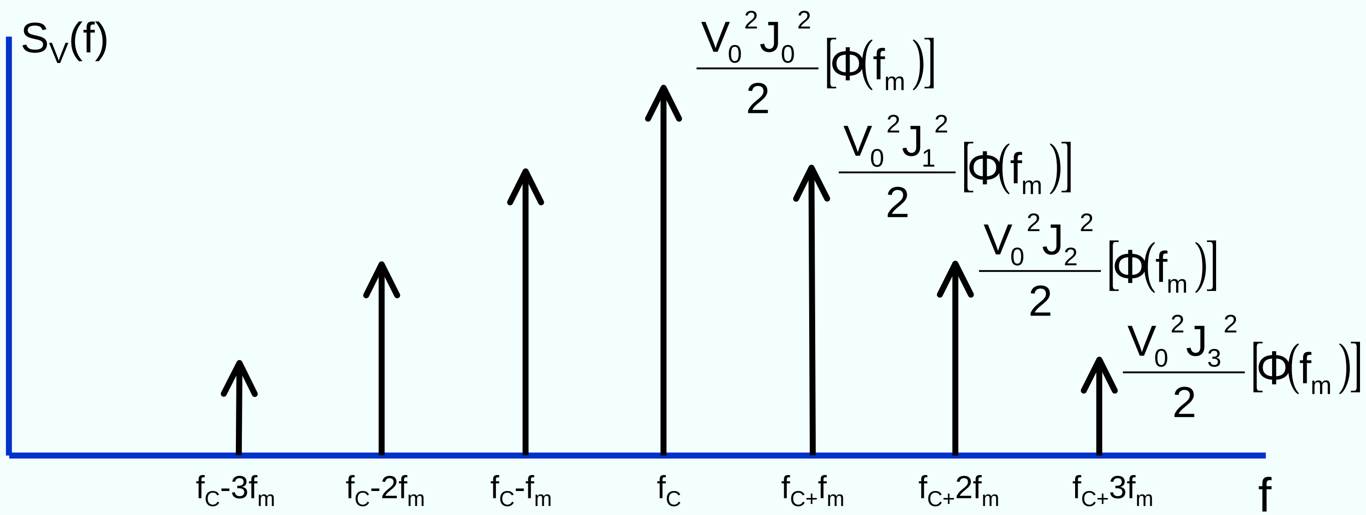
$$\text{SSB Power Ratio} = L(f_m) = \frac{\Phi^2(f_m)}{4} = \frac{S_\phi(f_m)}{2}$$

$S_\phi(f)$, $S_V(f)$ and L (f)

$$\Phi(t) = \Phi(f_m)\cos(2\pi f_m t)$$

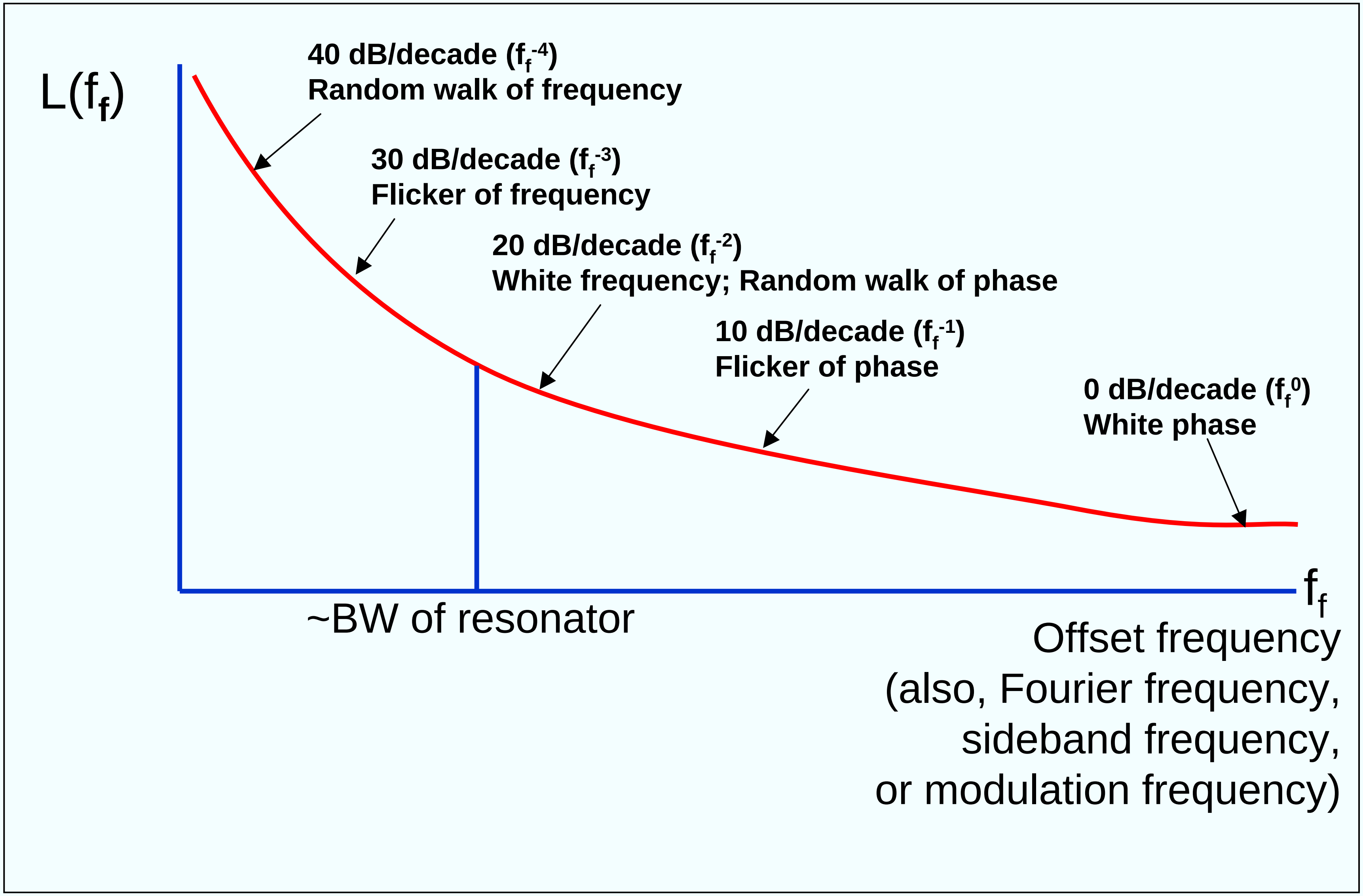


$$V(t) = V_0\cos[2\pi f_C t + \Phi(f_m)]$$



$$\text{SSB Power Ratio} = \frac{V_0^{\,2} J_1^{\,2}[\Phi(f_m)]}{V_0^{\,2} J_0^{\,2}[\Phi(f_m)] + 2\sum_{i=1}^{\infty} J_i^2\left[\Phi\left(f_m\right)\right]} \cong L(f_m) \equiv \frac{S\varphi(f_m)}{2}$$

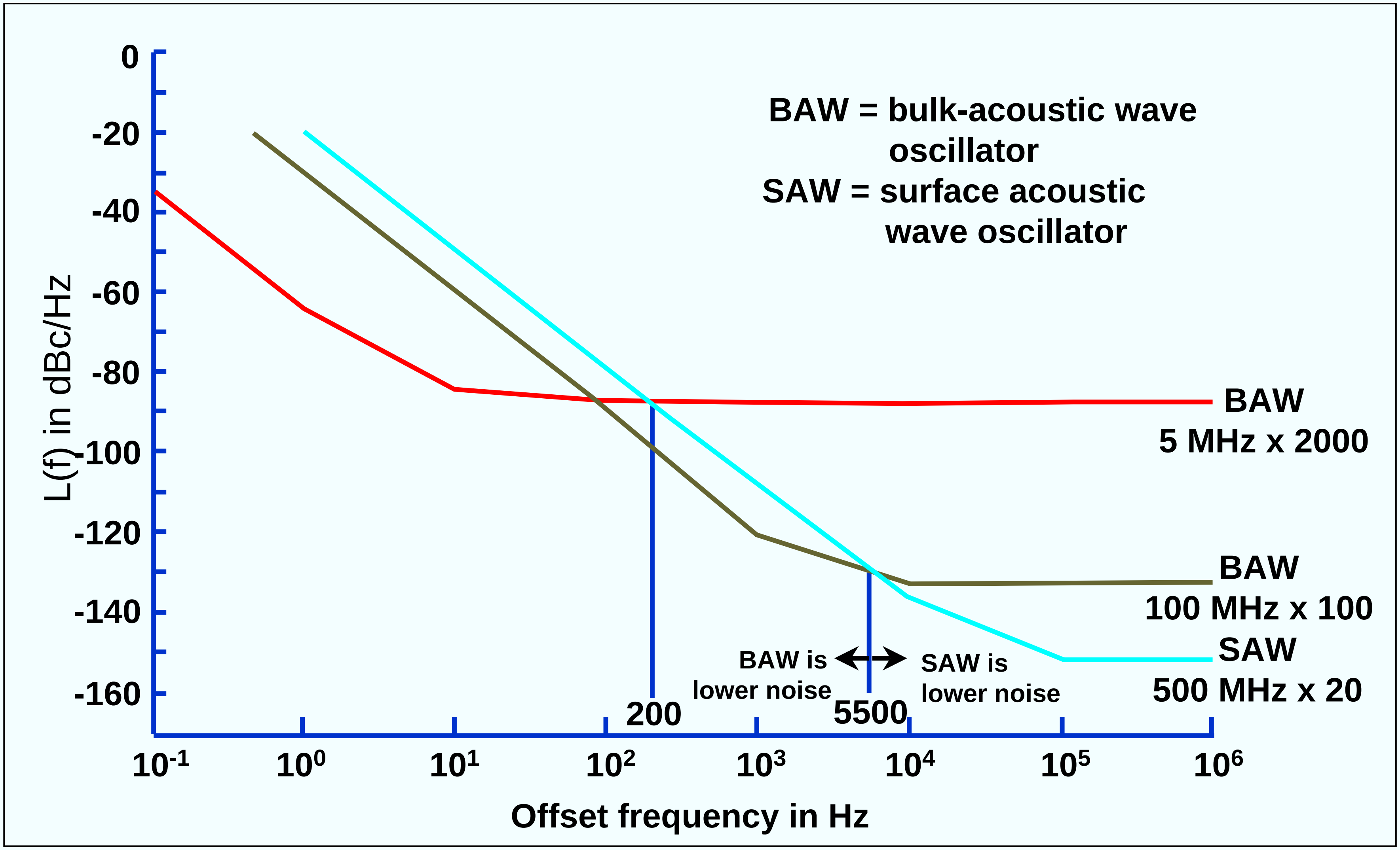
Types of Phase Noise



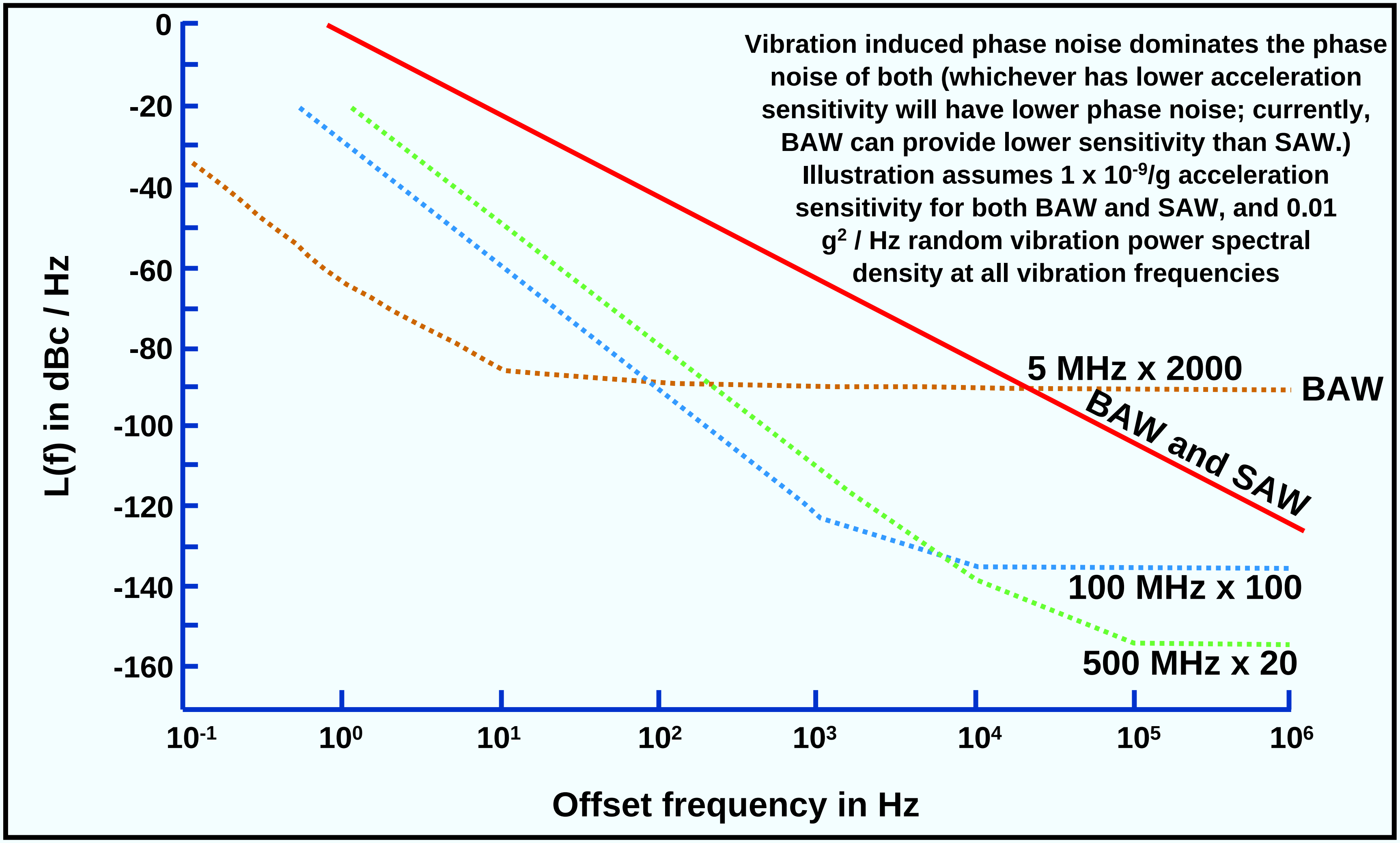
Noise in Crystal Oscillators

- The resonator is the primary noise source close to the carrier; the oscillator sustaining circuitry is the primary source far from the carrier.
- Frequency multiplication by N increases the phase noise by N^2 (i.e., by 20log N, in dB's).
- Vibration-induced "noise" dominates all other sources of noise in many applications (see acceleration effects section, later).
- Close to the carrier (within BW of resonator), $S_y(f)$ varies as 1/f, $S_\phi(f)$ as $1/f^3$, where f = offset from carrier frequency, ν. $S_\phi(f)$ also varies as $1/Q^4$, where Q = unloaded Q. Since $Q_{max}\nu$ = const., $S_\phi(f) \propto \nu^4$. $(Q_{max}\nu)_{BAW} = 1.6 \times 10^{13}$ Hz; $(Q_{max}\nu)_{SAW} = 1.05 \times 10^{13}$ Hz.
- In the time domain, noise floor is $\sigma_y(\tau) \geq (2.0 \times 10^{-7})Q^{-1} \approx 1.2 \times 10^{-20}\nu$, ν in Hz. In the regions where $\sigma_y(\tau)$ varies as τ^{-1} and $\tau^{-1/2}$ ($\tau^{-1/2}$ occurs in atomic frequency standards), $\sigma_y(\tau) \propto (QS_R)^{-1}$, where S_R is the signal-to-noise ratio; i.e., the higher the Q and the signal-to-noise ratio, the better the short term stability (and the phase noise far from the carrier, in the frequency domain).
- It is the loaded Q of the resonator that affects the noise when the oscillator sustaining circuitry is a significant noise source.
- Noise floor is limited by Johnson noise; noise power, kT = -174 dBm/Hz at 290°K.
- Higher signal level improves the noise floor but not the close-in noise. (In fact, high drive levels generally degrade the close-in noise, for reasons that are not fully understood.)
- Low noise SAW vs. low noise BAW multiplied up: BAW is lower noise at f < ~1 kHz, SAW is lower noise at f > ~1 kHz; can phase lock the two to get the best of both.

Low-Noise SAW and BAW Multiplied to 10 GHz (in a nonvibrating environment)



Low-Noise SAW and BAW Multiplied to 10 GHz (in a vibrating environment)



Effects of Frequency Multiplication

Input		Output
$f_i \equiv f_{in}$		$f_o \equiv f_{out} = Mf_i$
Δf_i		$\Delta f_o = M\Delta f_i$
$\dfrac{\Delta f_i}{f_i} \equiv y$		$\dfrac{\Delta f_o}{f_o} = \dfrac{\Delta f_i}{f_i}$
$\Delta\varphi_i$	$\rightarrow$ $\boxed{\mathbf{f_i \times M = f_o}}$ $\rightarrow$	$\Delta\phi_o = M\Delta\varphi_i$
$L(f)_i$	**Noiseless Multiplier**	$L(f)_o = L(f)_i + 20 \log M$
$S_\varphi(f)_i$		$S_\varphi(f)_o = M^2 S_\phi(f)_i$
$S_y(f)_i$		$S_y(f)_o = S_y(f)_i$
$\sigma_y(\tau)_i$		$\sigma_y(\tau)_o = \sigma_y(\tau)_i$

Note that $y = \dfrac{\Delta f}{f}$, $S_y(f)$, and $\sigma_y(\tau)$ are unaffected by frequency multiplication.

TCXO Noise

The short term stabilities of TCXOs are temperature (T) dependent, and are generally worse than those of OCXOs, for the following reasons:

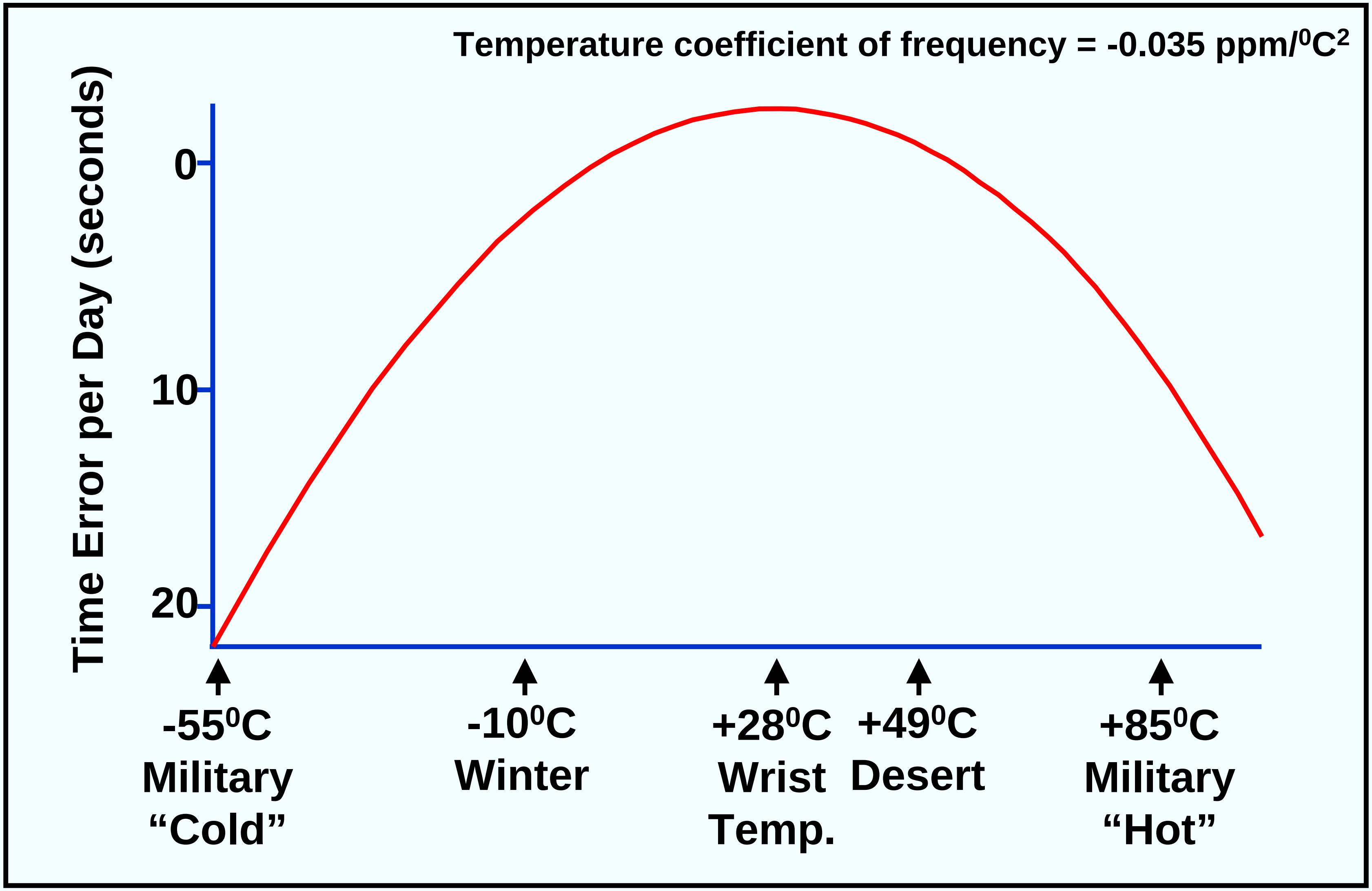
- The slope of the TCXO crystal's frequency (f) vs. T varies with T. For example, the f vs. T slope may be near zero at ~20°C, but it will be ~1ppm/°C at the T extremes. T fluctuations will cause small f fluctuations at laboratory ambient T's, so the stability can be good there, but millidegree fluctuations will cause ~10^{-9} f fluctuations at the T extremes. The TCXO's f vs. T slopes also vary with T; the zeros and maxima can be at any T, and the maximum slopes can be on the order of 1 ppm/°C.

- AT-cut crystals' thermal transient sensitivity makes the effects of T fluctuations depend not only on the T but also on the rate of change of T (whereas the SC-cut crystals typically used in precision OCXOs are insensitive to thermal transients). Under changing T conditions, the T gradient between the T sensor (thermistor) and the crystal will aggravate the problems.

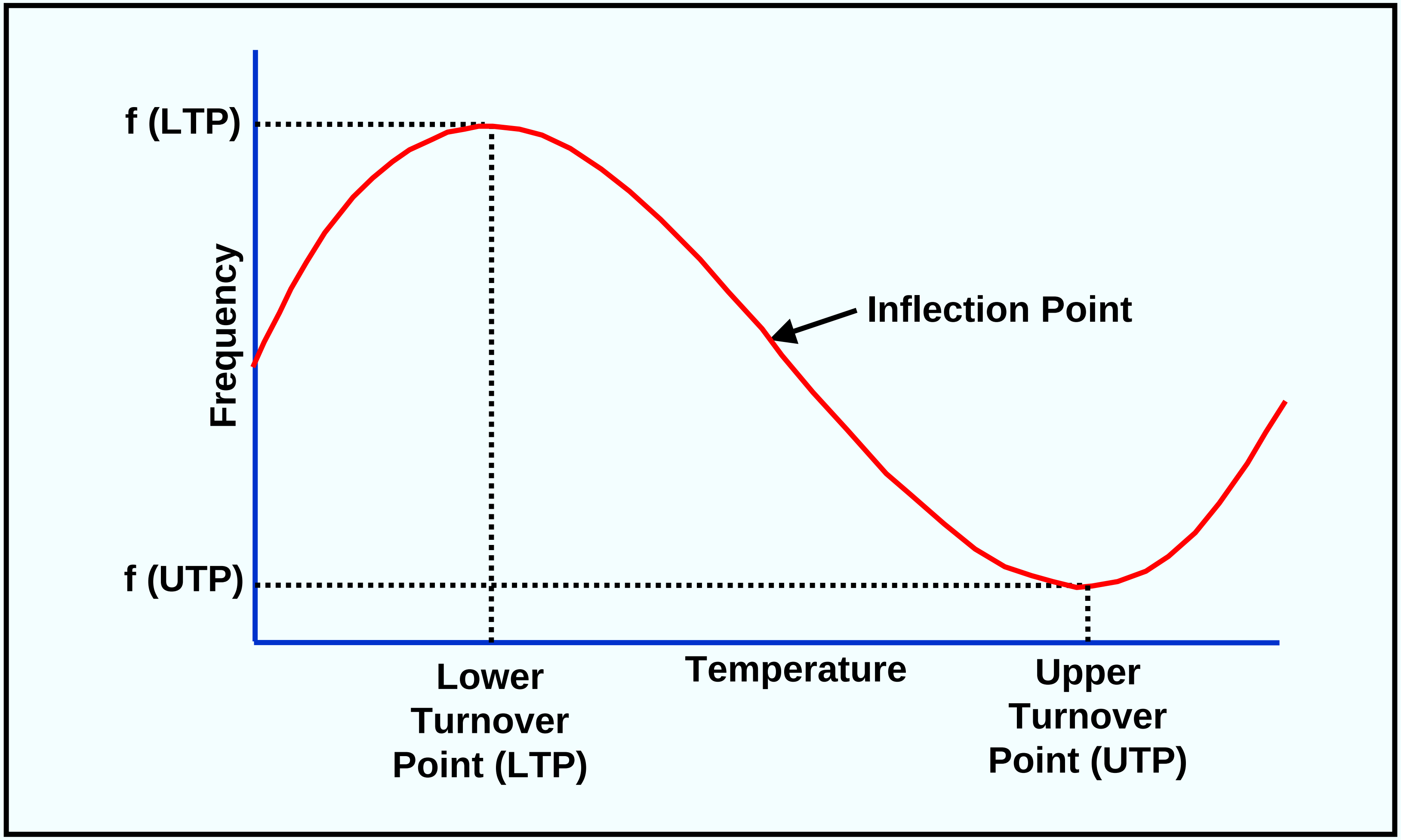
- TCXOs typically use fundamental mode AT-cut crystals which have lower Q and larger C_1 than the crystals typically used in OCXOs. The lower Q makes the crystals inherently noisier, and the larger C_1 makes the oscillators more susceptible to circuitry noise.

- AT-cut crystals' f vs. T often exhibit activity dips (see "Activity Dips" later in this chapter). At the T's where the dips occur, the f vs. T slope can be very high, so the noise due to T fluctuations will also be very high, e.g., 100x degradation of $\sigma_y(\tau)$ and 30 dB degradation of phase noise are possible. Activity dips can occur at any T.

Quartz Wristwatch Accuracy vs. Temperature



Frequency vs. Temperature Characteristics

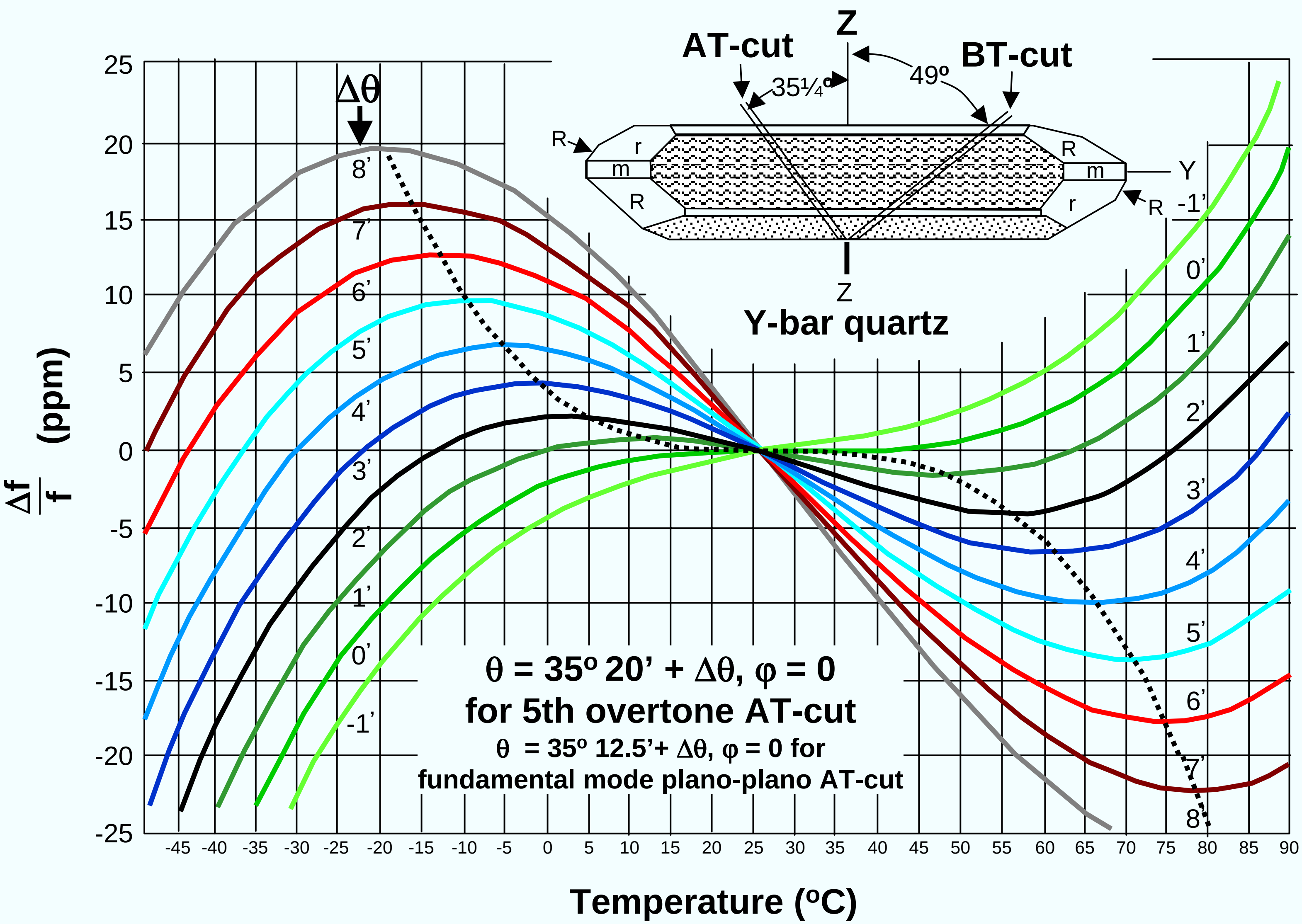


Resonator f vs. T Determining Factors

- **Primary:** Angles of cut

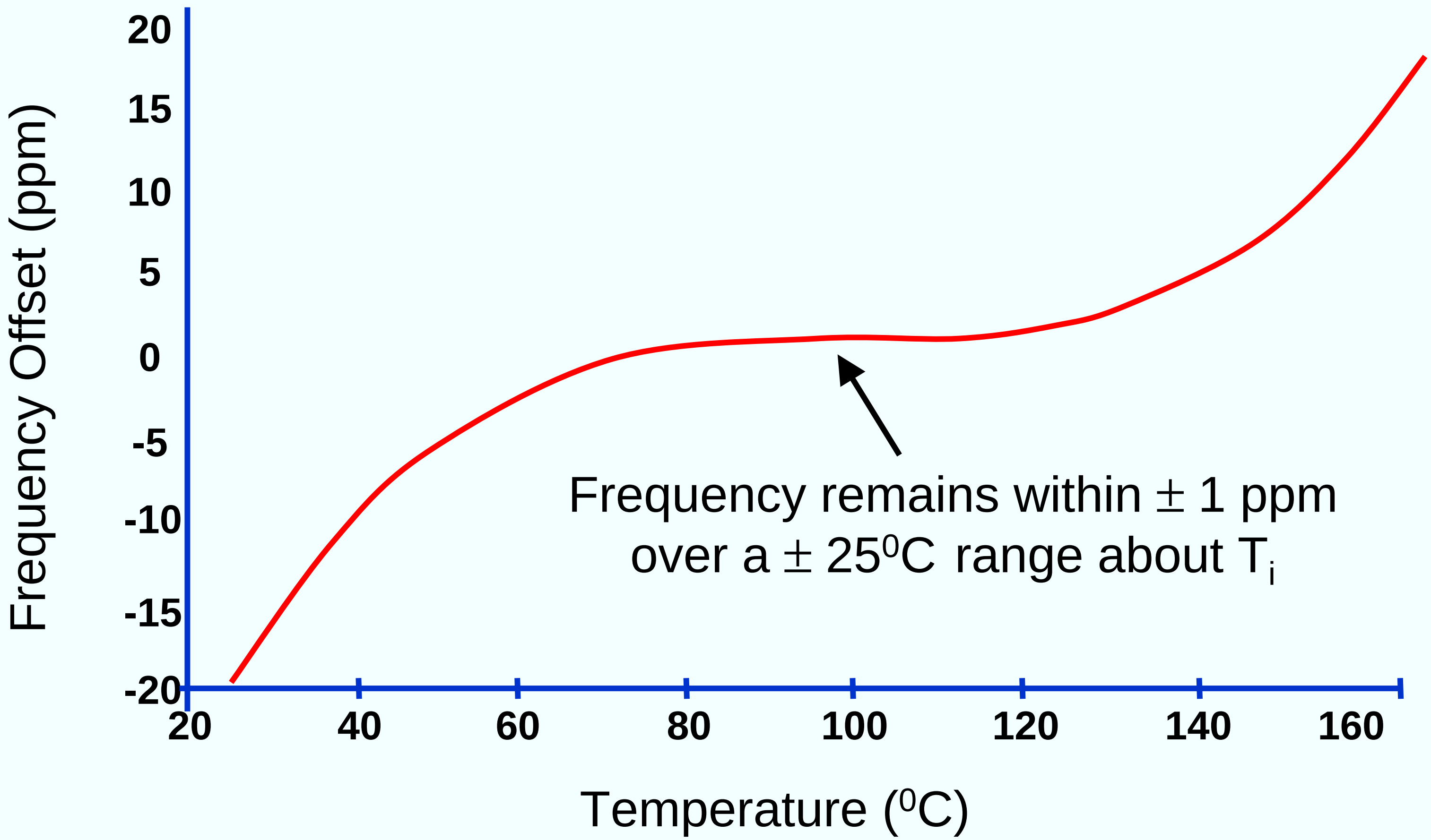
- **Secondary:**
  - Overtone
  - Blank geometry (contour, dimensional ratios)
  - Material impurities and strains
  - Mounting & bonding stresses (magnitude and direction)
  - Electrodes (size, shape, thickness, density, stress)
  - Drive level
  - Interfering modes
  - Load reactance (value & temperature coefficient)
  - Temperature rate of change
  - Thermal history
  - Ionizing radiation

Frequency-Temperature vs. Angle-of-Cut, AT-cut

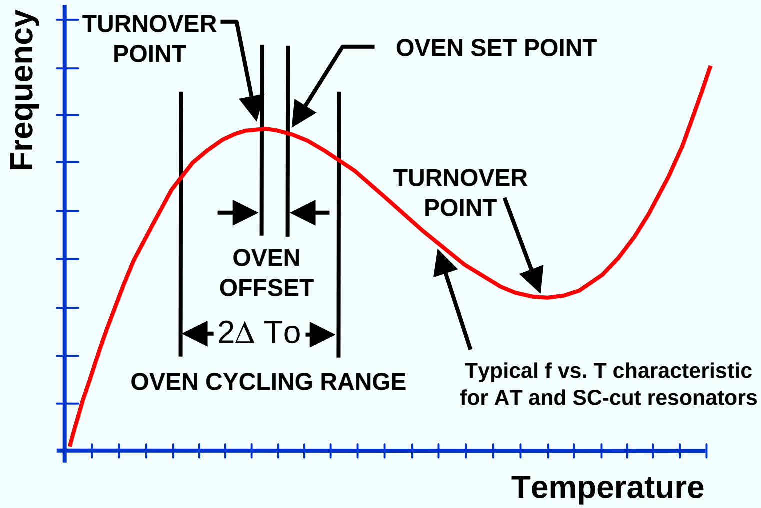


Desired f vs. T of SC-cut Resonator

for OCXO Applications



OCXO Oven's Effect on Stability



Oven Parameters vs. Stability for SC-cut Oscillator
Assuming $T_i - T_{LTP} = 10^0C$

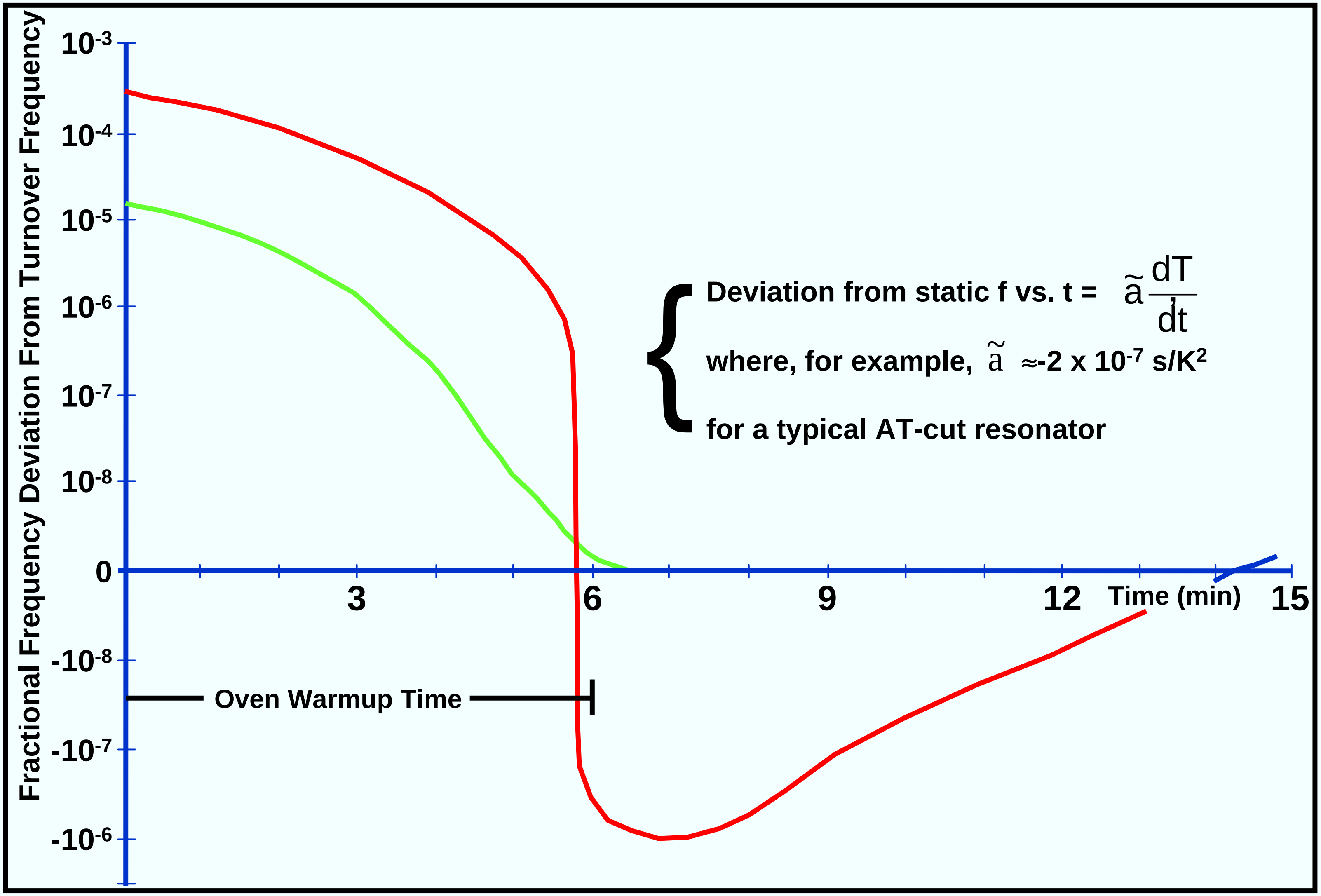
$T_i - T_{LTP} = 10^0C$		Oven Cycling Range (millidegrees)			
		10	1	0.1	0.01
Oven Offset (millidegrees)	100	4×10^{-12}	4×10^{-13}	4×10^{-14}	4×10^{-15}
	10	6×10^{-13}	4×10^{-14}	4×10^{-15}	4×10^{-16}
	1	2×10^{-13}	6×10^{-15}	4×10^{-16}	4×10^{-17}
	0.1	2×10^{-13}	2×10^{-15}	6×10^{-17}	4×10^{-18}
	0	2×10^{-13}	2×10^{-15}	2×10^{-17}	2×10^{-19}

A comparative table for AT and other non-thermal-transient compensated cuts of oscillators would not be meaningful because the dynamic f vs. T effects would generally dominate the static f vs. T effects.

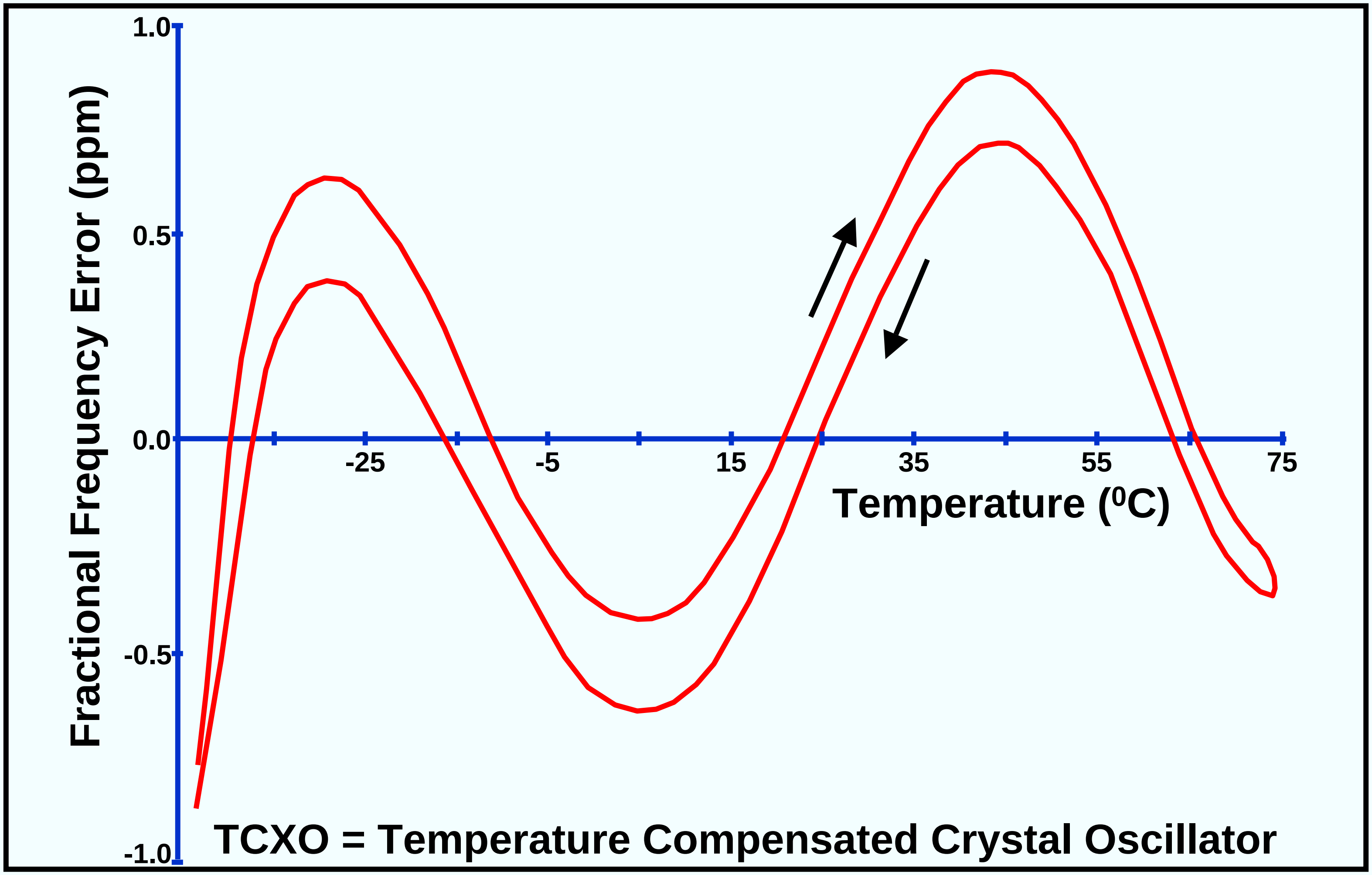
Oven Stability Limits

- Thermal gain of 10^5 has been achieved with a feed-forward compensation technique (i.e., measure outside T of case & adjust setpoint of the thermistor to anticipate and compensate). For example, with a 10^5 gain, if outside $\Delta T = 100^{o}C$, inside $\Delta T = 1$ mK.
- Stability of a good amplifier ~1μK/K
- Stability of thermistors ~1mK/year to 100mK/year
- Noise < 1μK (Johnson noise in thermistor + amplifier noise + shot noise in the bridge current)
- Quantum limit of temperature fluctuations ~ 1nK
- Optimum oven design can provide very high f vs. T stability

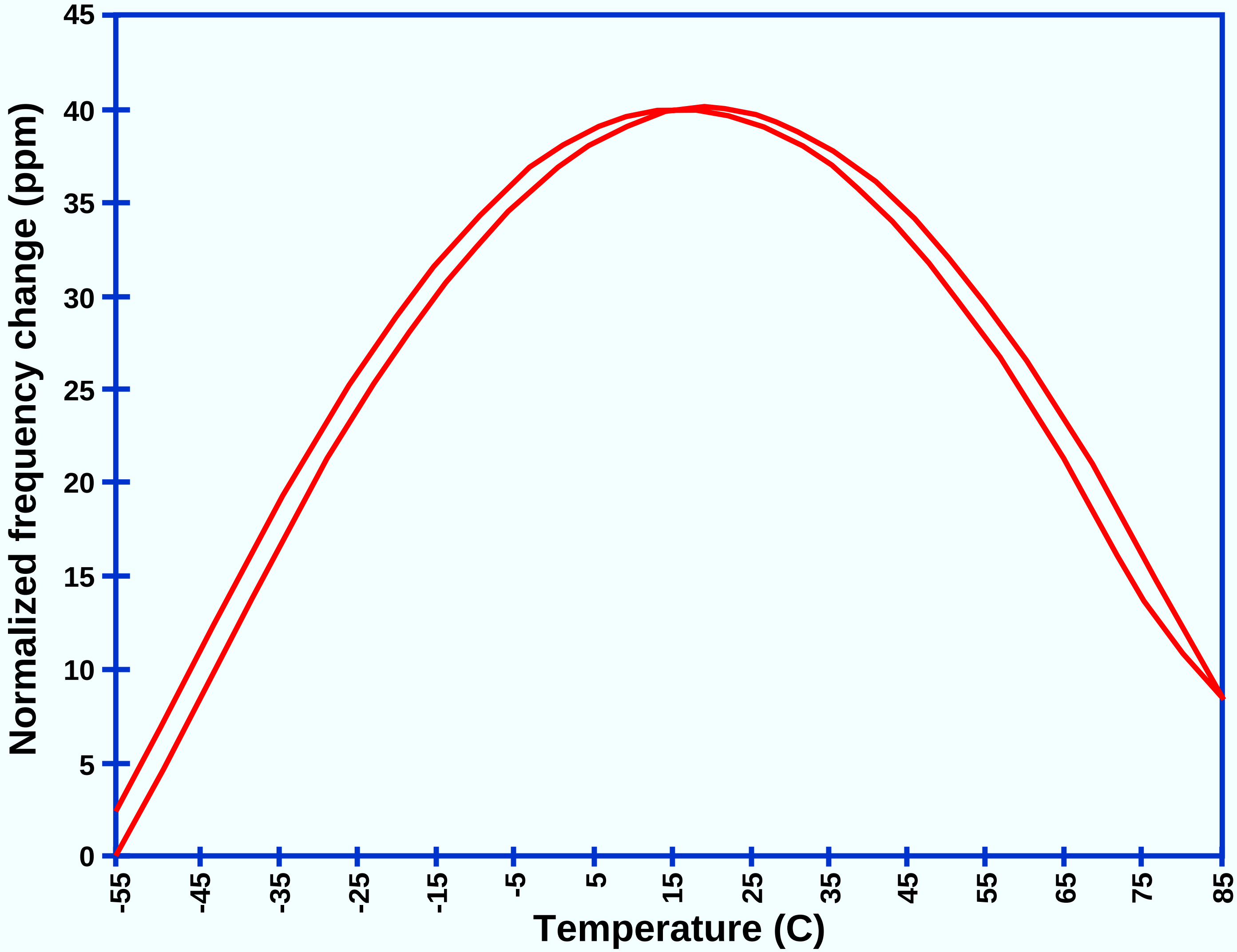
Warmup of AT- and SC-cut Resonators



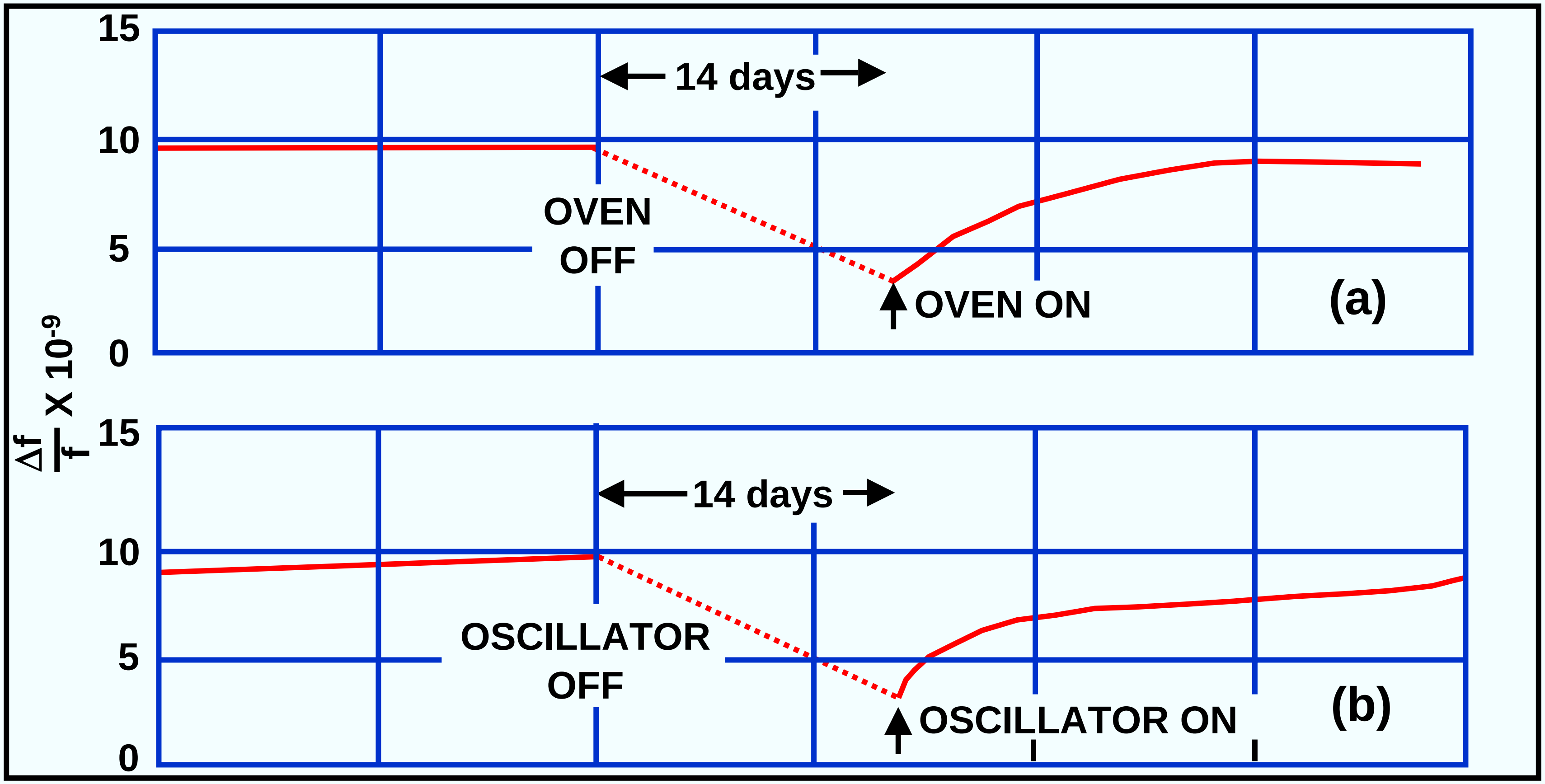
TCXO Thermal Hysteresis



Apparent Hysteresis

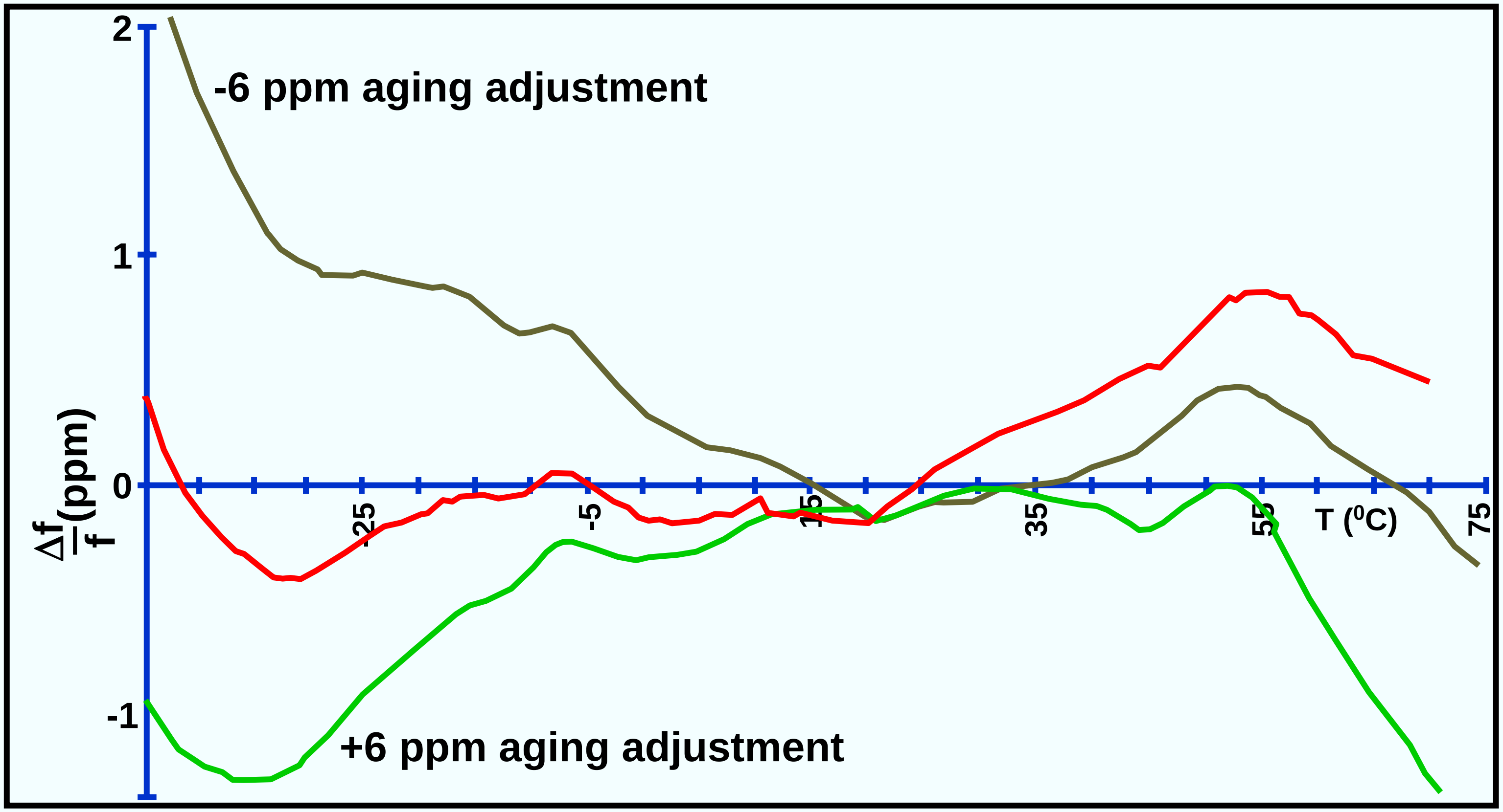


OCXO Retrace



In (a), the oscillator was kept on continuously while the oven was cycled off and on. In (b), the oven was kept on continuously while the oscillator was cycled off and on.

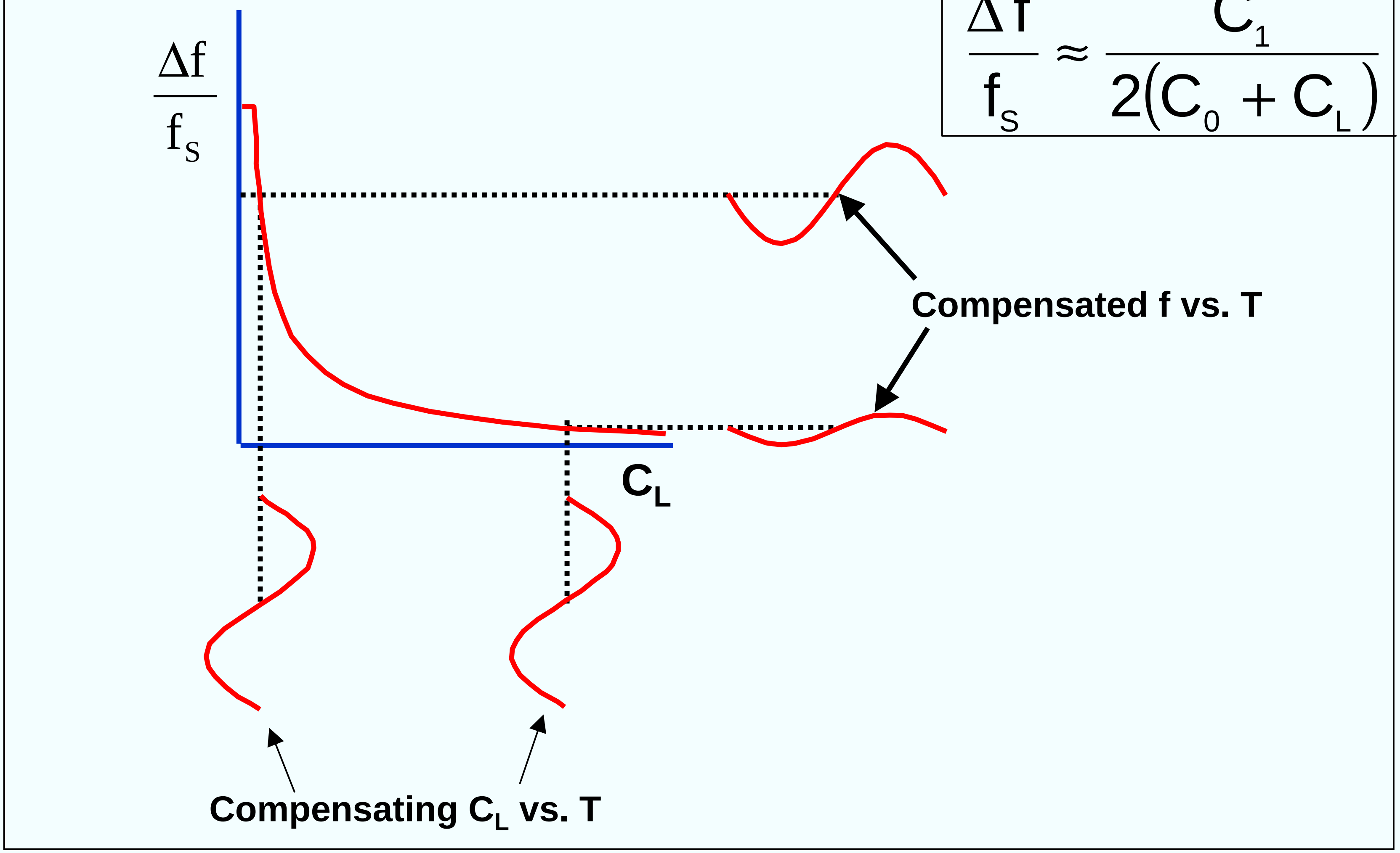
TCXO Trim Effect



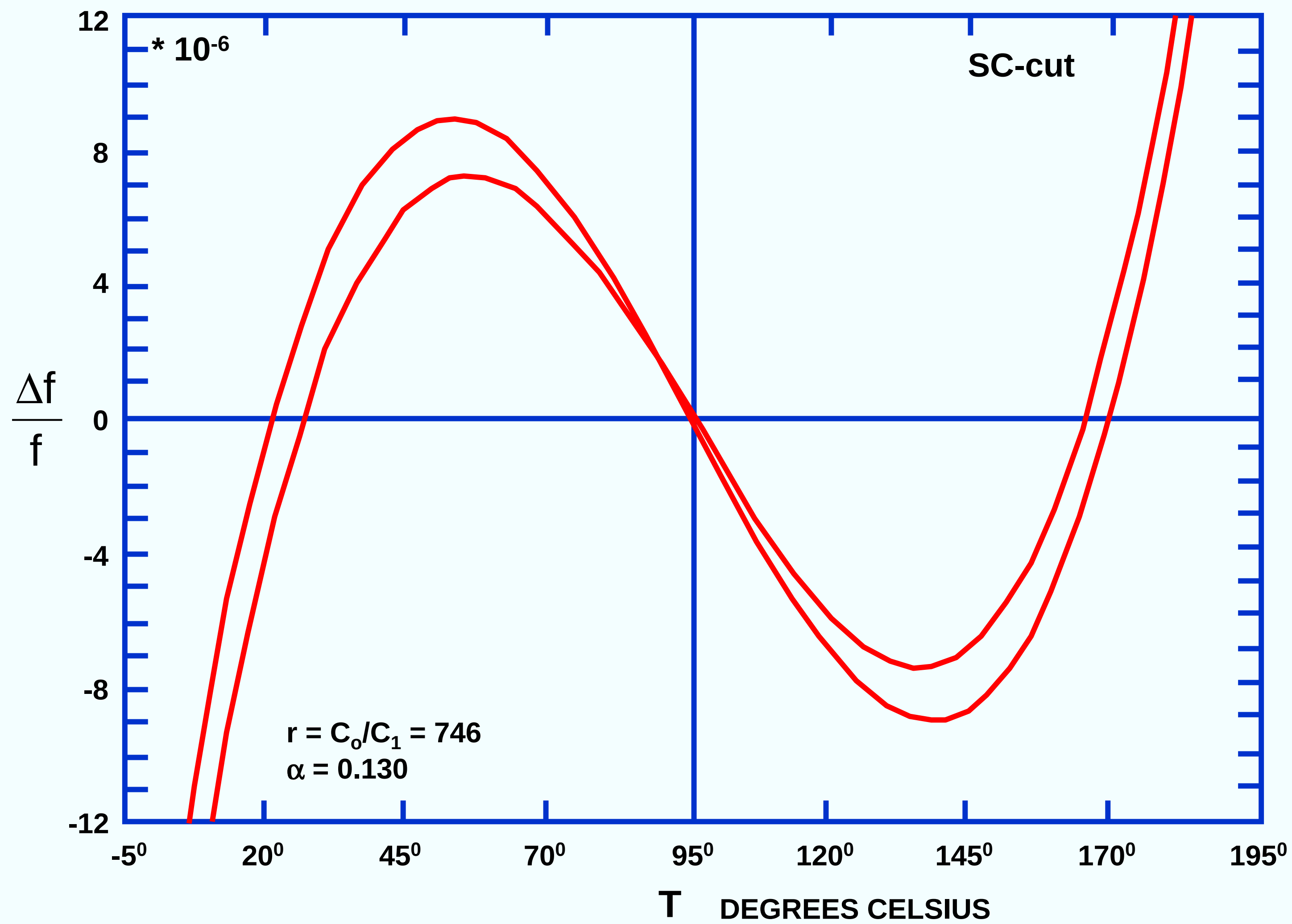
In TCXO's, temperature sensitive reactances are used to compensate for f vs. T variations. A variable reactance is also used to compensate for TCXO aging. The effect of the adjustment for aging on f vs. T stability is the "trim effect". Curves show f vs. T stability of a "0.5 ppm TCXO," at zero trim and at ±6 ppm trim. (Curves have been vertically displaced for clarity.)

Why the Trim Effect?

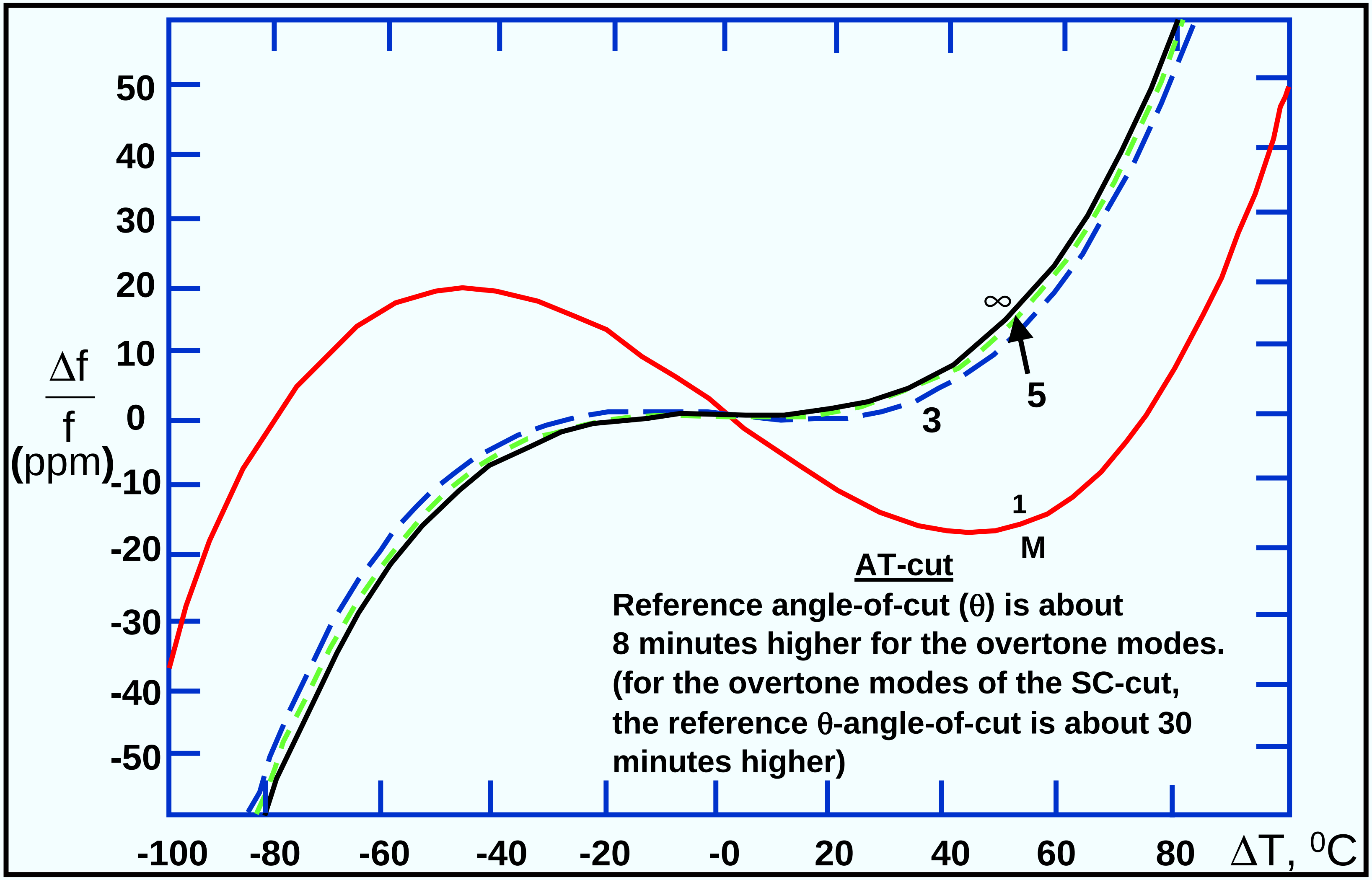
$$\frac{\Delta f}{f_S} \approx \frac{C_1}{2\left(C_0 + C_L\right)}$$



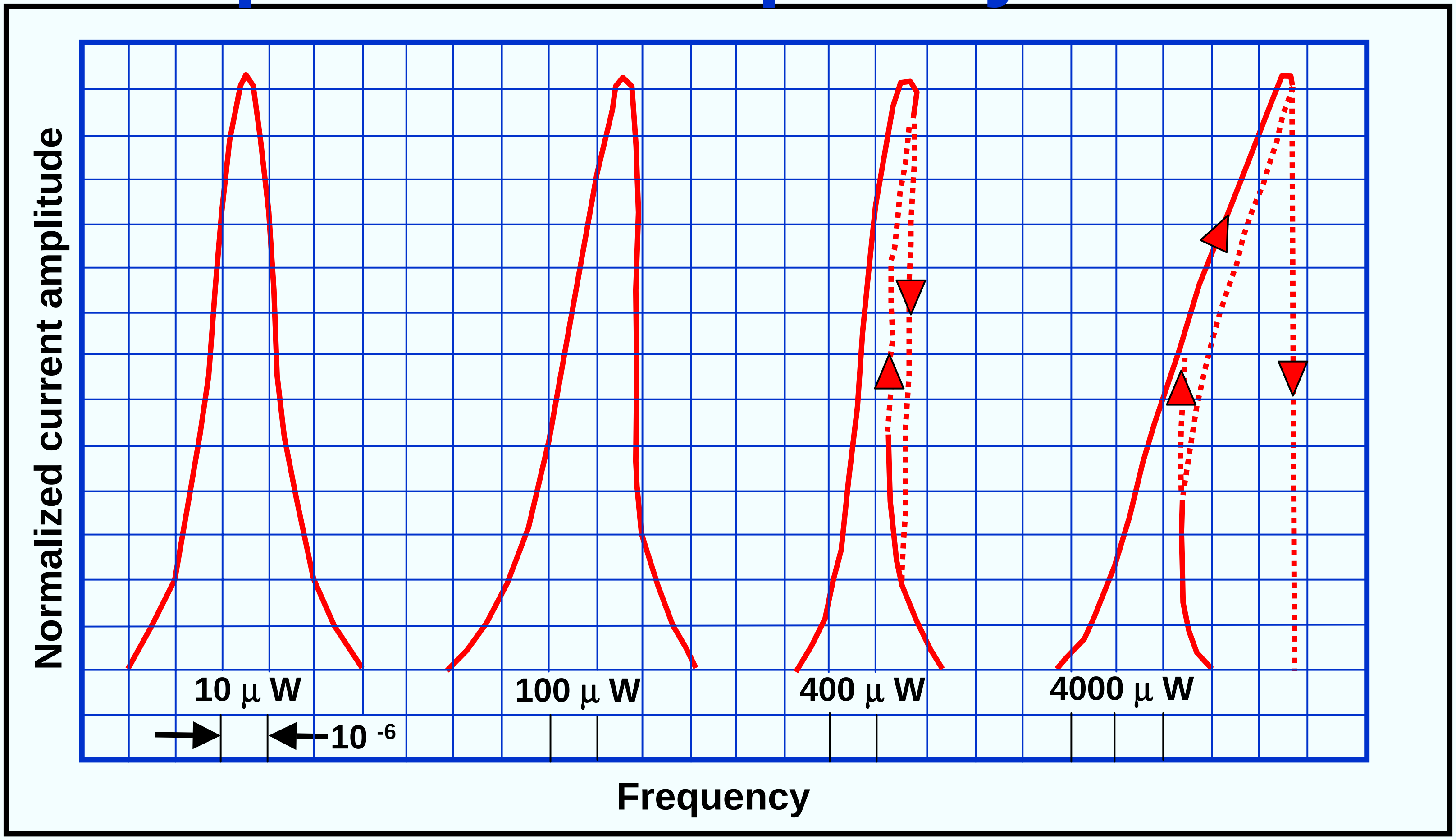
Effects of Load Capacitance on f vs. T



Effects of Harmonics on f vs. T

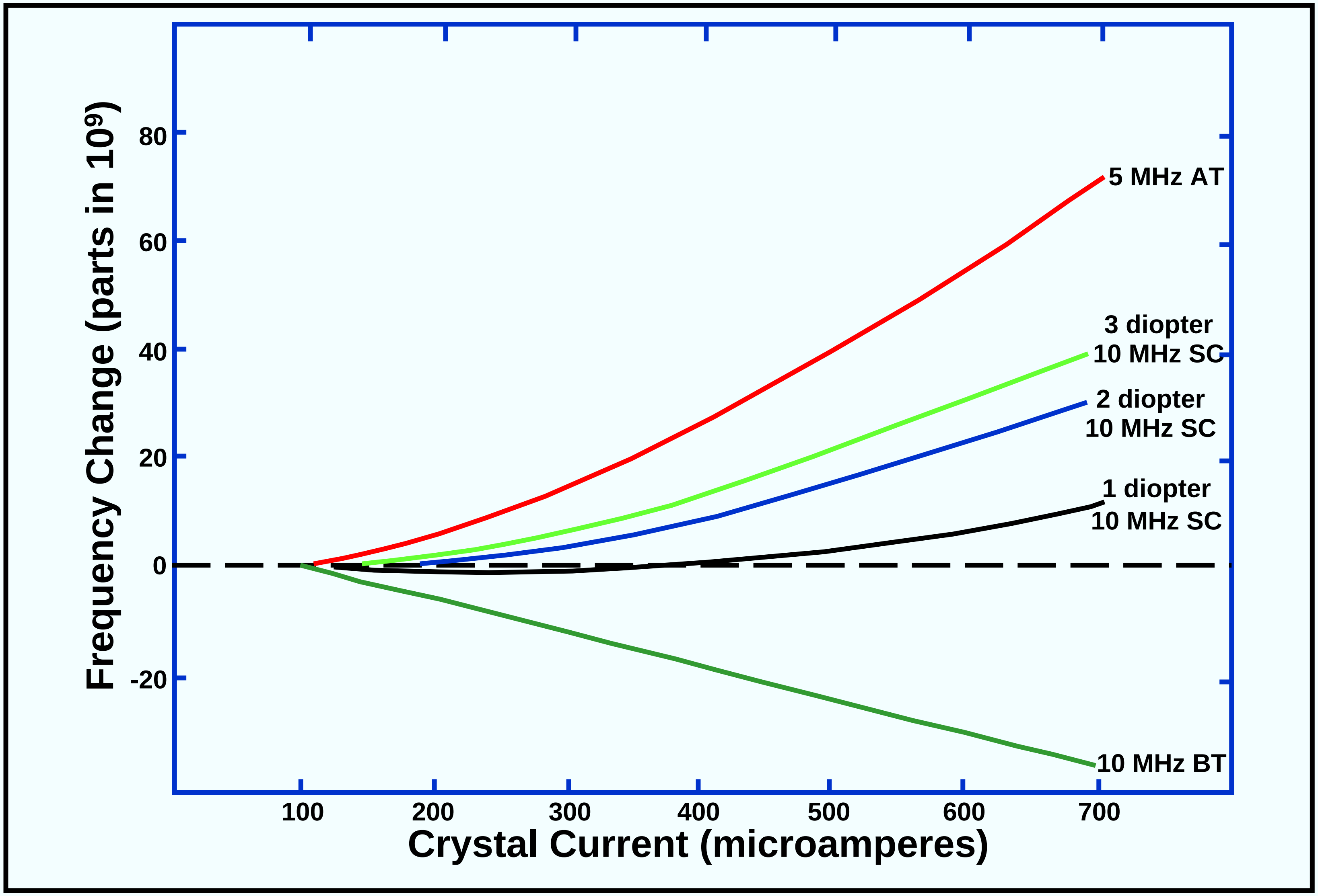


Amplitude - Frequency Effect

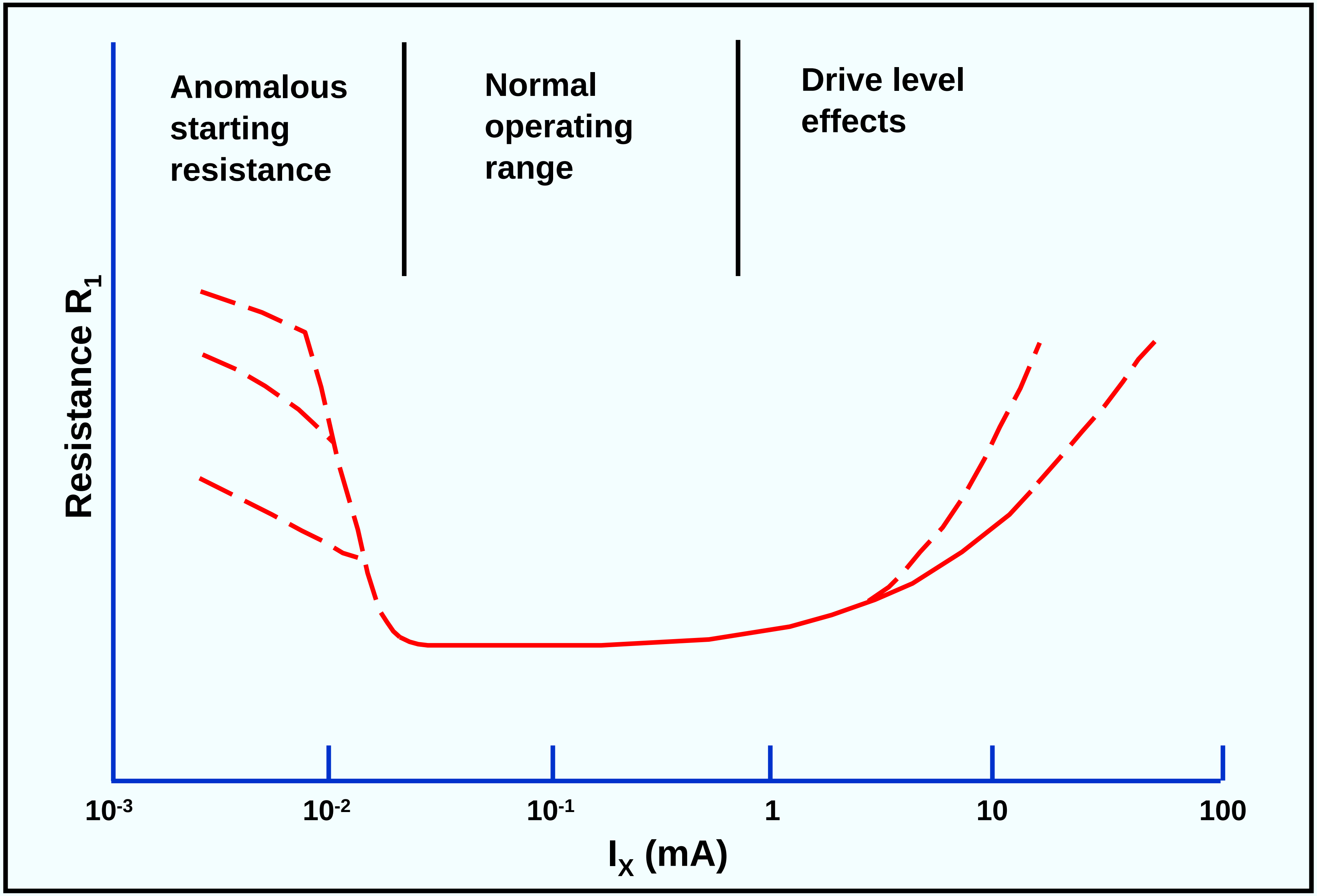


At high drive levels, resonance curves become asymmetric due to the nonlinearities of quartz.

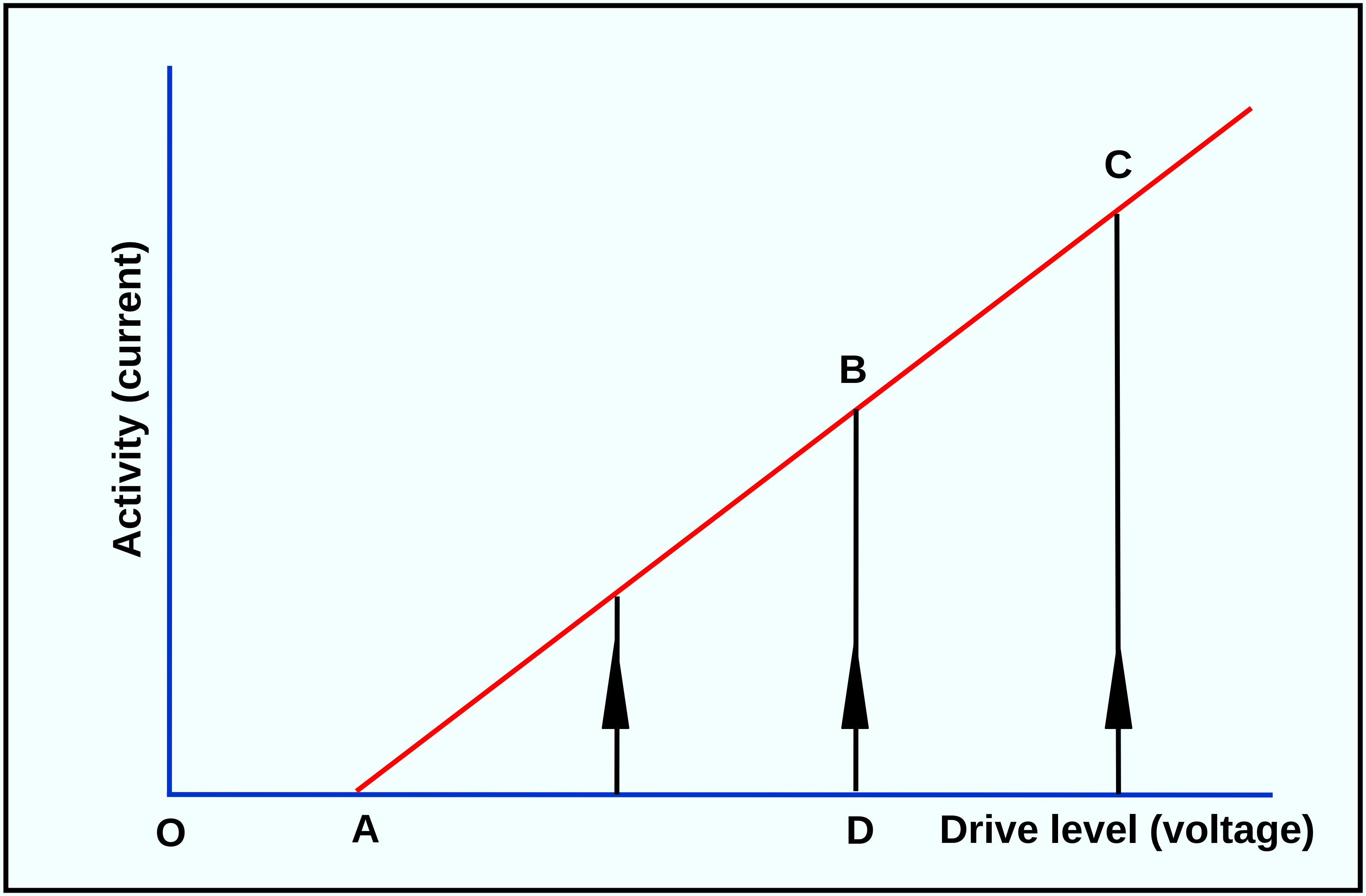
Frequency vs. Drive Level



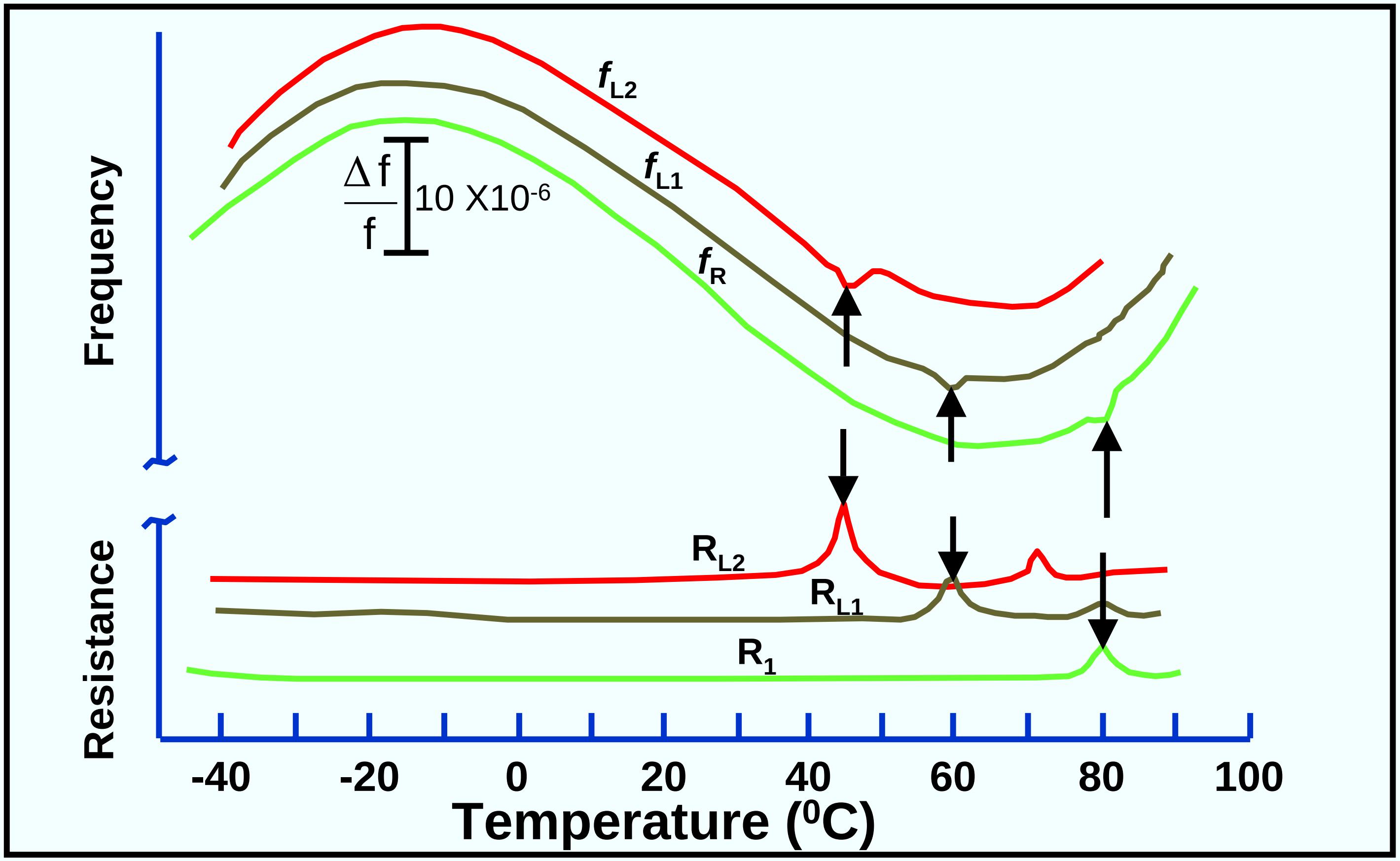
Drive Level vs. Resistance



Second Level of Drive Effect

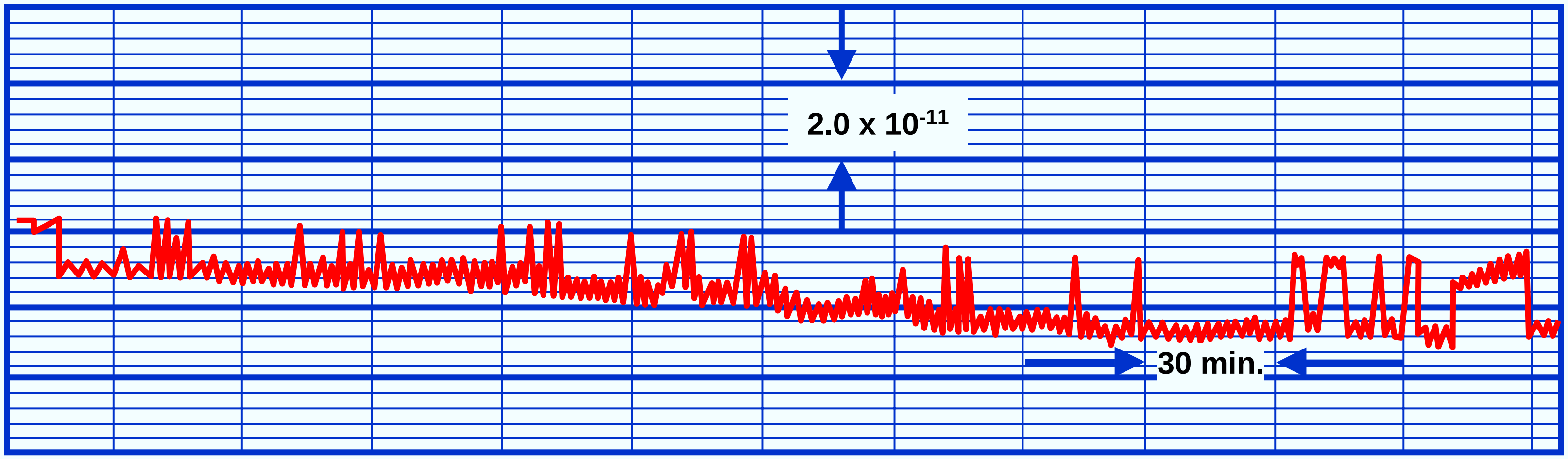


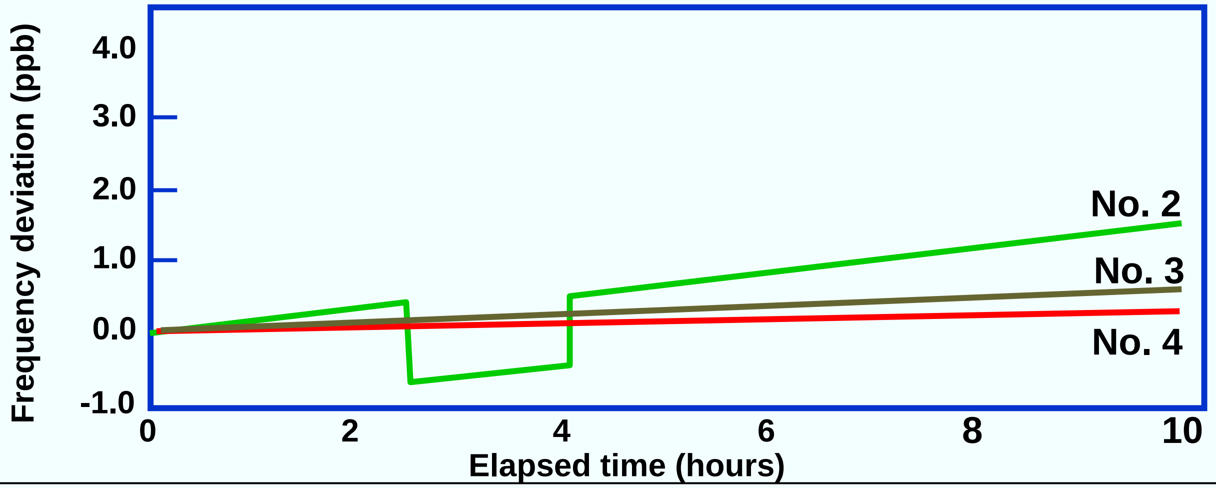
Activity Dips



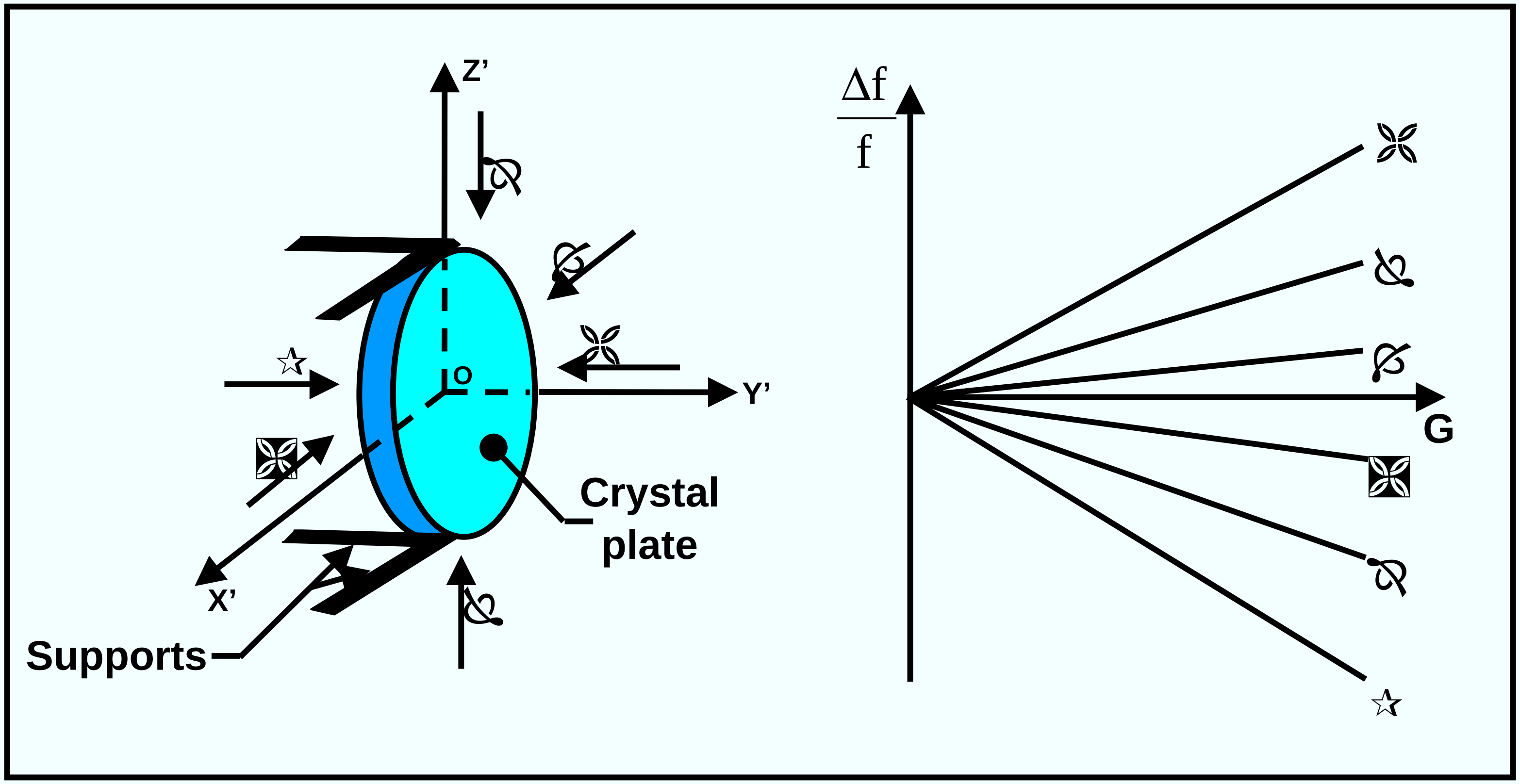
Activity dips in the f vs. T and R vs. T when operated with and without load capacitors. Dip temperatures are a function of C_L, which indicates that the dip is caused by a mode (probably flexure) with a large negative temperature coefficient.

Frequency Jumps



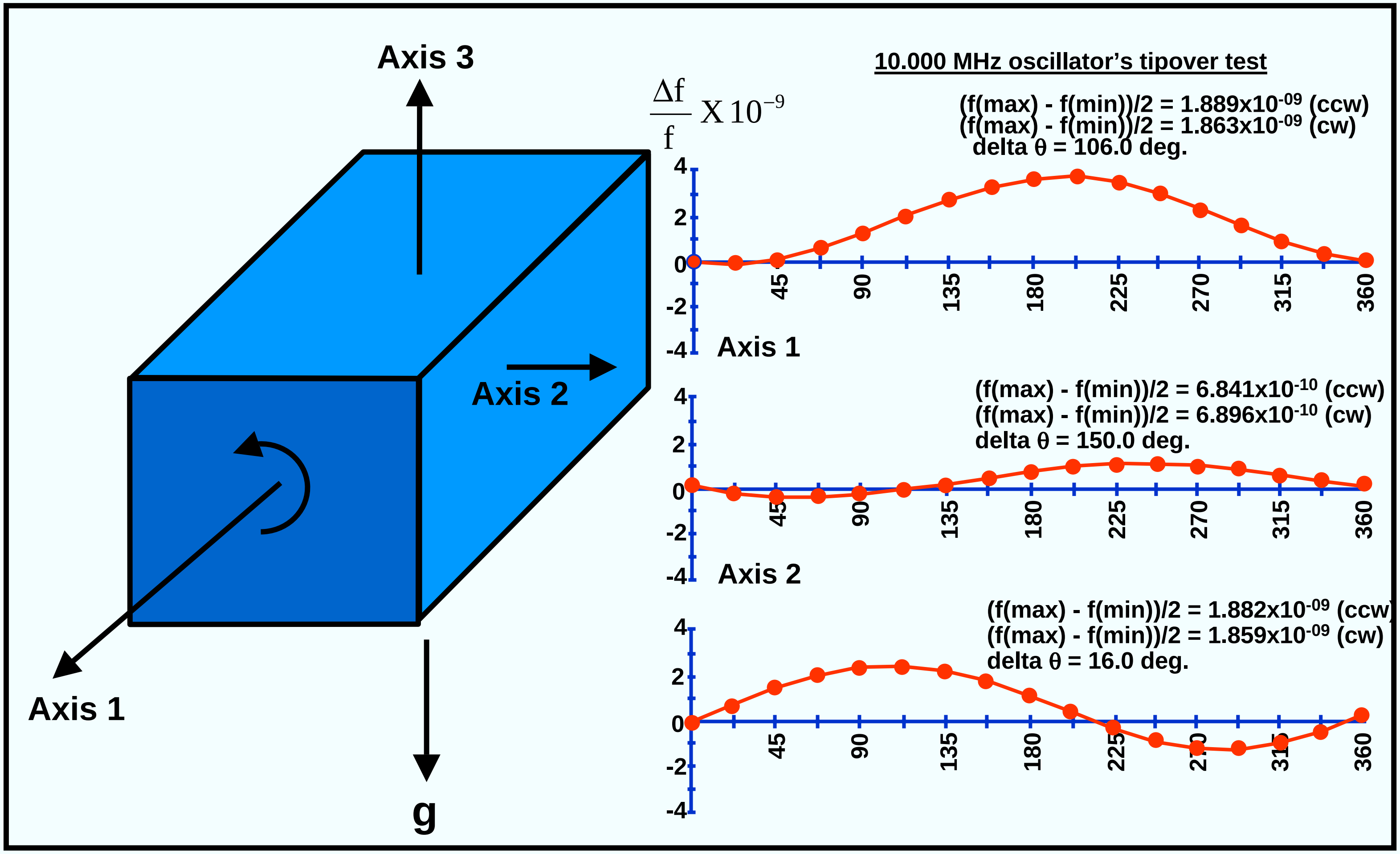


Acceleration vs. Frequency Change

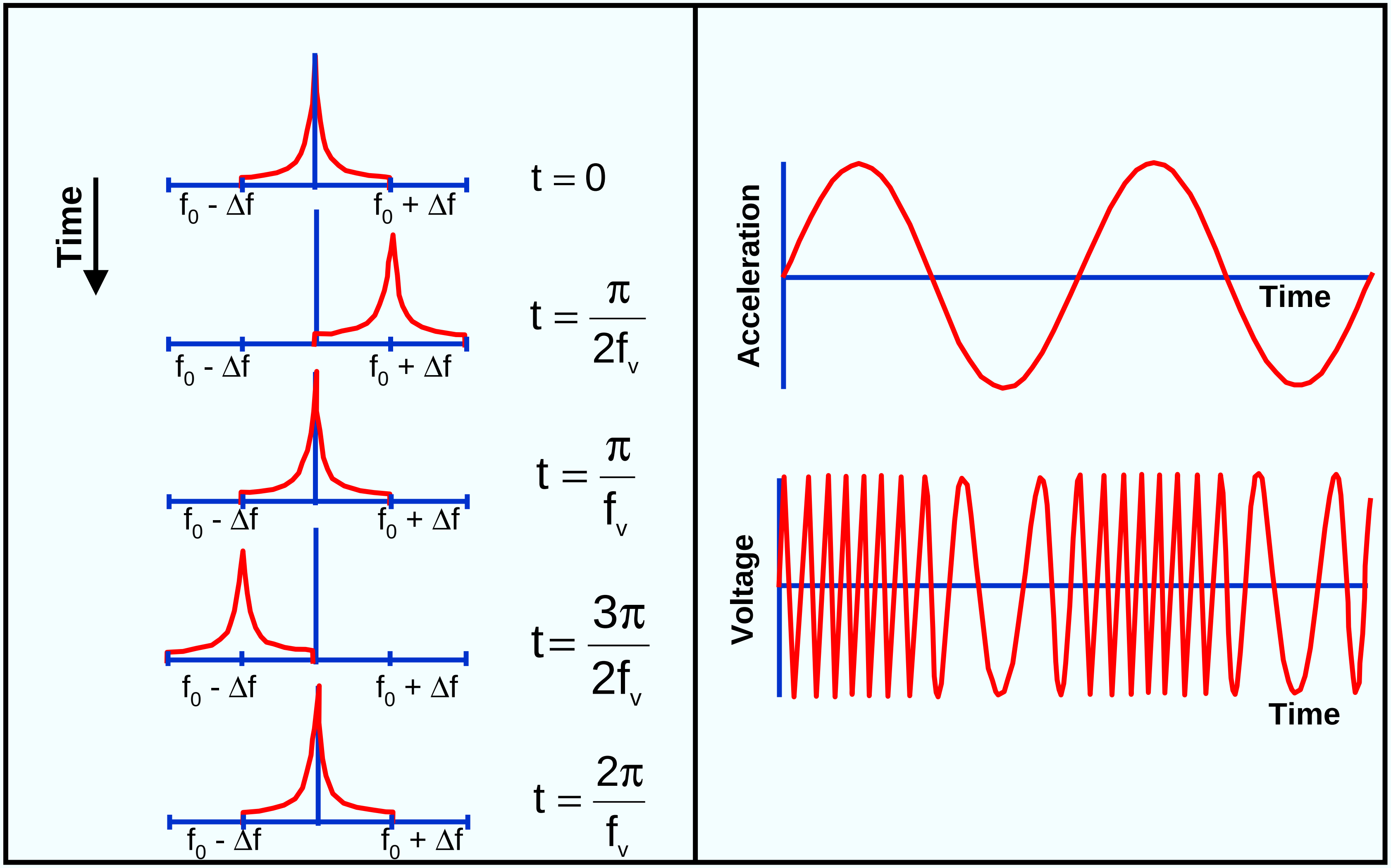


Frequency shift is a function of the magnitude and direction of the acceleration, and is usually linear with magnitude up to at least 50 g's.

2-g Tipover Test
(Δf vs. attitude about three axes)



Sinusoidal Vibration Modulated Frequency



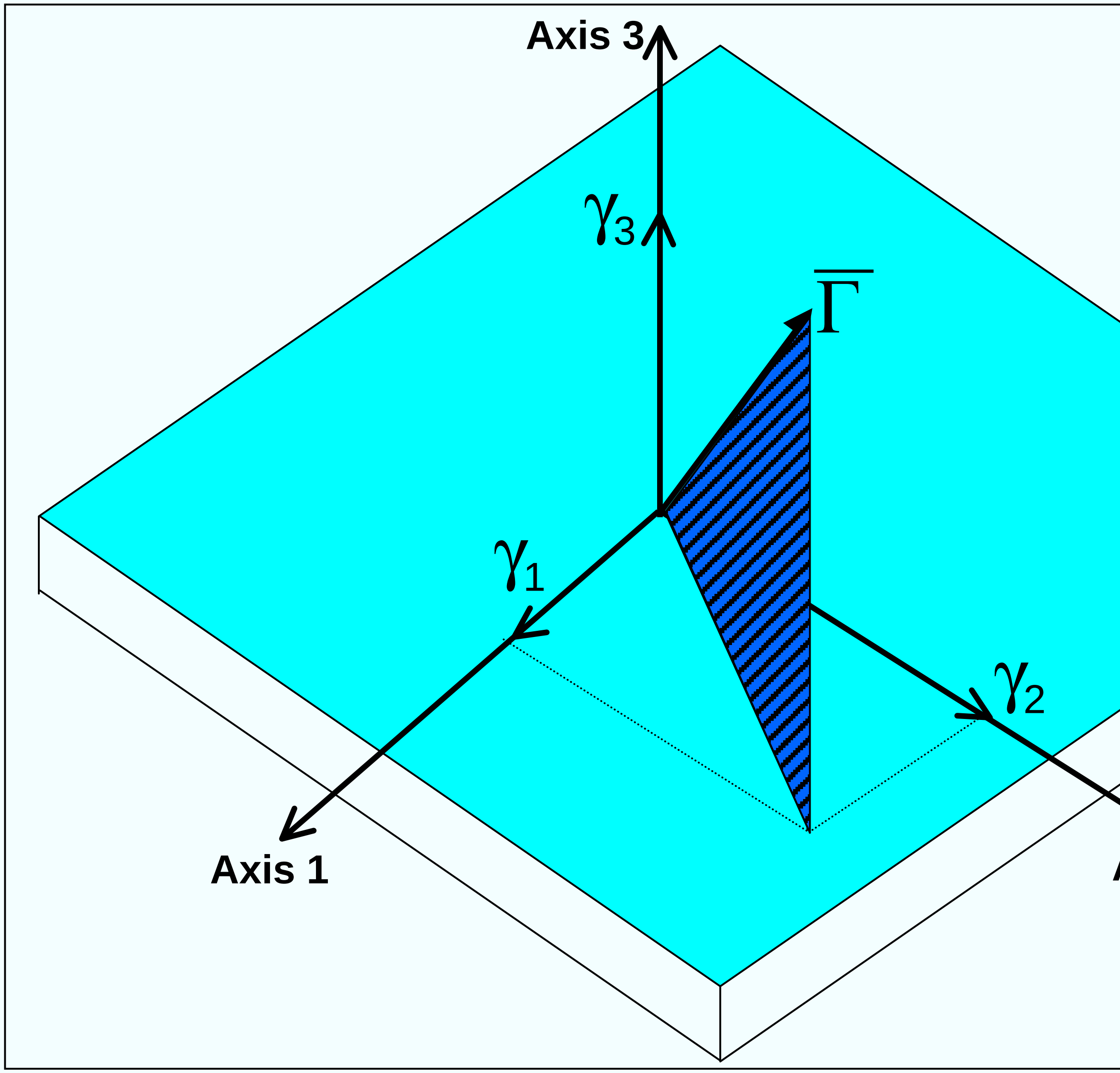
Acceleration Levels and Effects

Environment	Acceleration typical levels*, in g's	Δf $\times 10^{-11}$, for 1×10^{-9}/g oscillator
Buildings**, quiesent	0.02 rms	2
Tractor-trailer (3-80 Hz)	0.2 peak	20
Armored personnel carrier	0.5 to 3 rms	50 to 300
Ship - calm seas	0.02 to 0.1 peak	2 to 10
Ship - rough seas	0.8 peak	80
Propeller aircraft	0.3 to 5 rms	30 to 500
Helicopter	0.1 to 7 rms	10 to 700
Jet aircraft	0.02 to 2 rms	2 to 200
Missile - boost phase	15 peak	1,500
Railroads	0.1 to 1 peak	10 to 100

* Levels at the oscillator depend on how and where the oscillator is mounted
Platform resonances can greatly amplify the acceleration levels.

** Building vibrations can have significant effects on noise measurements

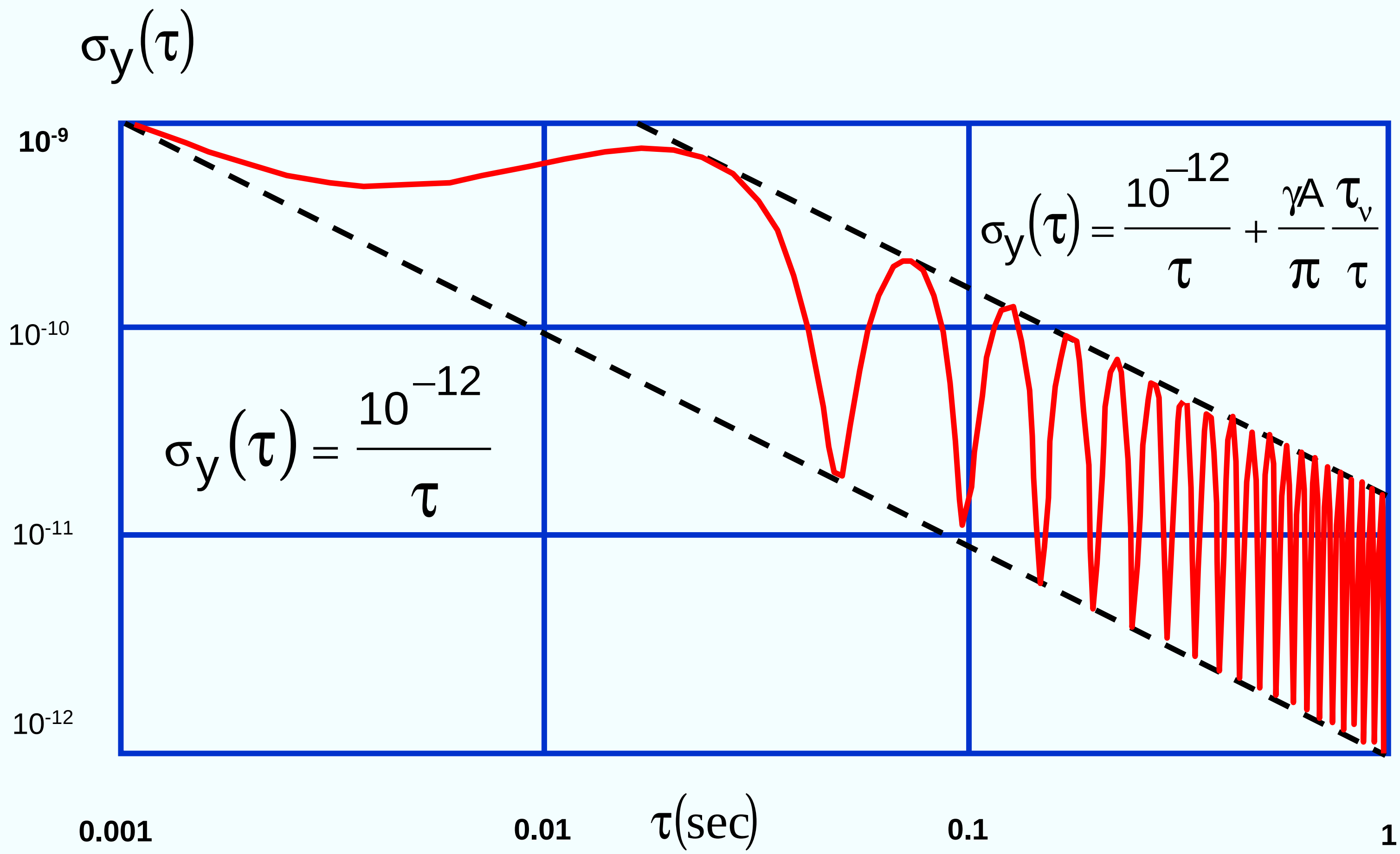
Acceleration Sensitivity Vector



$$\overline{\Gamma} = \gamma_1 \hat{i} + \gamma_2 \hat{j} + \gamma_3 \hat{k}$$

$$\Gamma = \sqrt{\gamma_1^2 + \gamma_2^2 + \gamma_3^2}$$

Vibration-Induced Allan Deviation Degradation



Example shown: f_v = 20, Hz A = 1.0 g along Γ, $|\Gamma|$ = 1 x 10^{-9}/g

Vibration-Induced Phase Excursion

The phase of a vibration modulated signal is

$$\varphi(t) = 2\pi f_0 t + \left(\frac{\Delta f}{f_v}\right)\sin(2\pi f_v t)$$

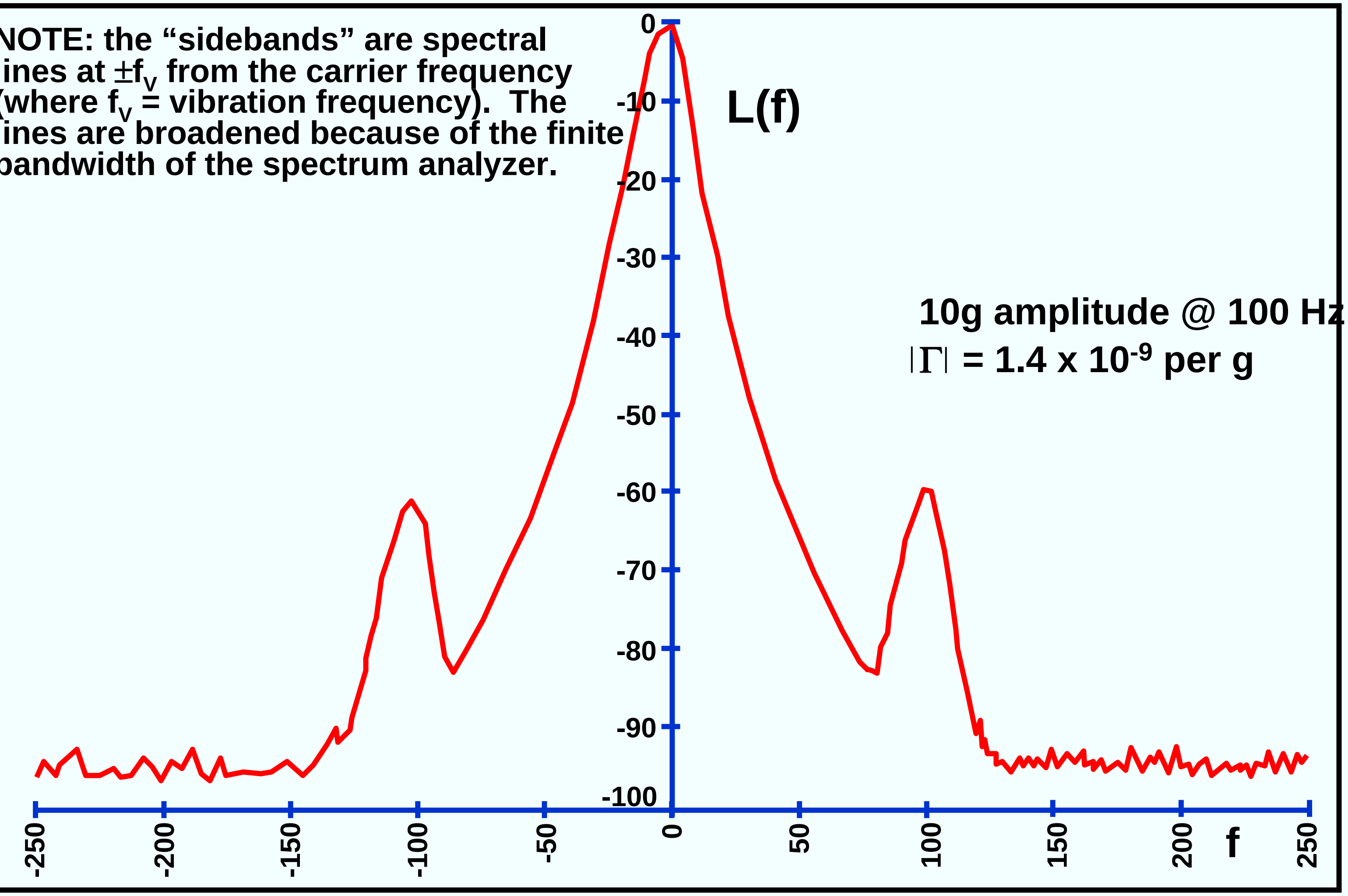
When the oscillator is subjected to a sinusoidal vibration, the peak phase excursion is

$$\Delta\varphi_{peak} = \frac{\Delta f}{f_v} = \frac{\left(\overline{\Gamma} \bullet \overline{A}\right) f_0}{f_v}$$

<u>Example</u>: if a 10 MHz, 1 x 10^{-9}/g oscillator is subjected to a 10 Hz sinusoidal vibration of amplitude 1g, the peak vibration-induced phase excursion is 1 x 10^{-3} radian. If this oscillator is used as the reference oscillator in a 10 GHz radar system, the peak phase excursion at 10GHz will be 1 radian. Such a large phase excursion can be catastrophic to the performance of many systems, such as those which employ phase locked loops (PLL) or phase shift keying (PSK).

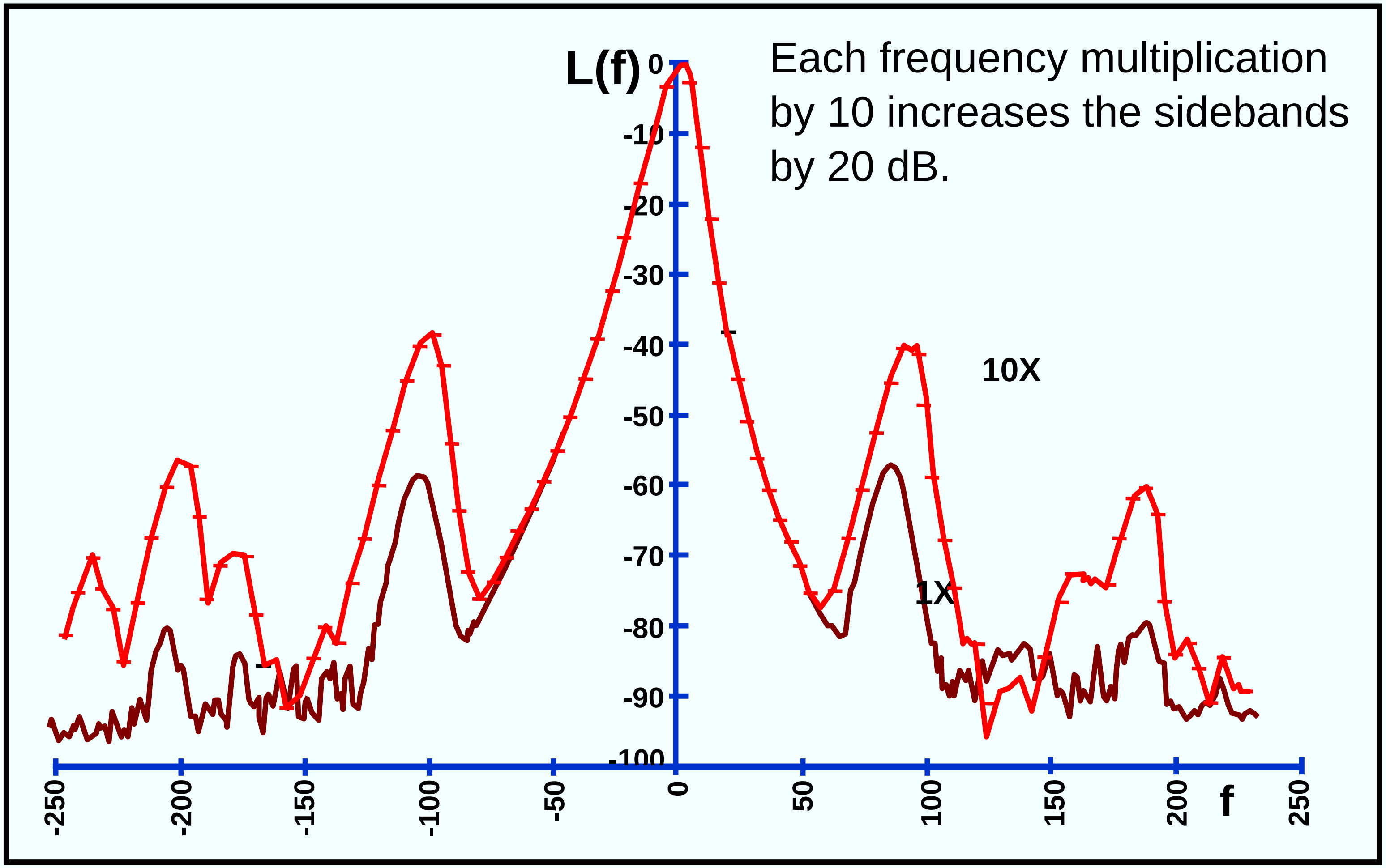
Vibration-Induced Sidebands

NOTE: the "sidebands" are spectral lines at $\pm f_V$ from the carrier frequency (where f_V = vibration frequency). The lines are broadened because of the finite bandwidth of the spectrum analyzer.



Vibration-Induced Sidebands

After Frequency Multiplication



Sine Vibration-Induced Phase Noise

Sinusoidal vibration produces spectral lines at $\pm f_v$ from the carrier, where f_v is the vibration frequency.

$$L'(f_v) = 20\log\left(\frac{\overline{\Gamma}\bullet\overline{A}f_0}{2f_v}\right)$$

e.g., if $|\overline{\Gamma}|$ = 1 x 10^{-9}/g and f_0 = 10 MHz, then even if the oscillator is completely **noise free at rest**, the phase "noise" i.e., the spectral lines, due solely to a sine vibration level of 1g will be;

Vibr. freq., f_v, in Hz	L'**(f_v)**, in dBc
1	-46
10	-66
100	-86
1,000	-106
10,000	-126

Random Vibration-Induced Phase Noise

Random vibration's contribution to phase noise is given by:

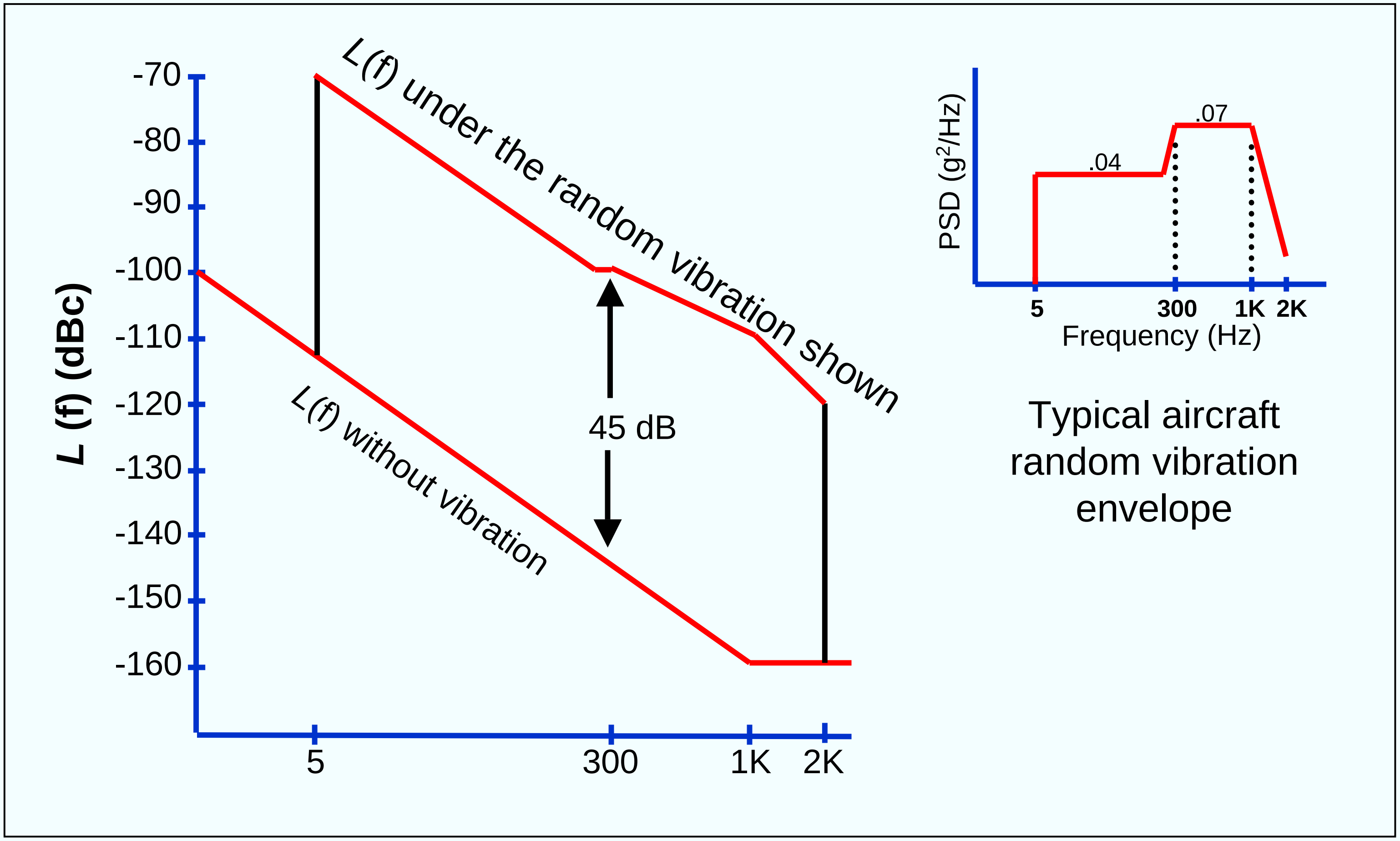
$$\mathcal{L}(f) = 20 \log\left(\frac{\bar{\Gamma} \bullet \bar{A} f_0}{2f}\right), \quad \text{where } |\bar{A}| = [(2)(PSD)]^{1/2}$$

e.g., if $|\bar{\Gamma}|$ = 1 x 10**$^{-9}$**/g and f_0 = 10 MHz, then even if the oscillator is completely **noise free at rest**, the phase "noise" i.e., the spectral lines, due solely to a vibration PSD = 0.1 g^2/Hz will be:

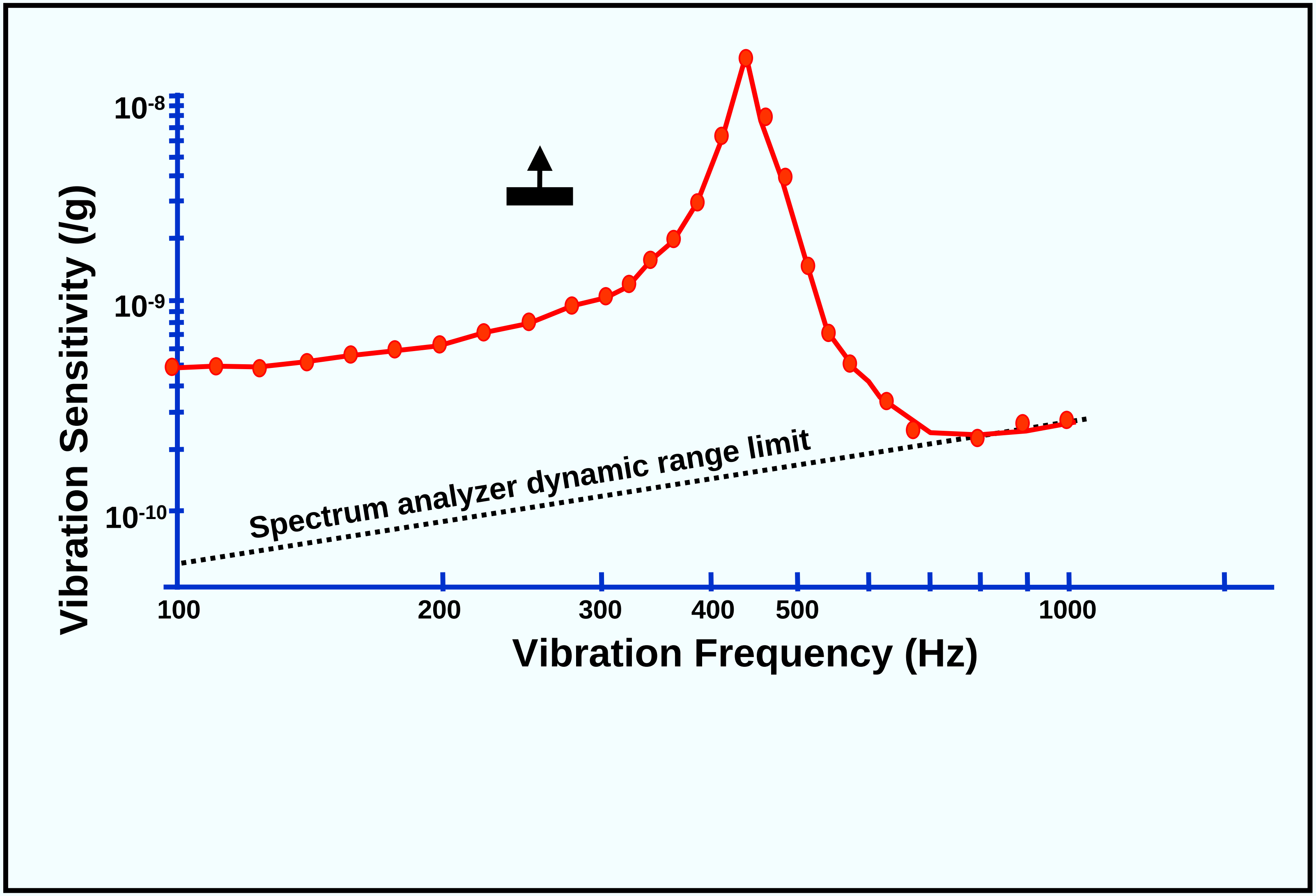
Offset freq., f, in Hz	L'**(f)**, in dBc/Hz
1	-53
10	-73
100	-93
1,000	-113
10,000	-133

Random-Vibration-Induced Phase Noise

Phase noise under vibration is for $\Gamma = 1 \times 10^{-9}$ per g and f = 10 MHz



Acceleration Sensitivity vs. Vibration Frequency



Acceleration Sensitivity of Quartz Resonators

Resonator acceleration sensitivities range from the low parts in 10^{10} per g for the best commercially available SC-cuts, to parts in 10^{7} per g for tuning-fork-type watch crystals. When a wide range of resonators were examined: AT, BT, FC, IT, SC, AK, and GT-cuts; 5 MHz 5th overtones to 500 MHz fundamental mode inverted mesa resonators; resonators made of natural quartz, cultured quartz, and swept cultured quartz; numerous geometries and mounting configurations (including rectangular AT-cuts); nearly all of the results were within a factor of three of $1\text{x}10^{-9}$ per g. On the other hand, the fact that a few resonators have been found to have sensitivities of less than $1\text{x}10^{-10}$ per g indicates that the observed acceleration sensitivities are not due to any inherent natural limitations.

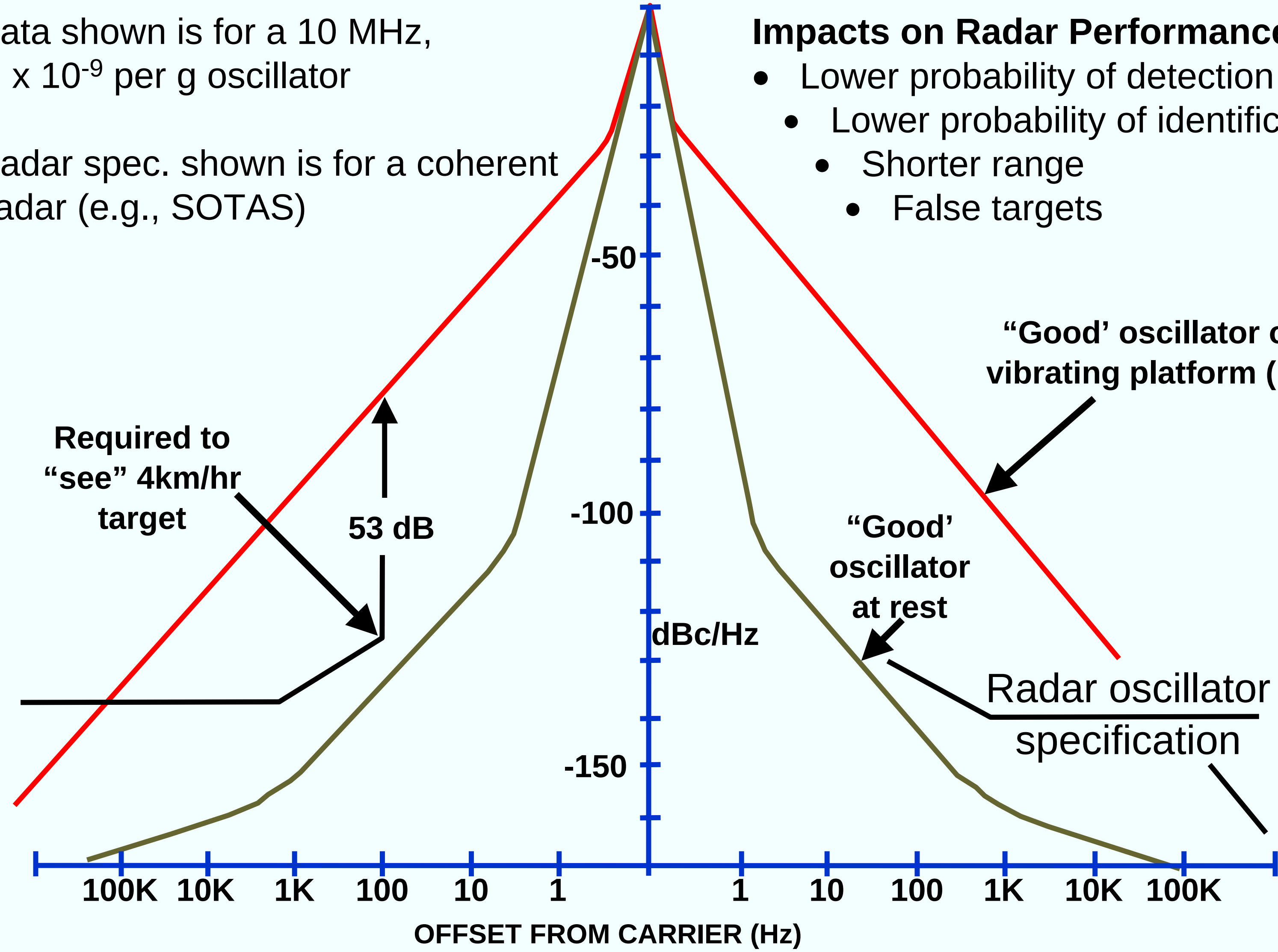
Theoretical and experimental evidence indicates that the major variables yet to be controlled properly are the mode shape and location (i.e., the amplitude of vibration distribution), and the strain distribution associated with the mode of vibration. Theoretically, when the mounting is completely symmetrical with respect to the mode shape, the acceleration sensitivity can be zero, but tiny changes from this ideal condition can cause a significant sensitivity. Until the acceleration sensitivity problem is solved, acceleration compensation and vibration isolation can provide lower than $1\text{x}10^{-10}$ per g, for a limited range of vibration frequencies, and at a cost.

Phase Noise Degradation Due to Vibration

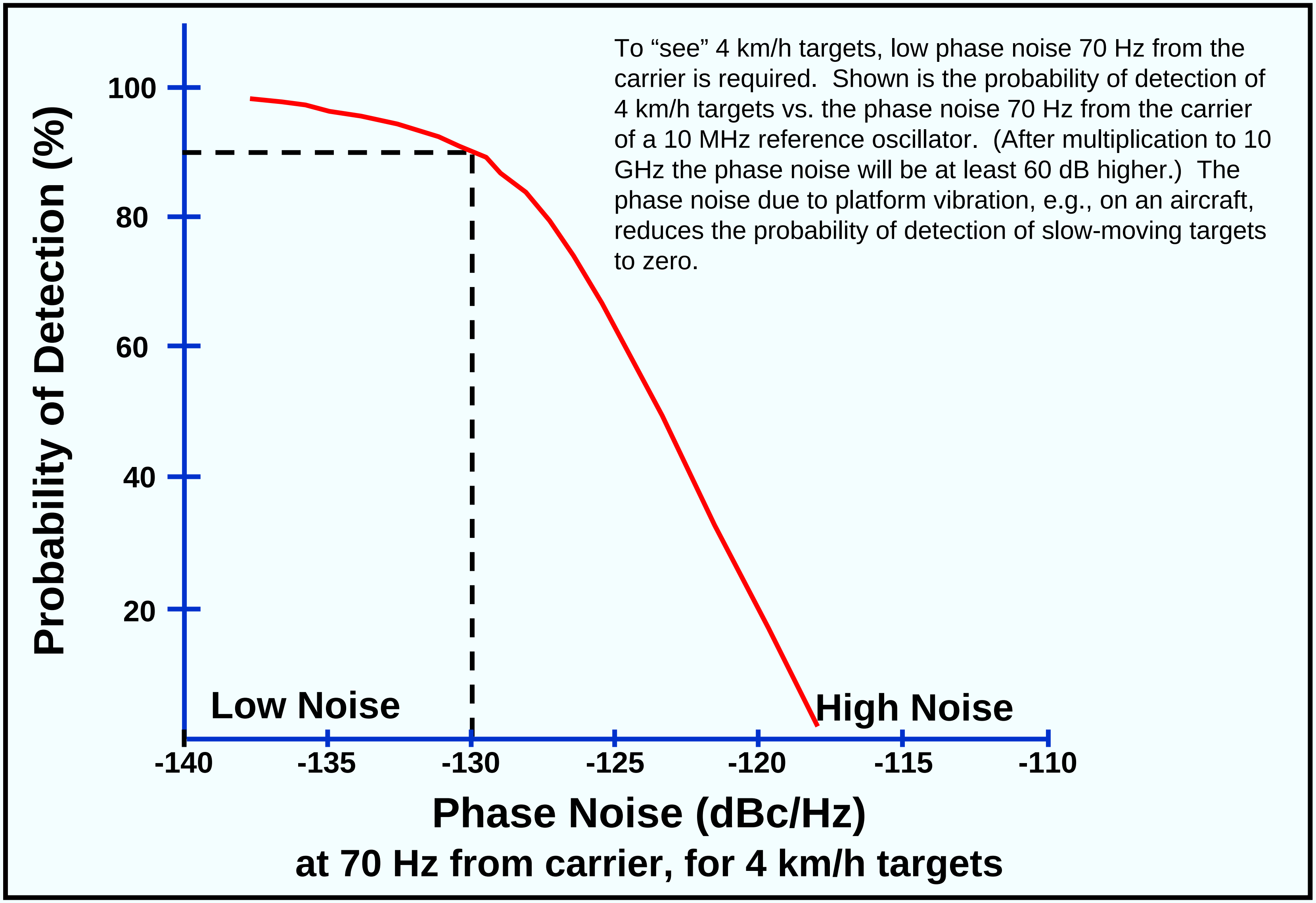
- Data shown is for a 10 MHz, 2×10^{-9} per g oscillator
- Radar spec. shown is for a coherent radar (e.g., SOTAS)

Impacts on Radar Performance

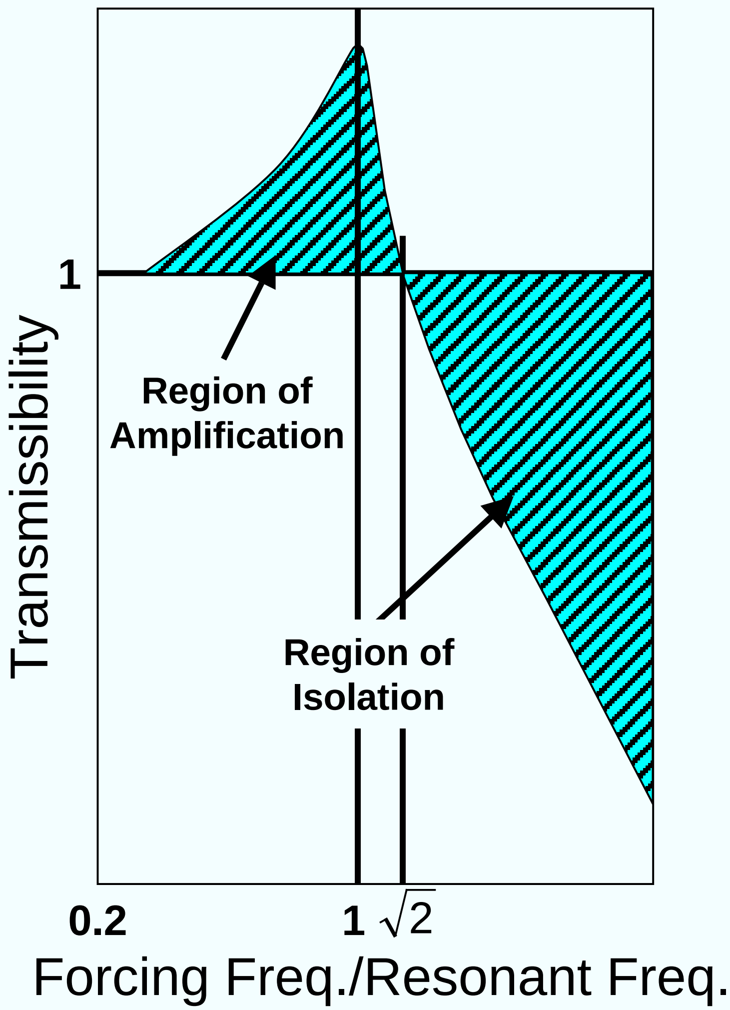
- Lower probability of detection
- Lower probability of identification
- Shorter range
- False targets



Coherent Radar Probability of Detection



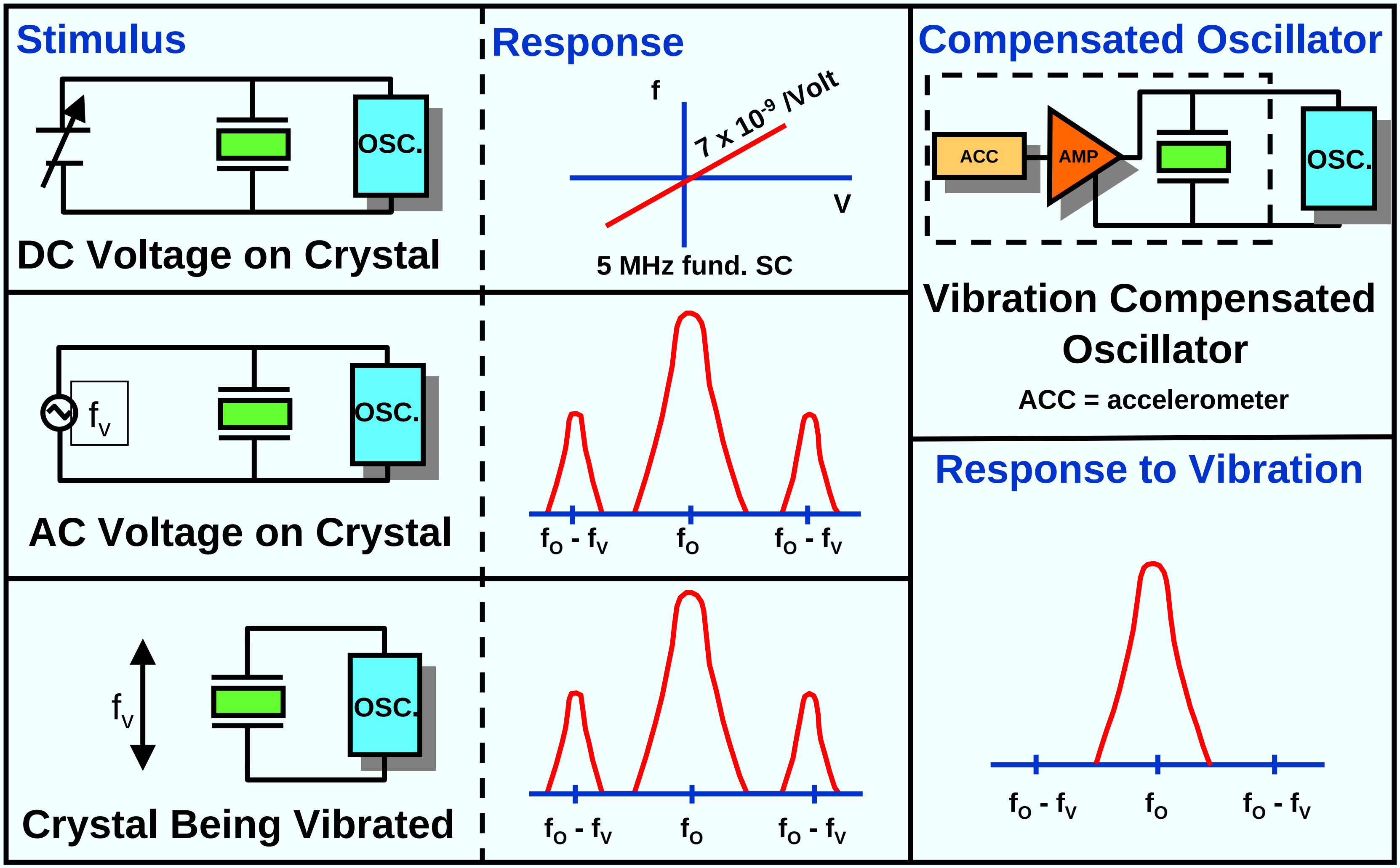
Vibration Isolation



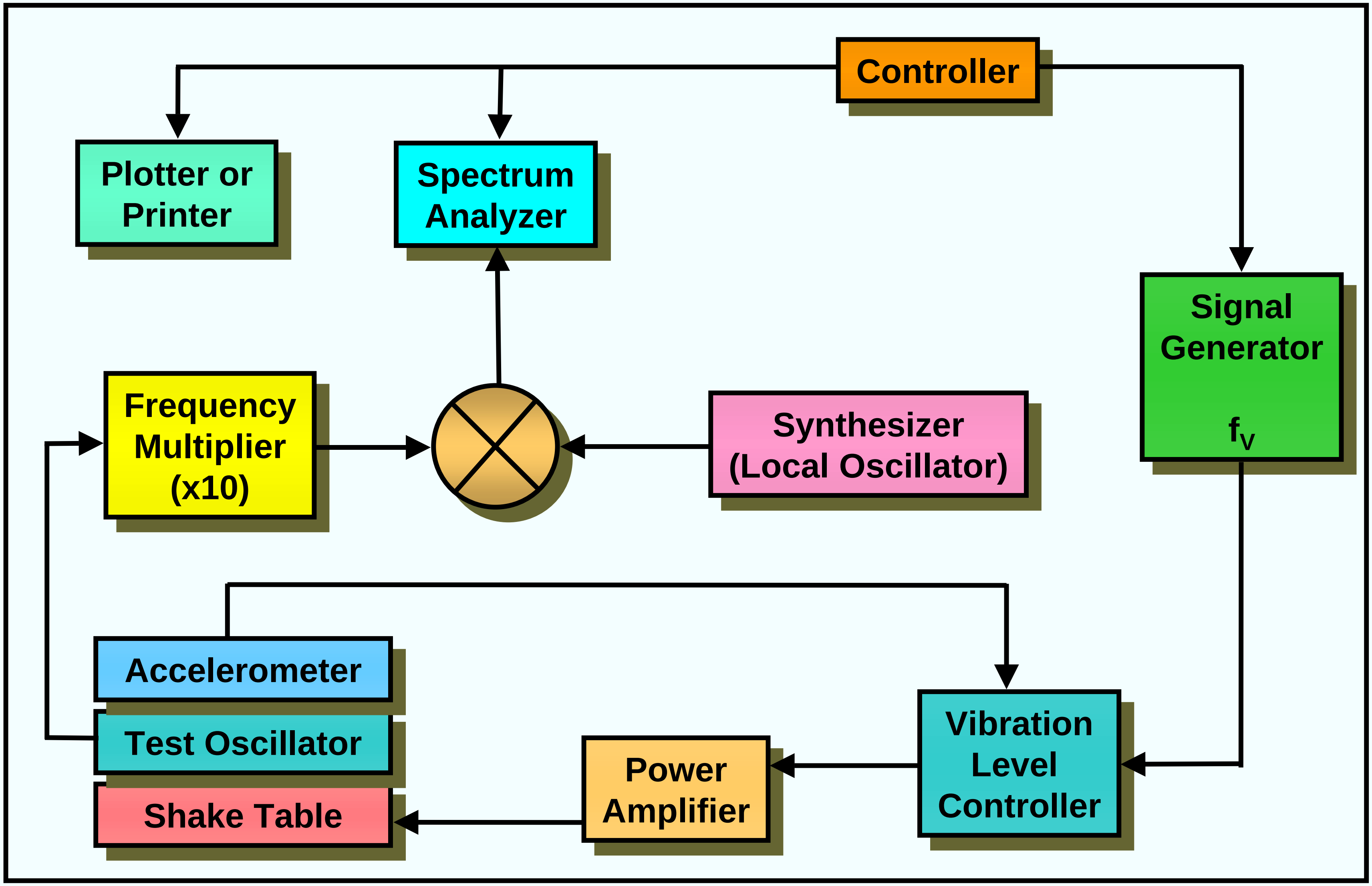
Limitations

- Poor at low frequencies
- Adds size, weight and cost
- Ineffective for acoustic noise

Vibration Compensation

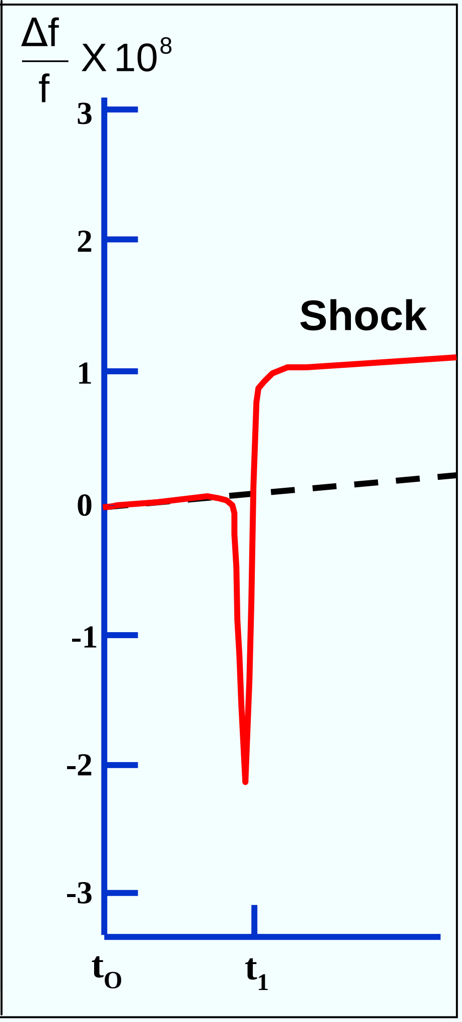


Vibration Sensitivity Measurement System

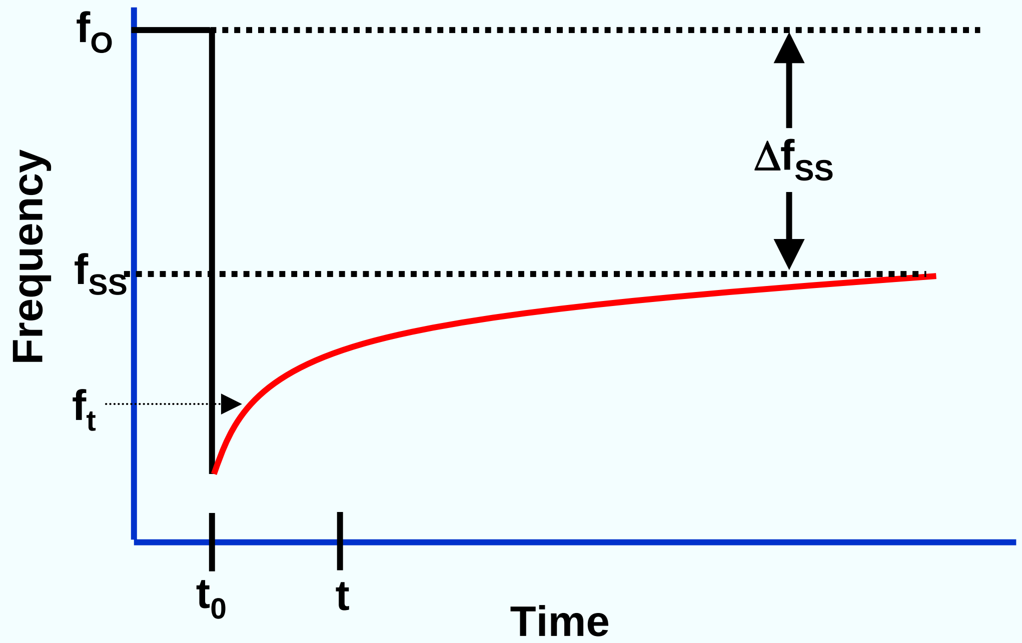


Shock

The frequency excursion during a shock is due to the resonator's stress sensitivity. The magnitude of the excursion is a function of resonator design, and of the shock induced stresses on the resonator (resonances in the mounting structure will amplify the stresses.) The permanent frequency offset can be due to: shock induced stress changes, the removal of (particulate) contamination from the resonator surfaces, and changes in the oscillator circuitry. Survival under shock is primarily a function of resonator surface imperfections. Chemical-polishing-produced scratch-free resonators have survived shocks up to 36,000 g in air gun tests, and have survived the shocks due to being fired from a 155 mm howitzer (16,000 g, 12 ms duration).



Radiation-Induced Frequency Shifts



f_O = original, preirradiation frequency

f_{SS} = steady-state frequency (0.2 to 24 hours after exposure)

Δf_{SS} = steady-state frequency offset

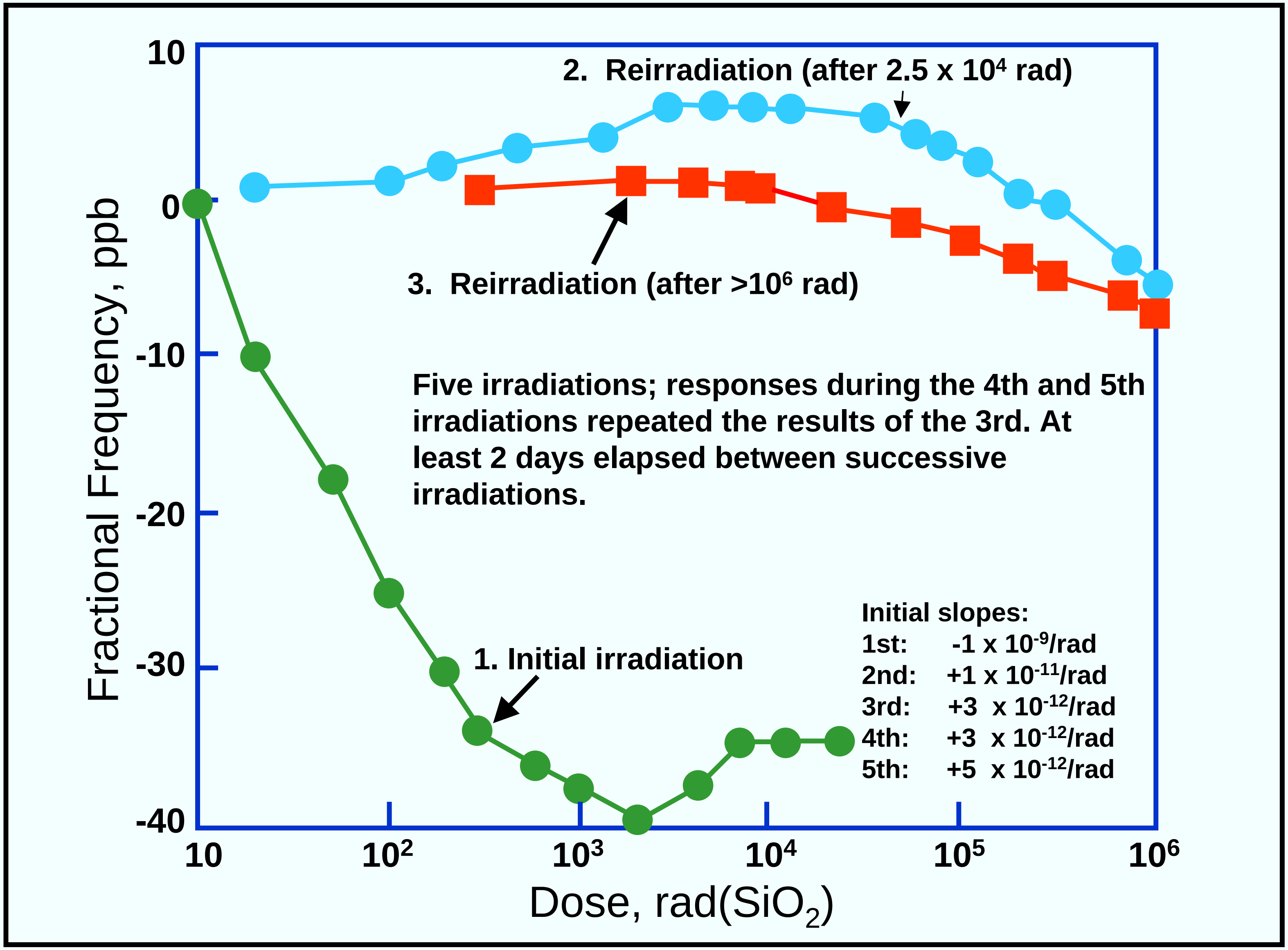
f_T = frequency at time t

Δf_{SS}/rad* =
- 10^{-11} for natural quartz (and R increase can stop the oscillation)
- 10^{-12} for cultured quartz
- 10^{-13} for swept cultured quartz

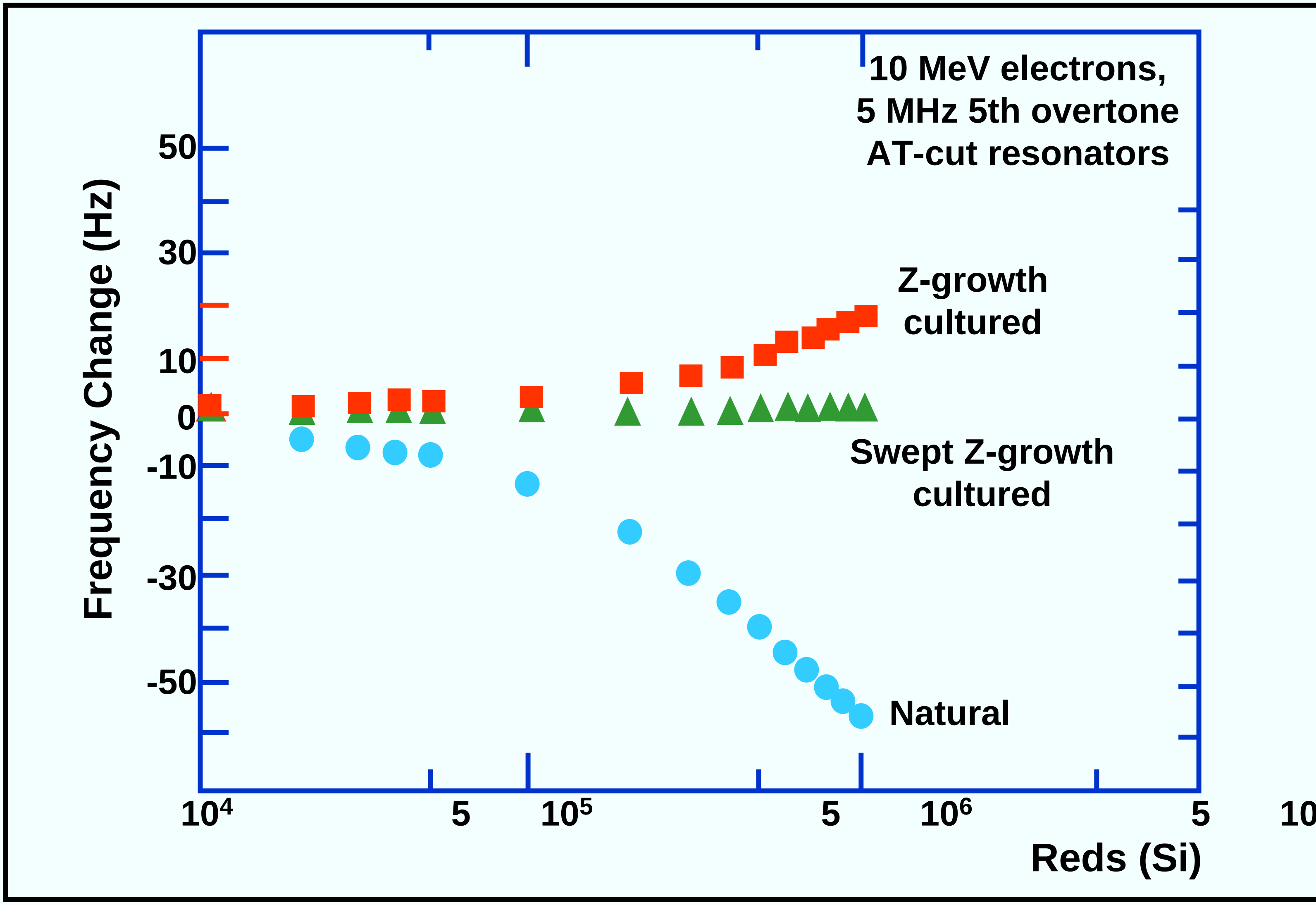
* for a 1 megarad dose (the coefficients are dose dependent)

Idealized frequency vs. time behavior for a quartz resonator following a pulse of ionizing radiation.

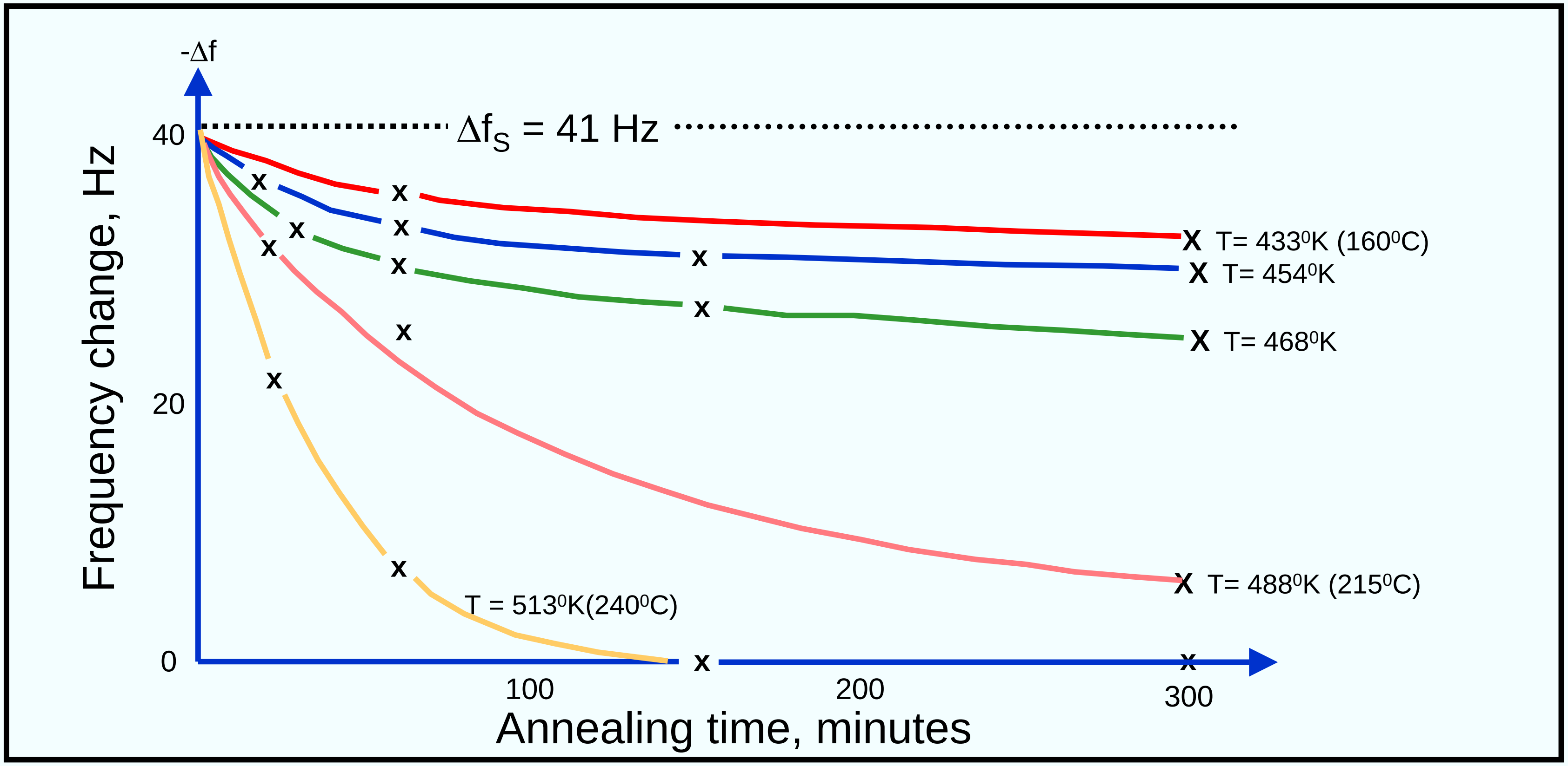
Effects of Repeated Radiations



Radiation Induced Δf vs. Dose and Quartz Type

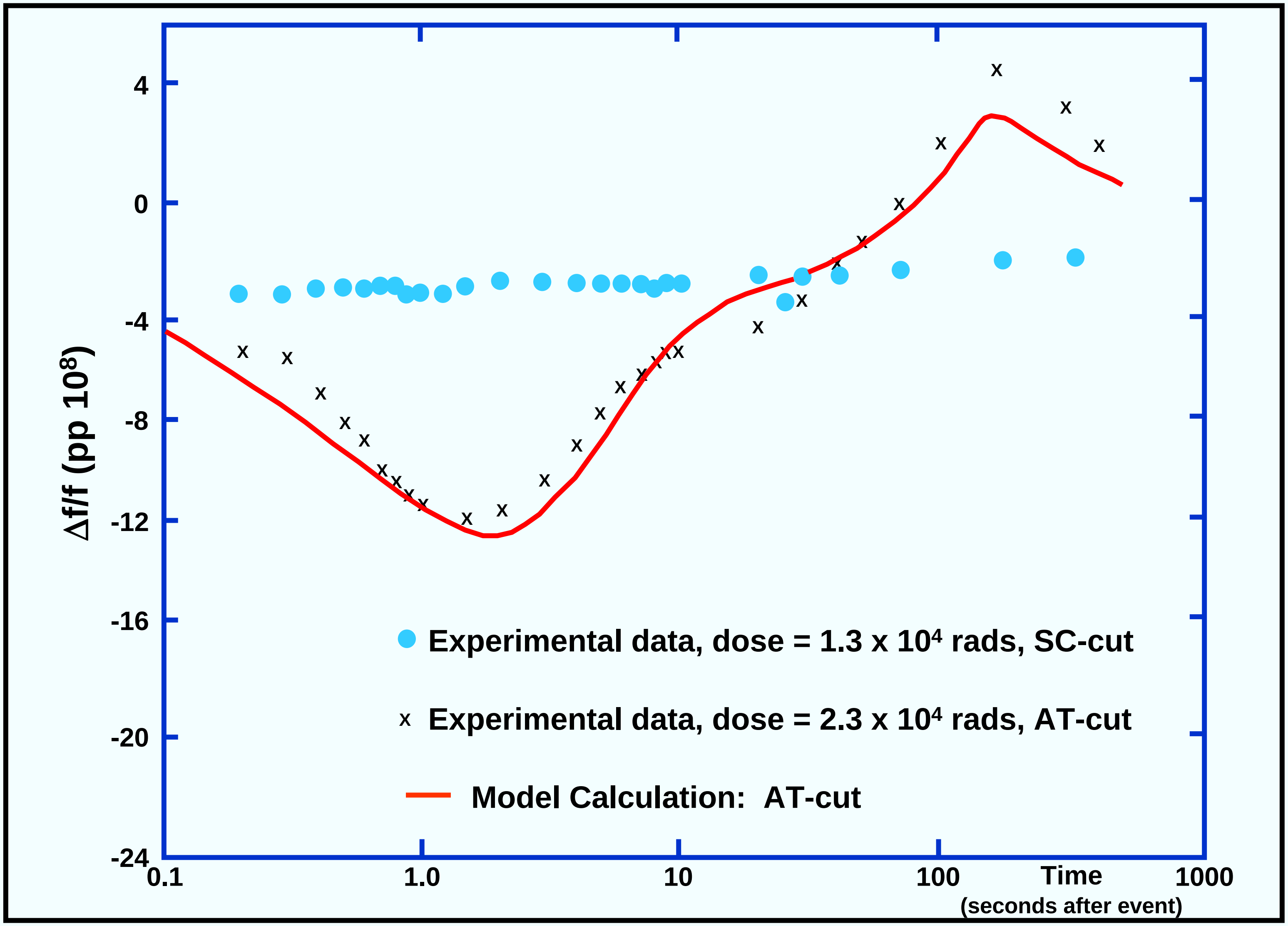


Annealing of Radiation Induced f Changes

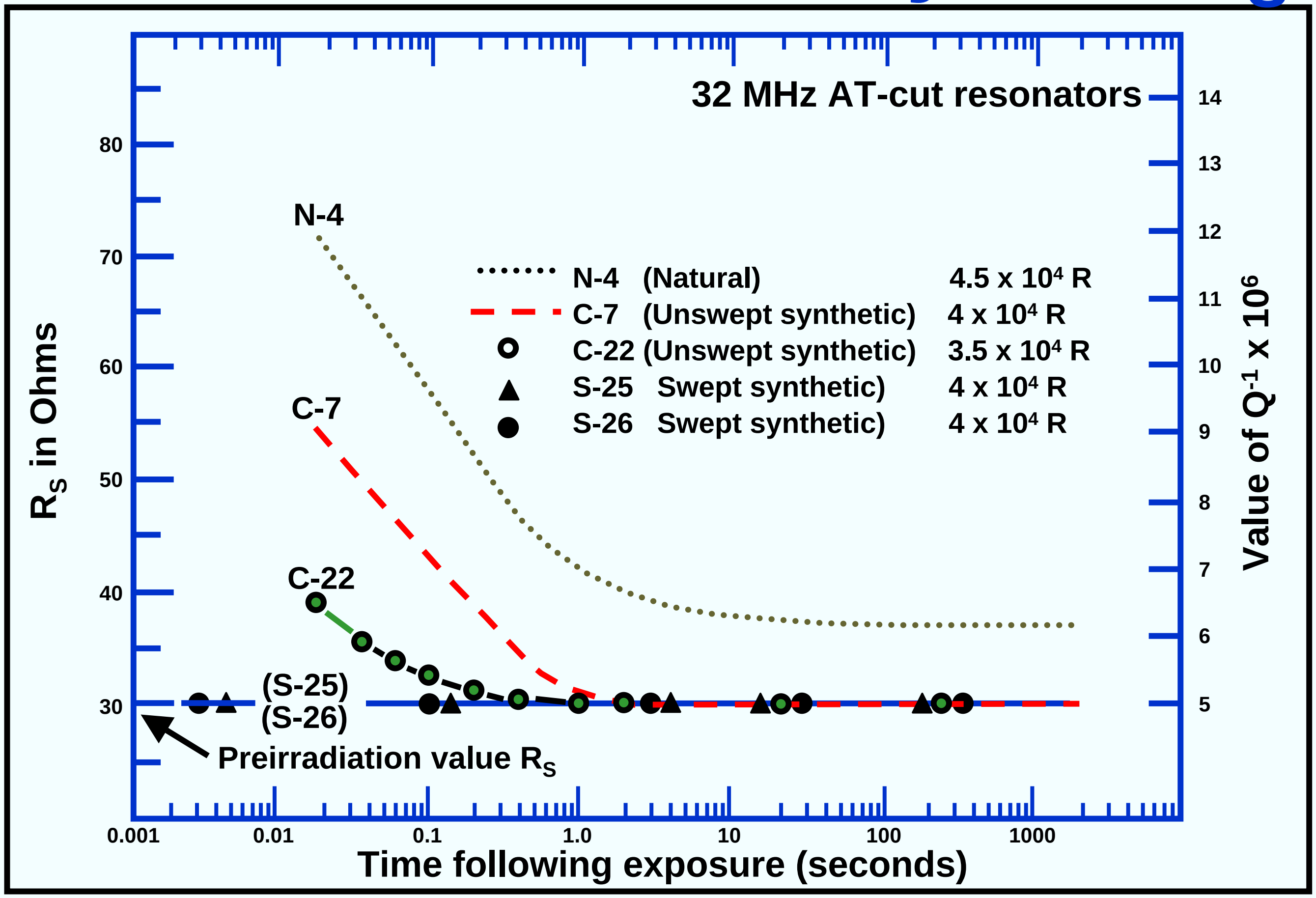


- For a 4 MHz AT-cut resonator, X-ray dose of 6×10^6 rads produced Δf = 41 Hz.
- Activiation energies were calculated from the temperature dependence of the annealing curves. The experimental results can be reproduced by two processes, with activation energies $E_1 = 0.3 \pm 0.1$ eV and $E_2 = 1.3 \pm 0.3$eV.
- Annealing was complete in less than 3 hours at > 240^0C.

Transient Δf After a Pulse of γ Radiation

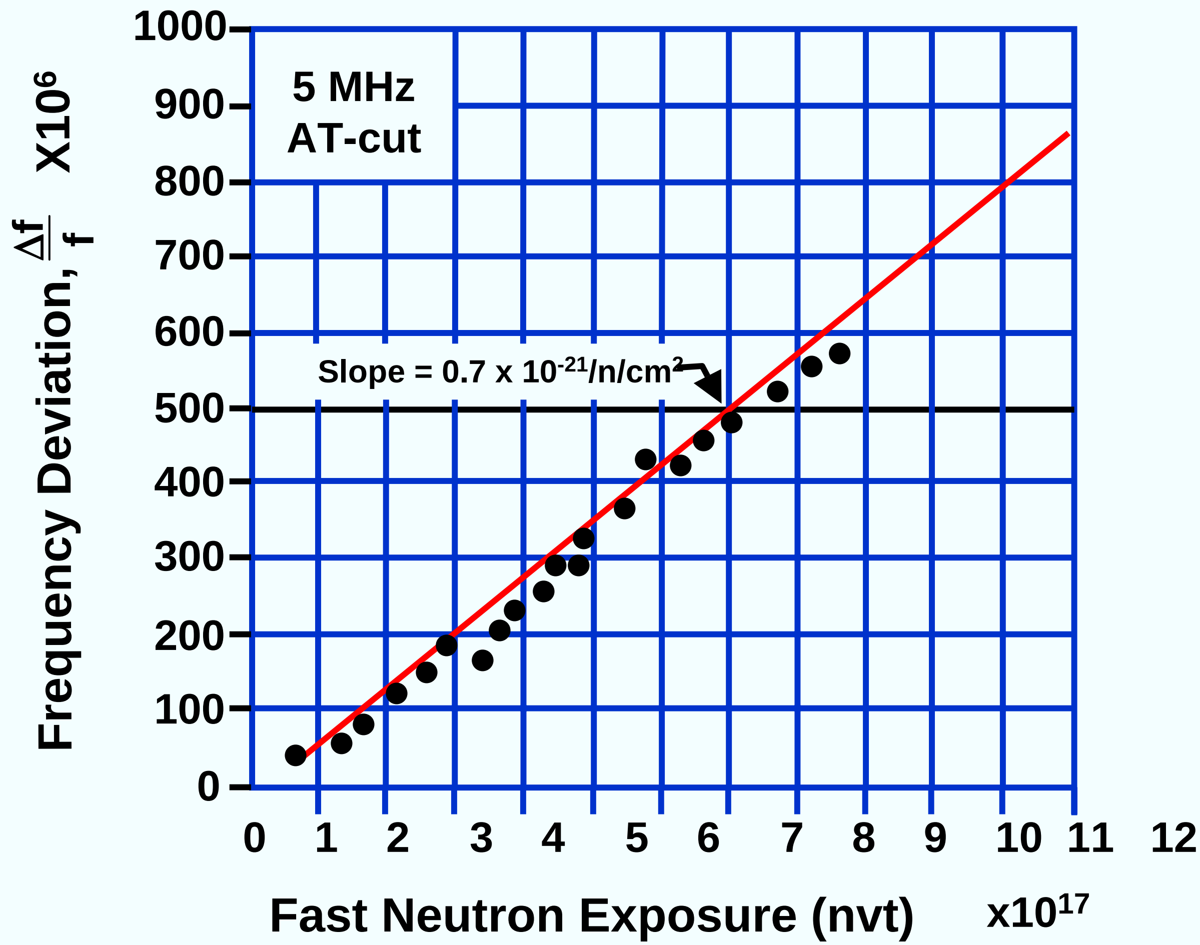


Effects of Flash X-rays on R_S

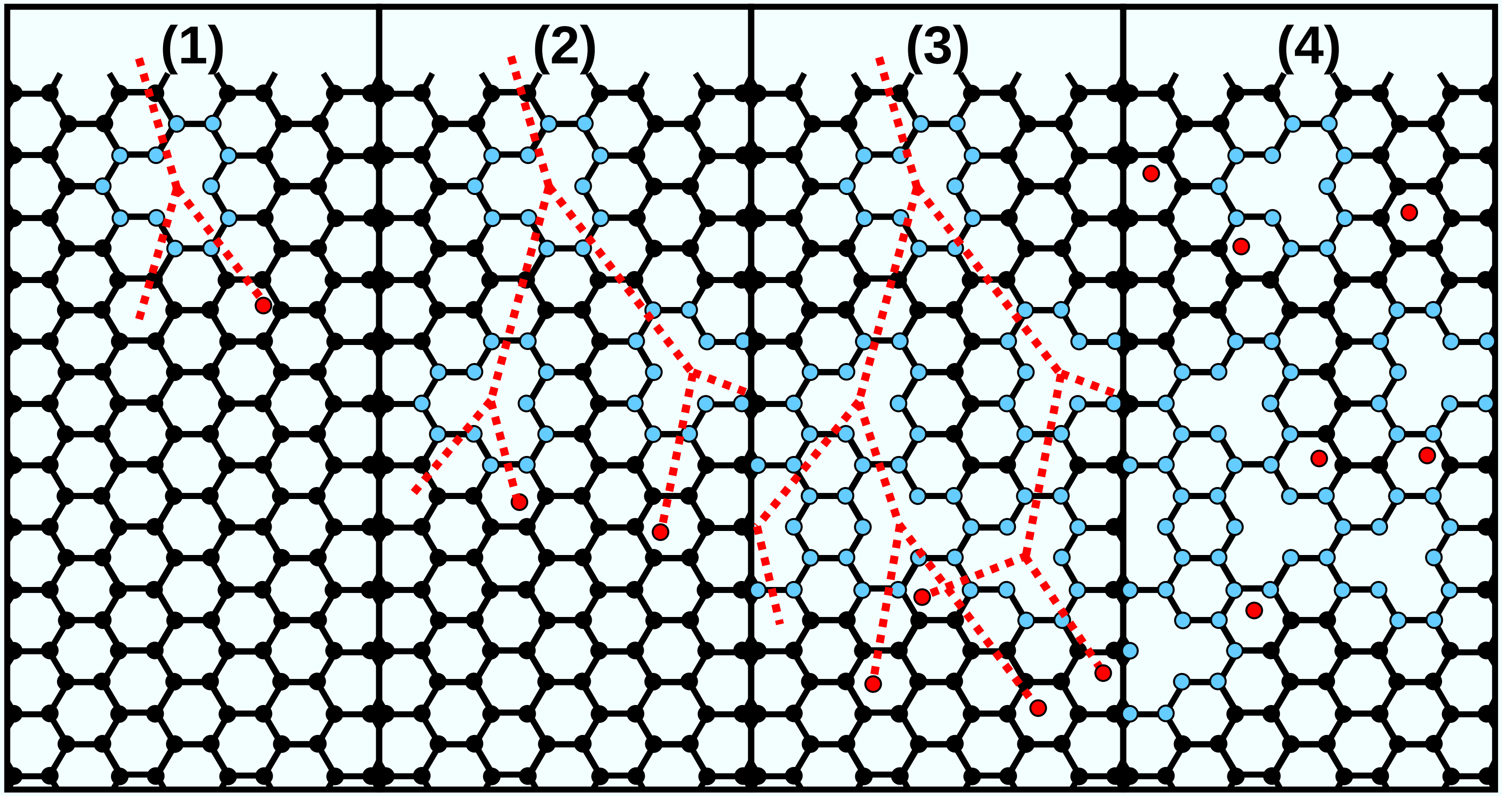


The curves show the series resonance resistance, R_S, vs. time following a 4 x 10^4 rad pulse. Resonators made of swept quartz show no change in R_S from the earliest measurement time (1 ms) after exposure, at room temperature. Large increases in R_S (i.e., large decrease in the Q) will stop the oscillation.

Frequency Change due to Neutrons



Neutron Damage



A fast neutron can displace about 50 to 100 atoms before it comes to rest. Most of the damage is done by the recoiling atoms. Net result is that each neutron can cause numerous vacancies and interstitials.

Summary - Steady-State Radiation Results

- Dose vs. frequency change is nonlinear; frequency change per rad is larger at low doses.
- At doses > 1 kRad, frequency change is quartz-impurity dependent. The ionizing radiation produces electron-hole pairs; the holes are trapped by the impurity Al sites while the compensating cation (e.g., Li^+ or Na^+) is released. The freed cations are loosely trapped along the optic axis. The lattice near the Al is altered, the elastic constant is changed; therefore, the frequency shifts. Ge impurity is also troublesome.
- At a 1 MRad dose, frequency change ranges from pp 10^{11} per rad for natural quartz to pp 10^{14} per rad for high quality swept quartz.
- Frequency change is negative for natural quartz; it can be positive or negative for cultured and swept cultured quartz.
- Frequency change saturates at doses >> 10^6 rads.
- Q degrades upon irradiation if the quartz contains a high concentration of alkali impurities; Q of resonators made of properly swept cultured quartz is unaffected.
- High dose radiation can also rotate the f vs. T characteristic.
- Frequency change anneals at T > 240°C in less than 3 hours.
- Preconditioning (e.g., with doses > 10^5 rads) reduces the high dose radiation sensitivities upon subsequent irradiations.
- At < 100 rad, frequency change is not well understood. Radiation induced stress relief & surface effects (adsorption, desorption, dissociation, polymerization and charging) may be factors.

Summary - Pulse Irradiation Results

- For applications requiring circuits hardened to pulse irradiation, quartz resonators are the least tolerant element in properly designed oscillator circuits.
- Resonators made of unswept quartz or natural quartz can experience a large increase in R_s following a pulse of radiation. The radiation pulse can stop the oscillation.
- Natural, cultured, and swept cultured AT-cut quartz resonators experience an initial negative frequency shift immediately after exposure to a pulse of X-rays (e.g., 10^4 to 10^5 Rad of flash X-rays), $\Delta f/f$ is as large as -3ppm at 0.02sec after burst of 10^{12} Rad/sec.
- Transient f offset anneals as $t^{-1/2}$; the nonthermal-transient part of the f offset is probably due to the diffusion and retrapping of hydrogen at the Al^{3+} trap.
- Resonators made of properly swept quartz experience a negligibly small change in R_s when subjected to pulsed ionizing radiation (therefore, the oscillator circuit does not require a large reserve of gain margin).
- SC-cut quartz resonators made of properly swept high Q quartz do not exhibit transient frequency offsets following a pulse of ionizing radiation.
- Crystal oscillators will stop oscillating during an intense pulse of ionizing radiation because of the large prompt photoconductivity in quartz and in the transistors comprising the oscillator circuit. Oscillation will start up within 15μsec after a burst if swept quartz is used in the resonator and the oscillator circuit is properly designed for the radiation environment.

Summary - Neutron Irradiation Results

- When a fast neutron (~MeV energy) hurtles into a crystal lattice and collides with an atom, it is scattered like a billiard ball. The recoiling atom, having an energy (~10^4 to 10^6 eV) that is much greater than its binding energy in the lattice, leaves behind a vacancy and, as it travels through the lattice, it displaces and ionizes other atoms. A single fast neutron can thereby produce numerous vacancies, interstitials, and broken interatomic bonds. Neutron damage thus changes both the elastic constants and the density of quartz. Of the fast neutrons that impinge on a resonator, most pass through without any collisions, i.e., without any effects on the resonator. The small fraction of neutrons that collide with atoms in the lattice cause the damage.

- Frequency increases approximately linearly with fluence. For AT- and SC-cut resonators, the slopes range from +0.7 x 10^{-21}/n/cm^2, at very high fluences (10^{17} to 10^{18}n/cm^2) to 5 x 10^{-21}/n/cm^2 at 10^{12} to 10^{13}n/cm^2, and 8 x 10^{-21}/n/cm^{2}at 10^{10} to 10^{12}n/cm^2. Sensitivity probably depends somewhat on the quartz defect density and on the neutron energy distribution. (Thermonuclear neutrons cause more damage than reactor neutrons.)

- Neutron irradiation also rotates the frequency vs. temperature characteristic.

- When a heavily neutron irradiated sample was baked at 500°C for six days, 90% of the neutron-induced frequency shift was removed (but the 10% remaining was still 93 ppm).

Other Effects on Stability

- **Electric field -** affects doubly-rotated resonators; e.g., a voltage on the electrodes of a 5 MHz fundamental mode SC-cut resonator results in a $\Delta f/f = 7 \times 10^{-9}$ per volt. The voltage can also cause sweeping, which can affect the frequency (of all cuts), even at normal operating temperatures.
- **Magnetic field -** quartz is diamagnetic, however, magnetic fields can induce Eddy currents, and will affect magnetic materials in the resonator package and the oscillator circuitry. Induced ac voltages can affect varactors, AGC circuits and power supplies. Typical frequency change of a "good" quartz oscillator is $<<10^{-10}$ per gauss.
- **Ambient pressure (altitude) -** deformation of resonator and oscillator packages, and change in heat transfer conditions affect the frequency.
- **Humidity -** can affect the oscillator circuitry, and the oscillator's thermal properties, e.g., moisture absorbed by organics can affect dielectric constants.
- **Power supply voltage, and load impedance -** affect the oscillator circuitry, and indirectly, the resonator's drive level and load reactance. A change in load impedance changes the amplitude or phase of the signal reflected into the oscillator loop, which changes the phase (and frequency) of the oscillation. The effects can be minimized by using a (low noise) voltage regulator and buffer amplifier.
- **Gas permeation -** stability can be affected by excessive levels of atmospheric hydrogen and helium diffusing into "hermetically sealed" metal and glass enclosures (e.g., hydrogen diffusion through nickel resonator enclosures, and helium diffusion through glass Rb standard bulbs).

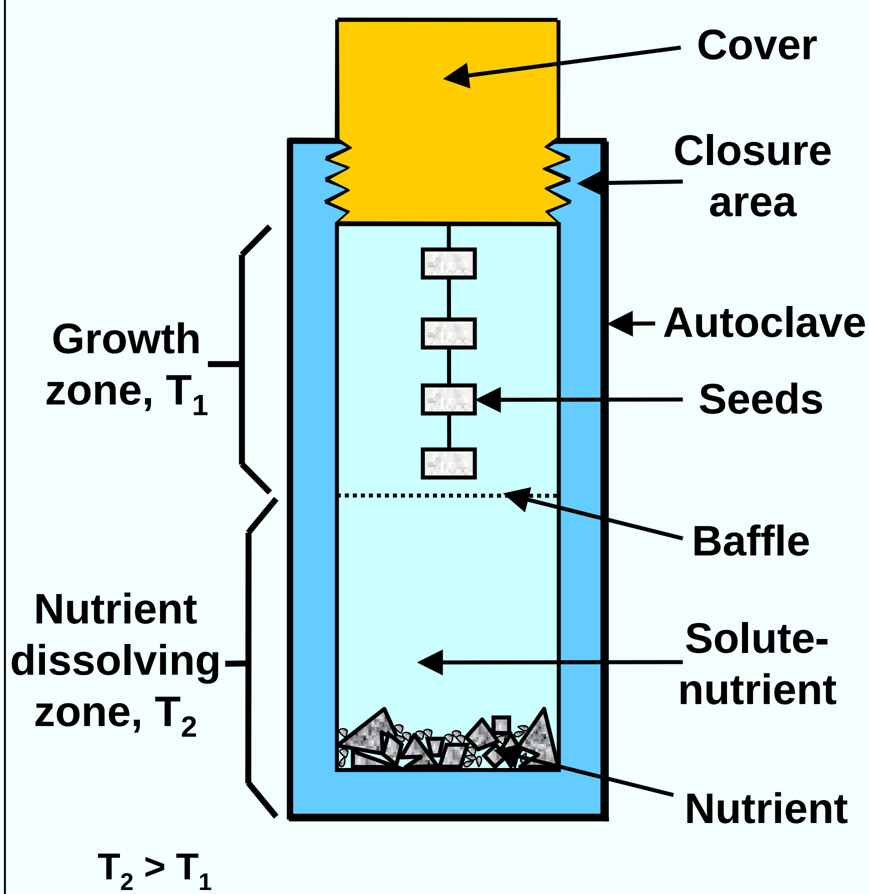
Interactions Among Influences

In attempting to measure the effect of a single influence, one often encounters interfering influences, the presence of which may or may not be obvious.

Measurement	Interfering Influence
Resonator aging	ΔT due to oven T (i.e., thermistor) aging Δ drive level due to osc. circuit aging
Short term stability	Vibration
Vibration sensitivity	Induced voltages due to magnetic fields
2-g tipover sensitivity	ΔT due to convection inside oven
Resonator f vs. T (static)	Thermal transient effect, humidity T-coefficient of load reactances
Radiation sensitivity	ΔT, thermal transient effect, aging

CHAPTER 5
Quartz Material Properties

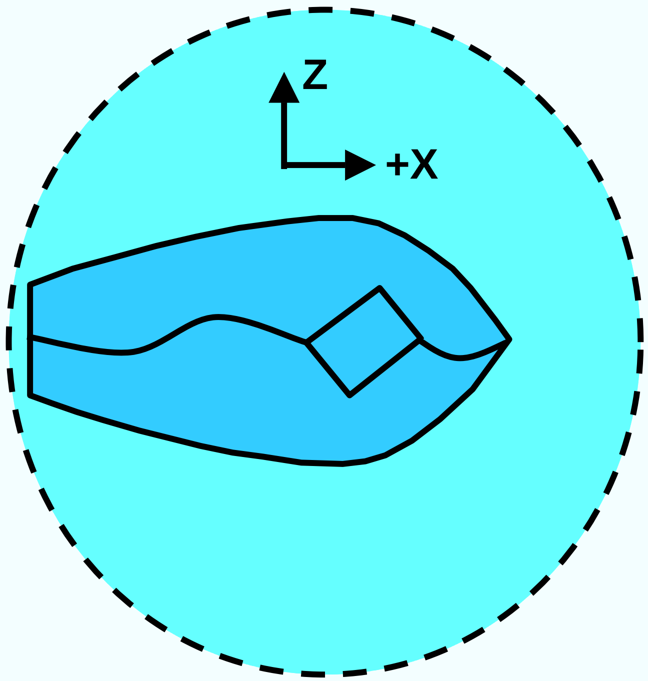
Hydrothermal Growth of Quartz



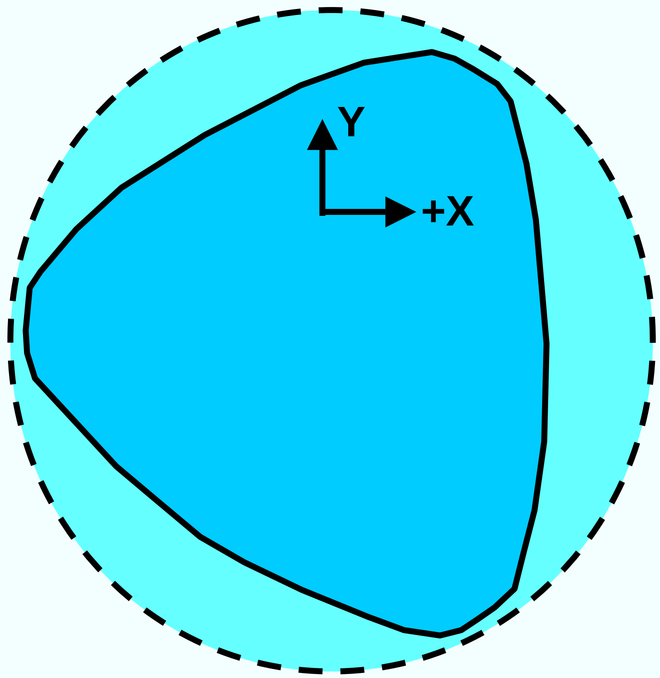
- The autoclave is filled to some predetermined factor with water plus mineralizer (NaOH or Na_2CO_3).
- The baffle localizes the temperature gradient so that each zone is nearly isothermal.
- The seeds are thin slices of (usually) Z-cut single crystals.
- The nutrient consists of small (~2½ to 4 cm) pieces of single-crystal quartz ("lascas").
- The temperatures and pressures are typically about 350^{0}C and 800 to 2,000 atmospheres; T_2 - T_1 is typically 4^{0}C to 10^{0}C.
- The nutrient dissolves slowly (30 to 260 days per run), diffuses to the growth zone, and deposits onto the seeds.

Deeply Dissolved Quartz Sphere

Anisotropic Etching



Looking along Y-axis



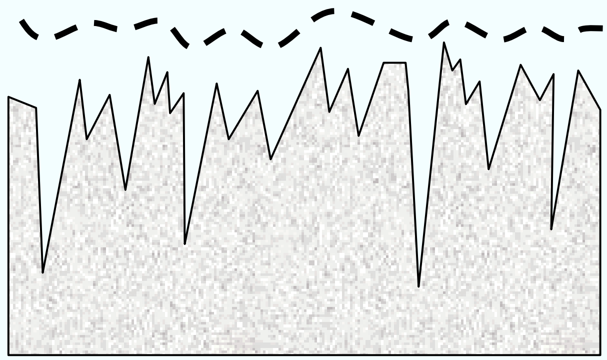
Looking along Z-axis

Etching & Chemical Polishing

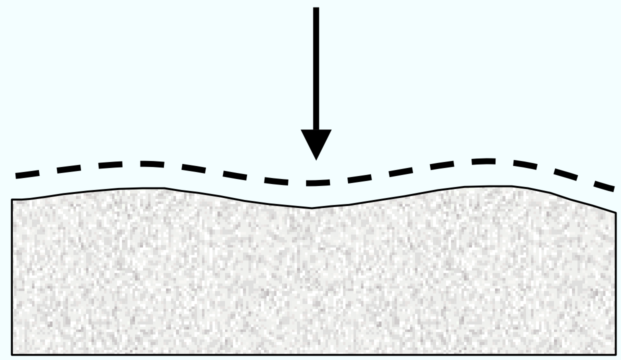
Etchant Must:
1. Diffuse to Surface
2. Be Adsorbed
3. React Chemically

Reaction Products Must:
4. Be Desorbed
5. Diffuse Away

Diffusion Controlled Etching:

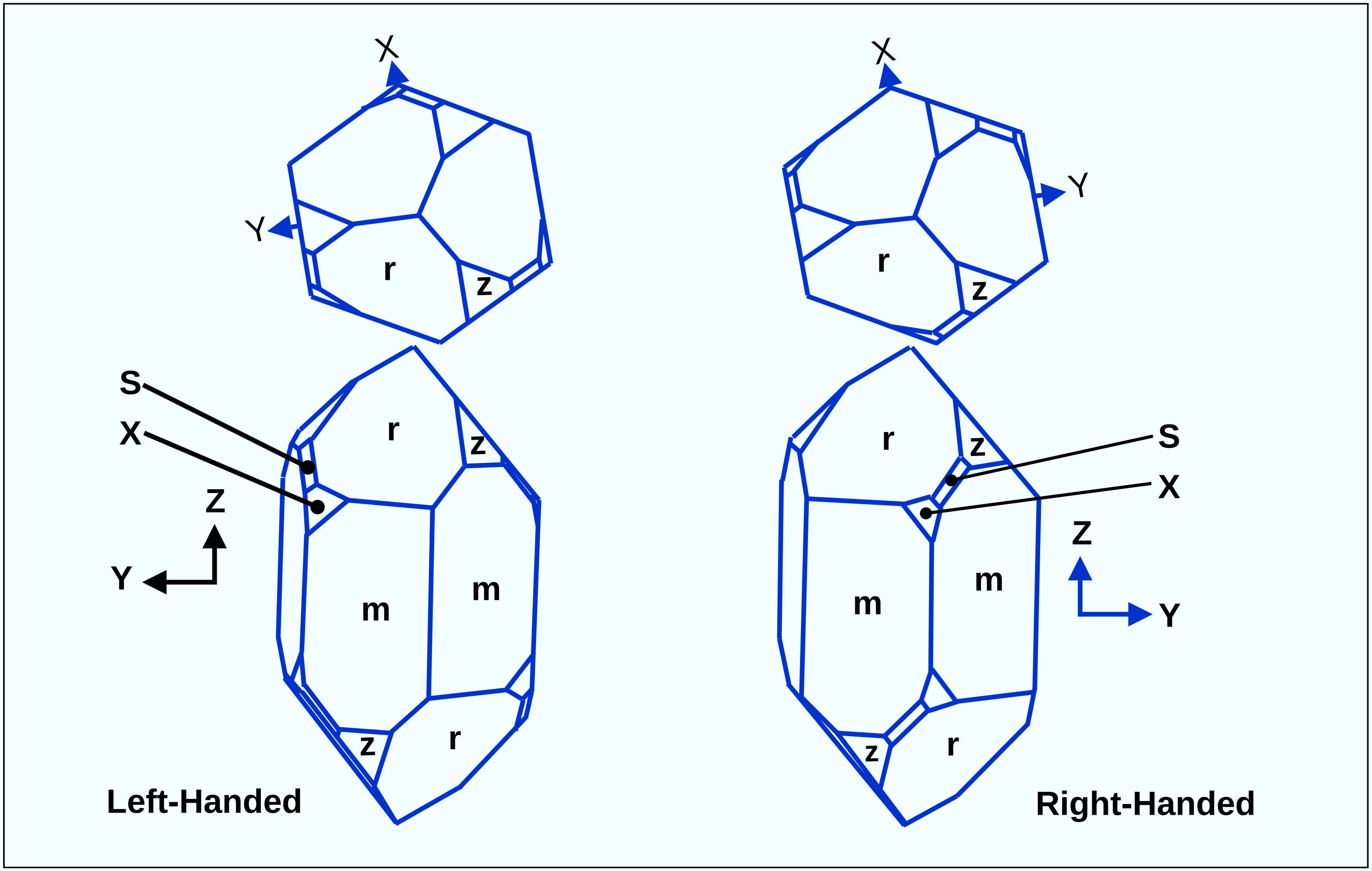


Lapped surface

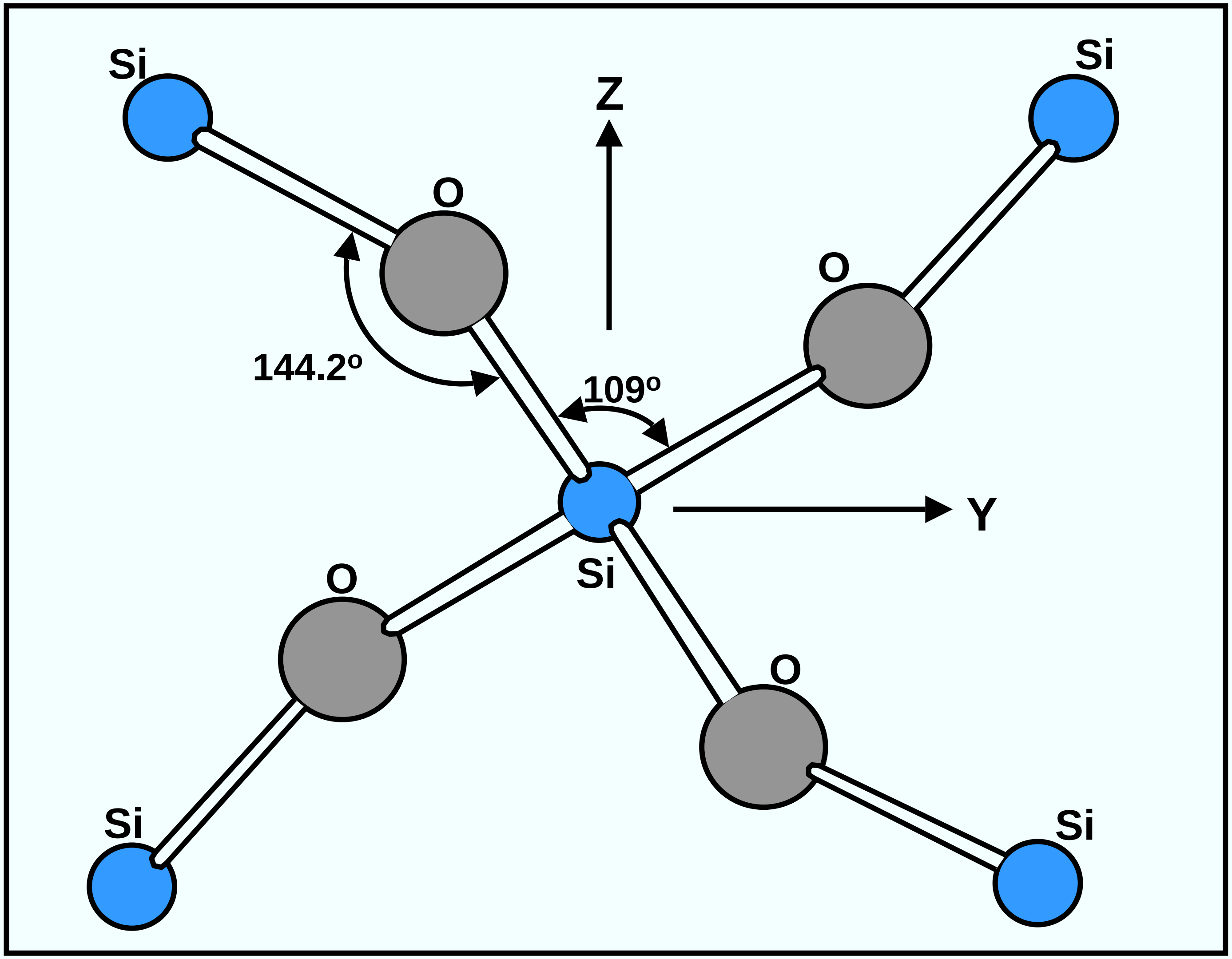


Chemically polished surface

Left-Handed and Right-Handed Quartz



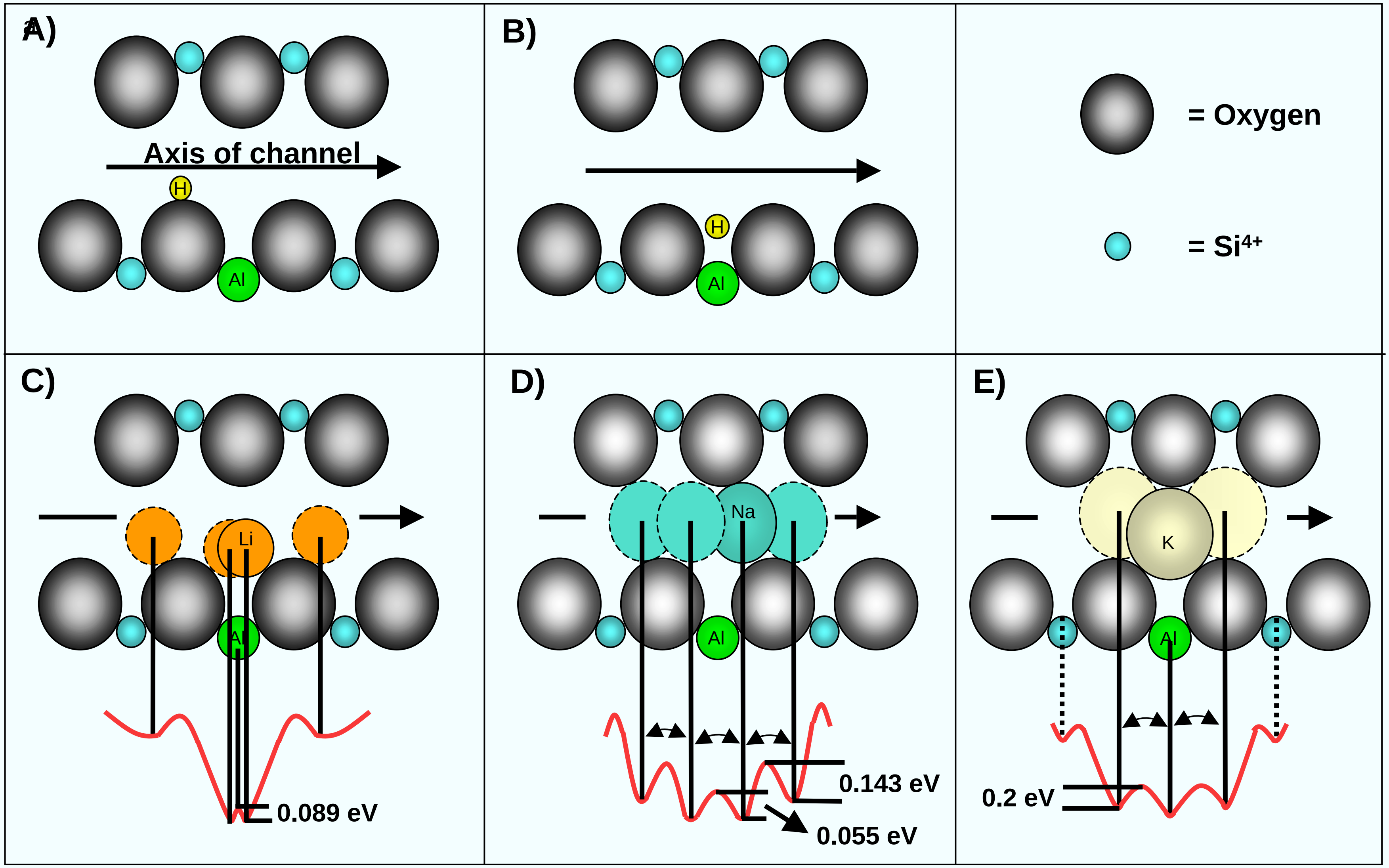
The Quartz Lattice



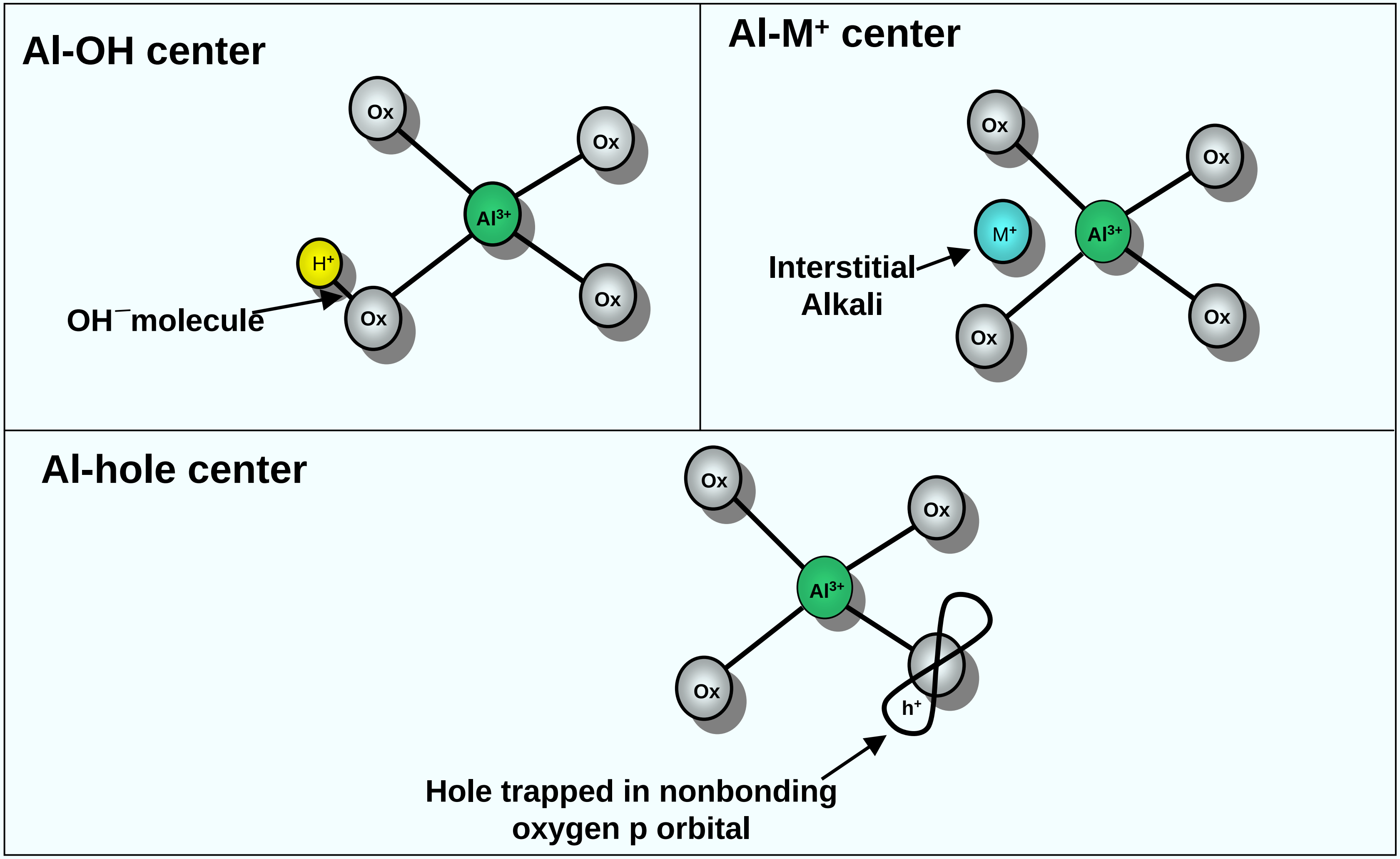
Quartz Properties' Effects on Device Properties

Quartz Property	Device and Device-Fabrication Property
Q	Oscillator short-term stability, phase noise close to carrier, long-term stability, filter loss
Purity (Al, Fe, Li, Na, K, -OH, H_2O)	Radiation hardness, susceptibility to twinning, optical characteristics
Crystalline Perfection, Strains	Sweepability, etchability for chem. polishing and photolithographic processing, optical properties, strength, aging(?), hysteresis(?)
Inclusions	High-temperature processing and applications, resonator Q, optical characteristics, etchability

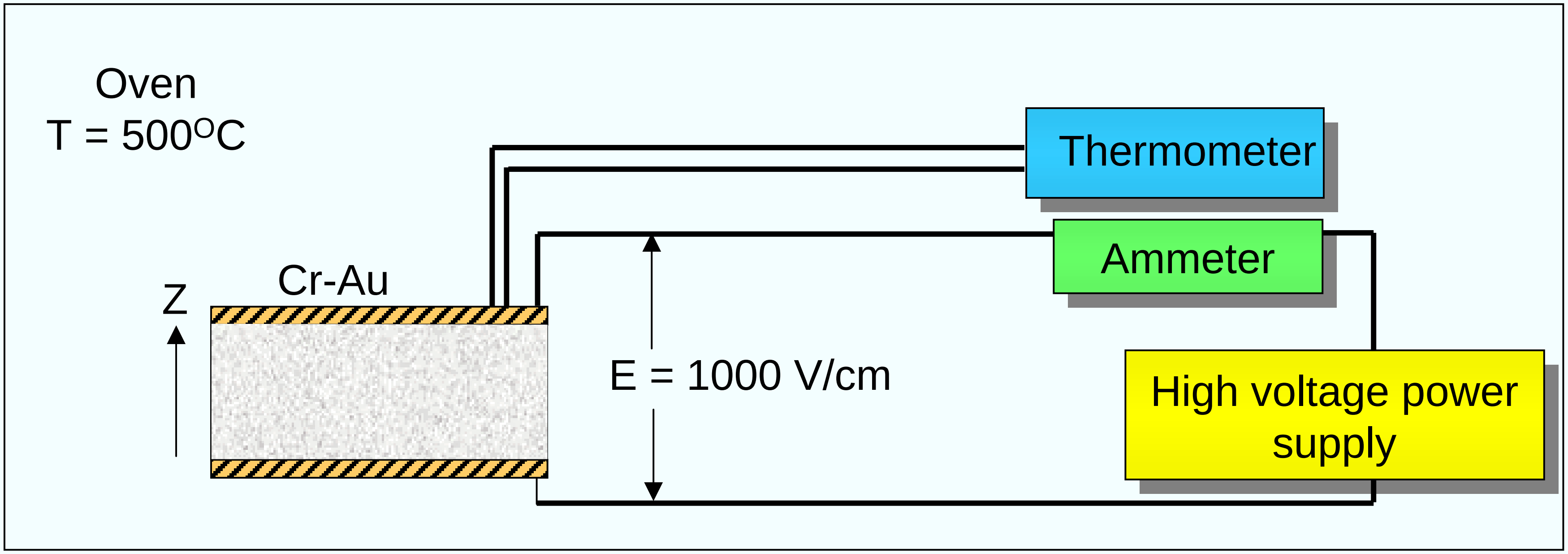
Ions in Quartz - Simplified Model



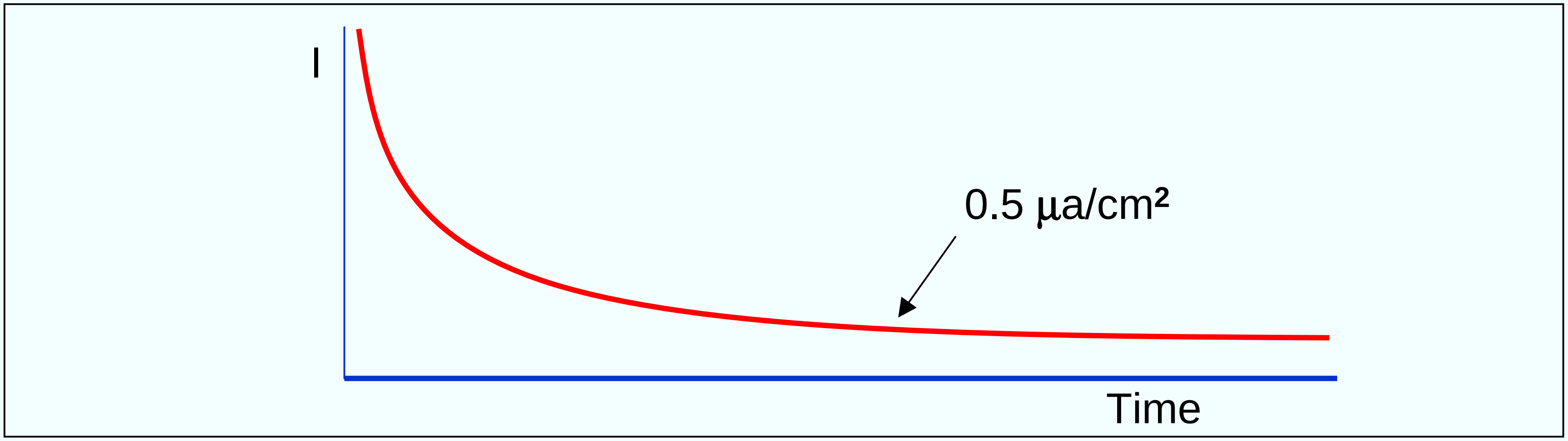
Aluminum Associated Defects



Sweeping



Oven
T = 500^{O}C
Z
Cr-Au
Thermometer
Ammeter
E = 1000 V/cm
High voltage power supply



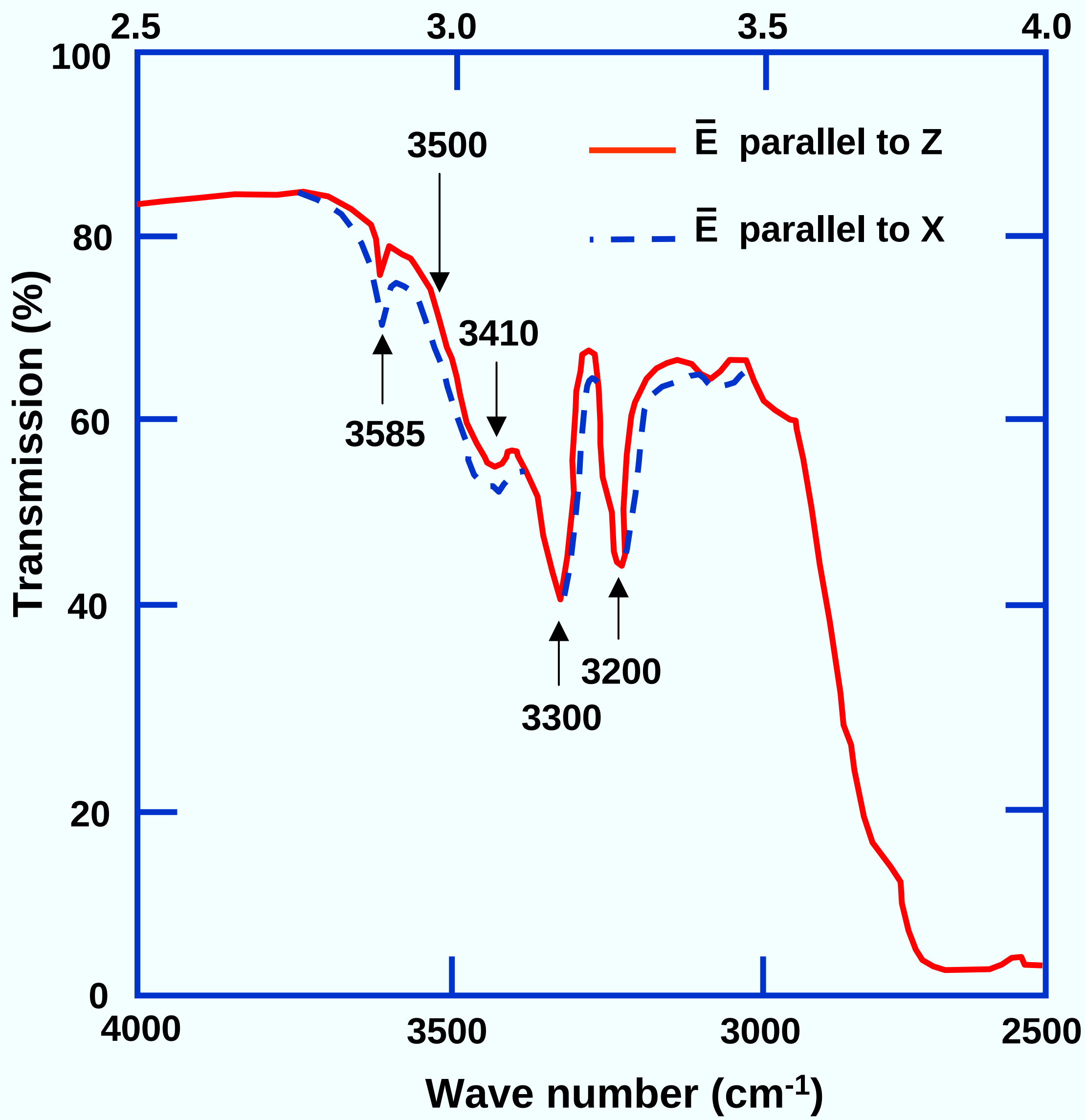
I
0.5 μa/cm²
Time

Quartz Quality Indicators

- **Infrared absorption coefficient***
- **Etch-channel density ***
- **Etch-pit density**
- **Inclusion density ***
- **Acoustic attenuation**
- **Impurity analysis**
- **X-ray topography**
- **UV absorption**
- **Birefringence along the optic axis**
- **Thermal shock induced fracture**
- **Electron spin resonance**
- **? ? ?**

* EIA Standard 477-1 contains standard test method for this quantity

Infrared Absorption of Quartz



Infrared Absorption Coefficient

One of the factors that determine the maximum achievable resonator Q is the OH content of the quartz. Infrared absorption measurements are routinely used to measure the intensities of the temperature-broadened OH defect bands. The **infrared absorption coefficient** α is defined by EIA Standard 477-1 as

$$\alpha = \frac{A\ (3500\ \mathrm{cm}^{-1}) - A\ (3800\ \mathrm{cm}^{-1})}{\text{Y-cut thickness in cm}}$$

where the A's are the logarithm (base 10) of the fraction of the incident beam absorbed at the wave numbers in the parentheses.

Grade	α, in cm^{-1}	Approx. max. Q*
A	0.03	3.0
B	0.045	2.2
C	0.060	1.8
D	0.12	1.0
E	0.25	0.5

* In millions, at 5 MHz (α is a quality indicator for unswept quartz only).

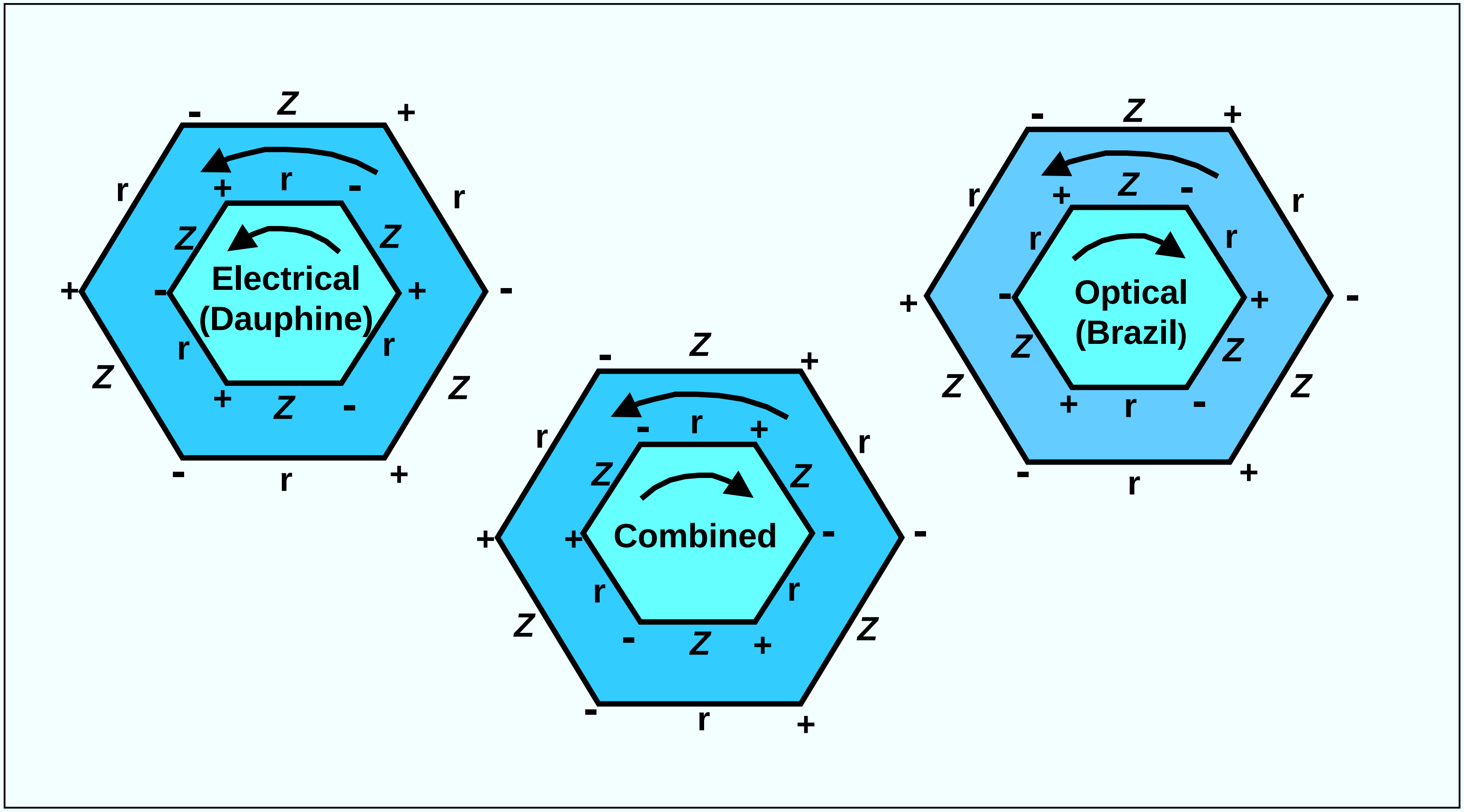
Quartz Twinning

Electrical twinning

Optical Twinning

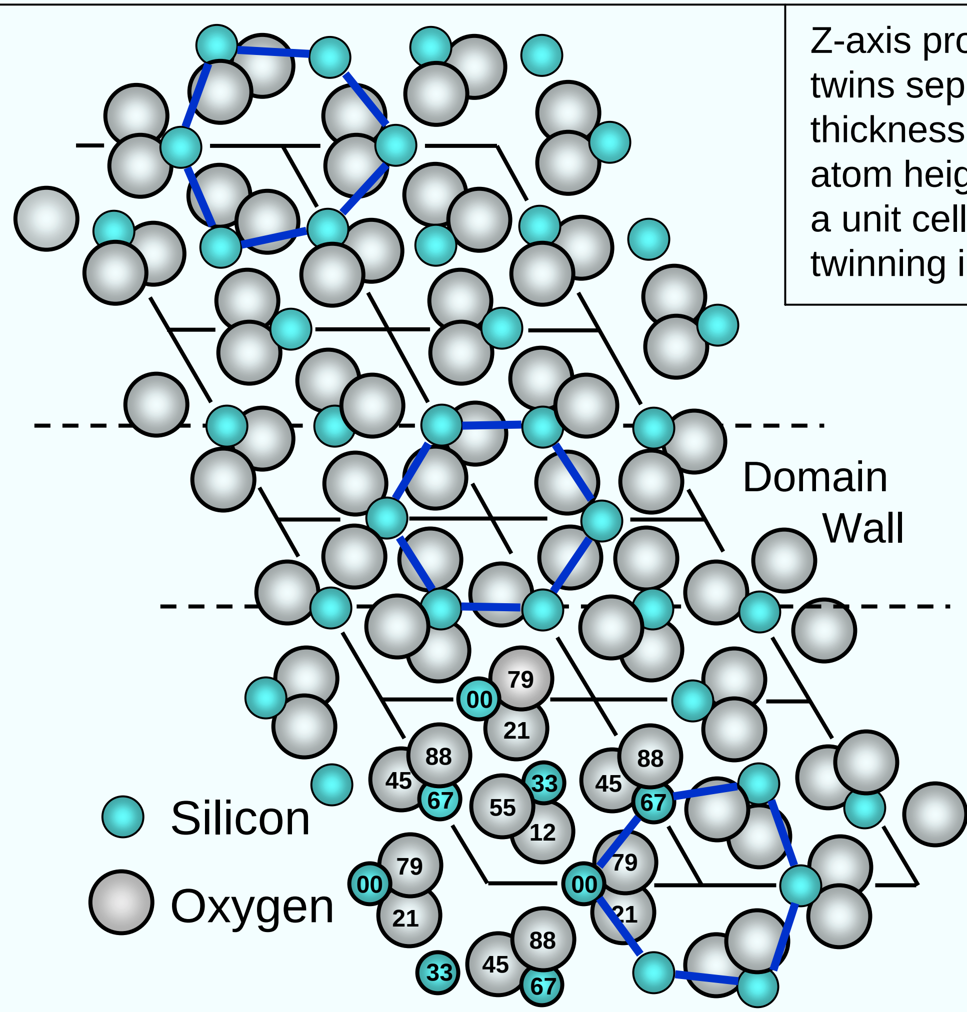
- The X-axes of quartz, the electrical axes, are parallel to the line bisecting adjacent prism faces; the +X-direction is positive upon extension due to tension.
- Electric twinning (also called Dauphiné twinning) consists of localized reversal of the X-axes. It usually consists of irregular patches, with irregular boundaries. It can be produced artificially by inversion from high quartz, thermal shock, high local pressure (even at room temperature), and by an intense electric field.
- In right-handed quartz, the plane of polarization is rotated clockwise as seen by looking toward the light source; in left handed, it is counterclockwise. Optically twinned (also called Brazil twinned) quartz contains both left and right-handed quartz. Boundaries between optical twins are usually straight.
- Etching can reveal both kinds of twinning.

Twinning - Axial Relationships



The diagrams illustrate the relationship between the axial system and hand of twinned crystals. The arrows indicate the hand.

Quartz Lattice and Twinning



Z-axis projection showing electric (Dauphiné) twins separated by a twin wall of one unit cell thickness. The numbers in the atoms are atom heights expressed in units of percent of a unit cell height. The atom shifts during twinning involve motions of <0.03 nm.

Quartz Inversion

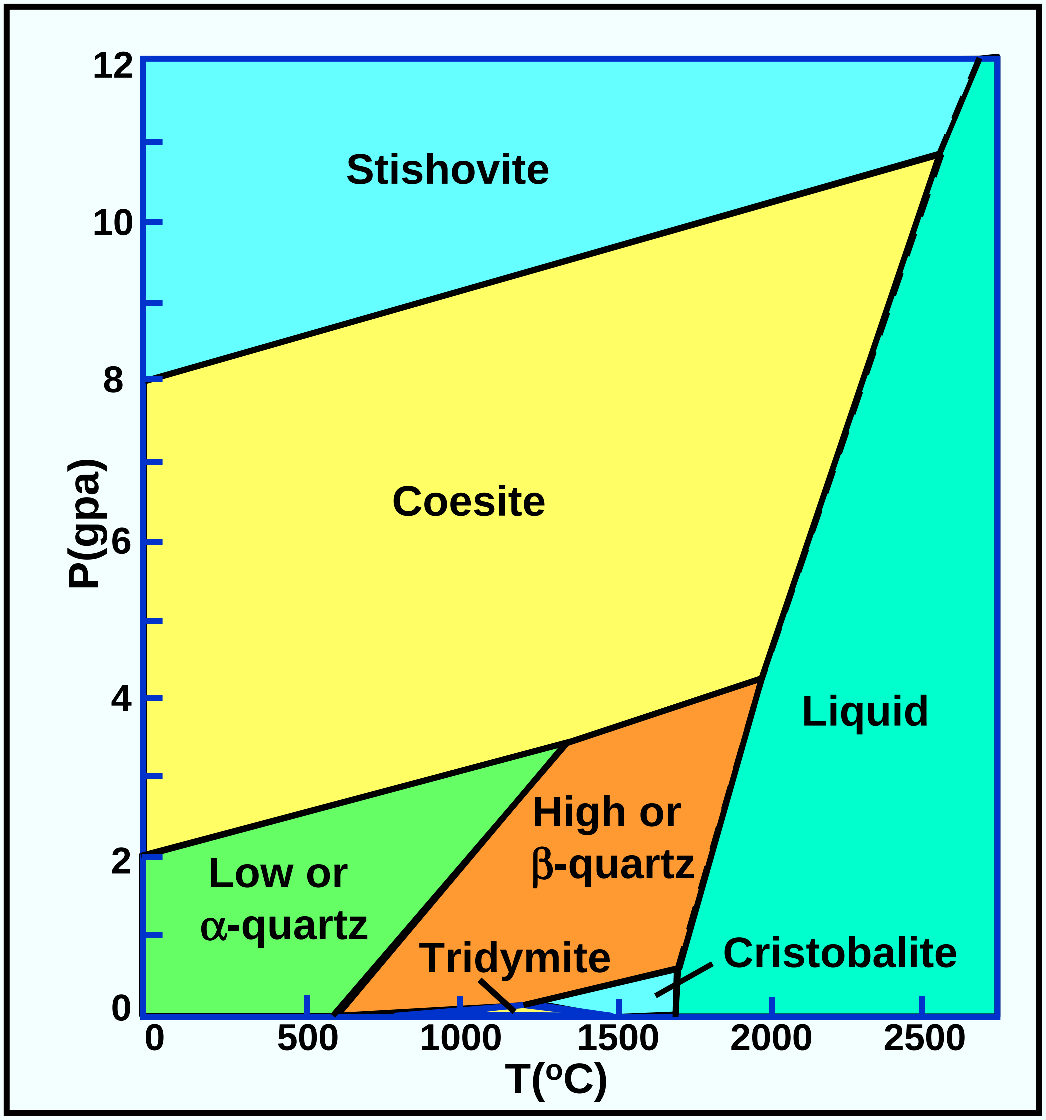
- **Quartz undergoes a high-low inversion (α - β transformation) at 573^0C. (It is 573^0C at 1 atm on rising temperature; it can be 1^0 to 2^0C lower on falling temperature.)**

- **Bond angles between adjoining (SiO_4) tetrahedra change at the inversion. Whereas low-quartz (α-quartz) is trigonal, high quartz (β -quartz) is hexagonal. Both forms are piezoelectric.**

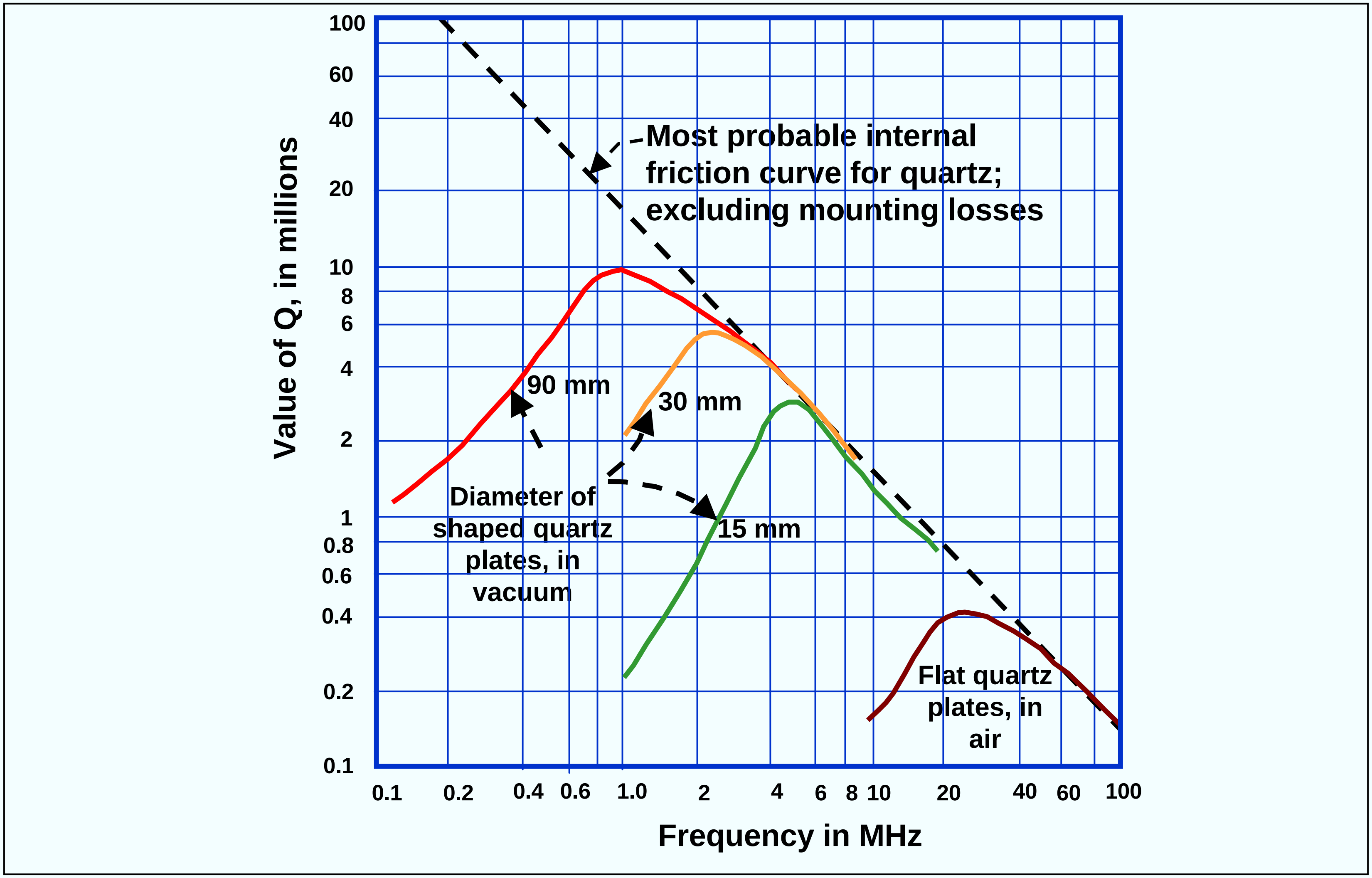
- **An abrupt change in nearly all physical properties takes place at the inversion point; volume increases by 0.86% during inversion from low to high quartz. The changes are reversible, although Dauphiné twinning is usually acquired upon cooling through the inversion point.**

- **Inversion temperature decreases with increasing Al and alkali content, increases with Ge content, and increases 1^0C for each 40 atm increase in hydrostatic pressure.**

Phase Diagram of Silica (SiO_2)



Internal Friction of Quartz



Empirically determined Q vs. frequency curves indicate that the maximum achievable Q times the frequency is a constant, e.g., 16 million for AT-cut resonators, when f is in MHz.

Langasite and Its Isomorphs

$La_3Ga_5SiO_{14}$	Langa**s**ite (LGS)
$La_3Ga_{5.5}Nb_{0.5}O_{14}$	Langa**n**ite (LGN)
$La_3Ga_{5.5}Ta_{0.5}O_{14}$	Langa**t**ate (LGT)

- Lower acoustic attenuation than quartz (higher Qf than AT- or SC-cut quartz)
- No phase transition (melts at ~1,400 ^oC vs. phase transition at 573 ^oC for quartz)
- Higher piezoelectric coupling than quartz
- Thicker than quartz at the same frequency
- Temperature-compensated

CHAPTER 6
Atomic Frequency Standards*

* There are two important reasons for including this chapter: 1. atomic frequency standards are one of the most important applications of precision quartz oscillators, and 2. those who study or use crystal oscillators ought to be aware of what is available in case they need an oscillator with better long-term stability than what crystal oscillators can provide.

Precision Frequency Standards

- **Quartz crystal resonator-based** (f ~ 5 MHz, Q ~ 10^6)
- **Atomic resonator-based**

  Rubidium cell (f_0 = 6.8 GHz, Q ~ 10^7)

  Cesium beam (f_0 = 9.2 GHz, Q ~ 10^8)

  Hydrogen maser (f_0 = 1.4 GHz, Q ~ 10^9)

  Trapped ions (f_0 > 10 GHz, Q > 10^{11})

  Cesium fountain (f_0 = 9.2 GHz, Q ~ 5 x 10^{11})

Atomic Frequency Standard Basic Concepts

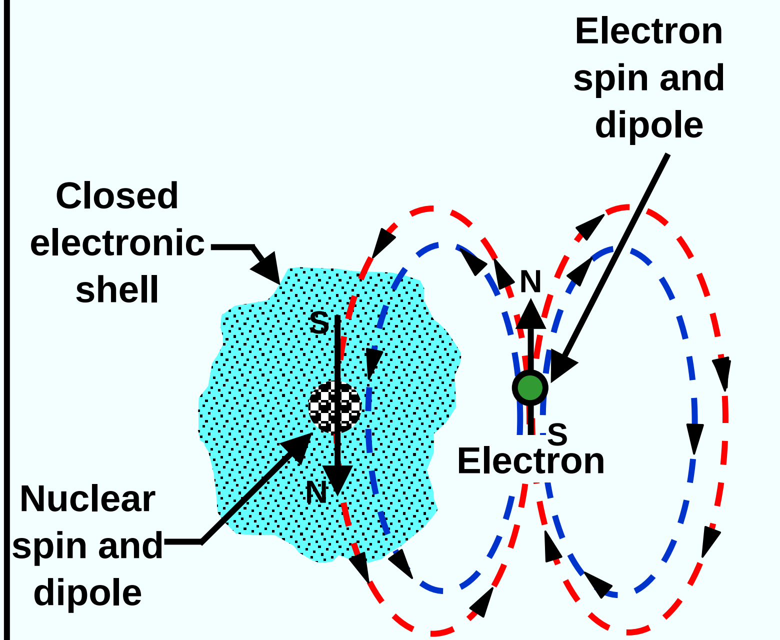
When an atomic system changes energy from an exited state to a lower energy state, a photon is emitted. The photon frequency ν is given by Planck's law

$$\nu = \frac{E_2 - E_1}{h}$$

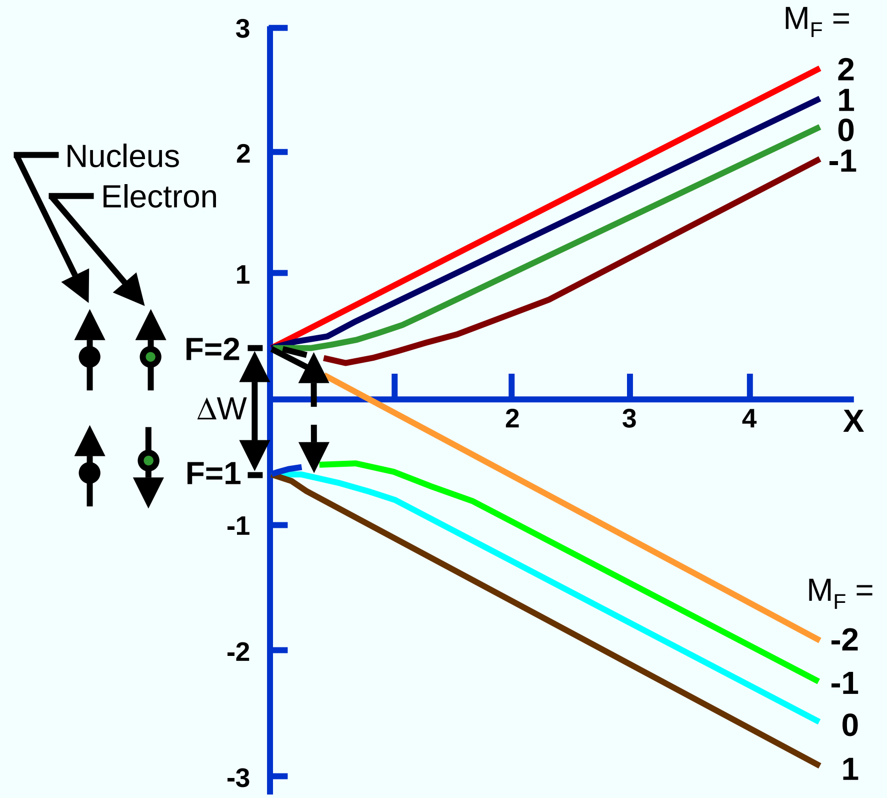
where E_2 and E_1 are the energies of the upper and lower states, respectively, and h is Planck's constant. An atomic frequency standard produces an output signal the frequency of which is determined by this intrinsic frequency rather than by the properties of a solid object and how it is fabricated (as it is in quartz oscillators).

The properties of isolated atoms at rest, and in free space, would not change with space and time. Therefore, the frequency of an ideal atomic standard would not change with time or with changes in the environment. Unfortunately, in real atomic frequency standards: 1) the atoms are moving at thermal velocities, 2) the atoms are not isolated but experience collisions and electric and magnetic fields, and 3) some of the components needed for producing and observing the atomic transitions contribute to instabilities.

Hydrogen-Like Atoms



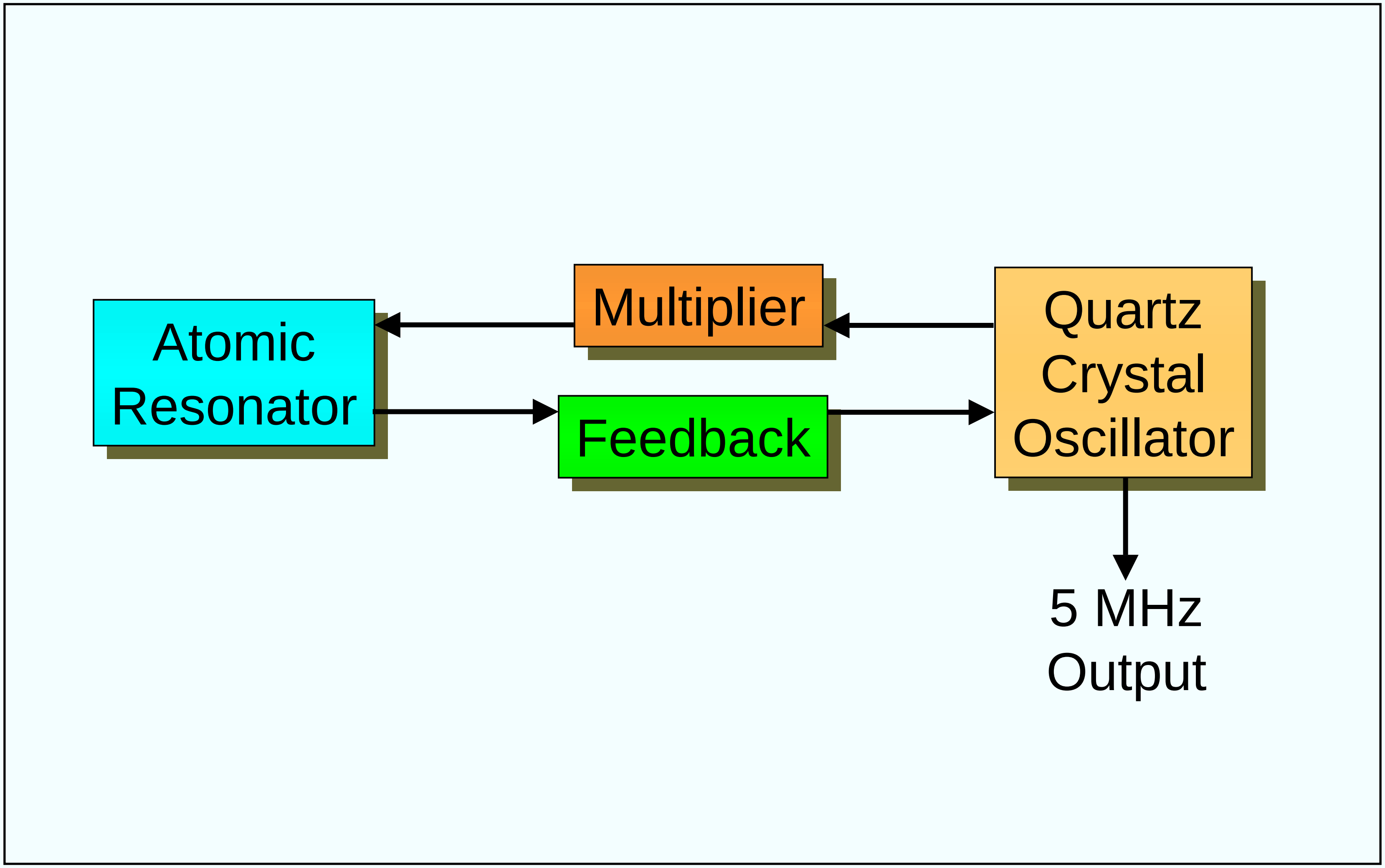
Hydrogen-like (or alkali) atoms



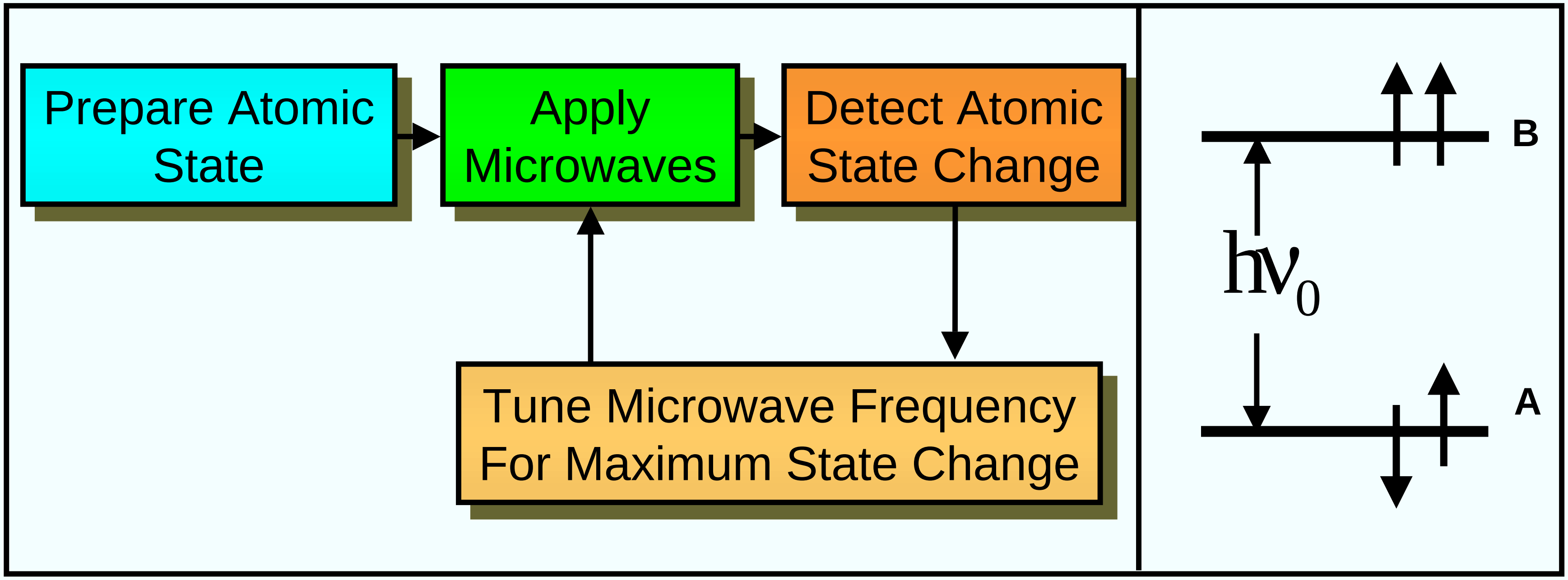
Hyperfine structure of ^{87}Rb, with nuclear spin I=3/2, $\nu_0=\Delta W/h=6{,}834{,}682{,}605$ Hz and $X=[(-\mu J/J)+(\mu I/I)]H_0/\Delta W$ calibrated in units of 2.44×10^3 Oe.

Atomic Frequency Standard

Block Diagram



Generalized Atomic Resonator

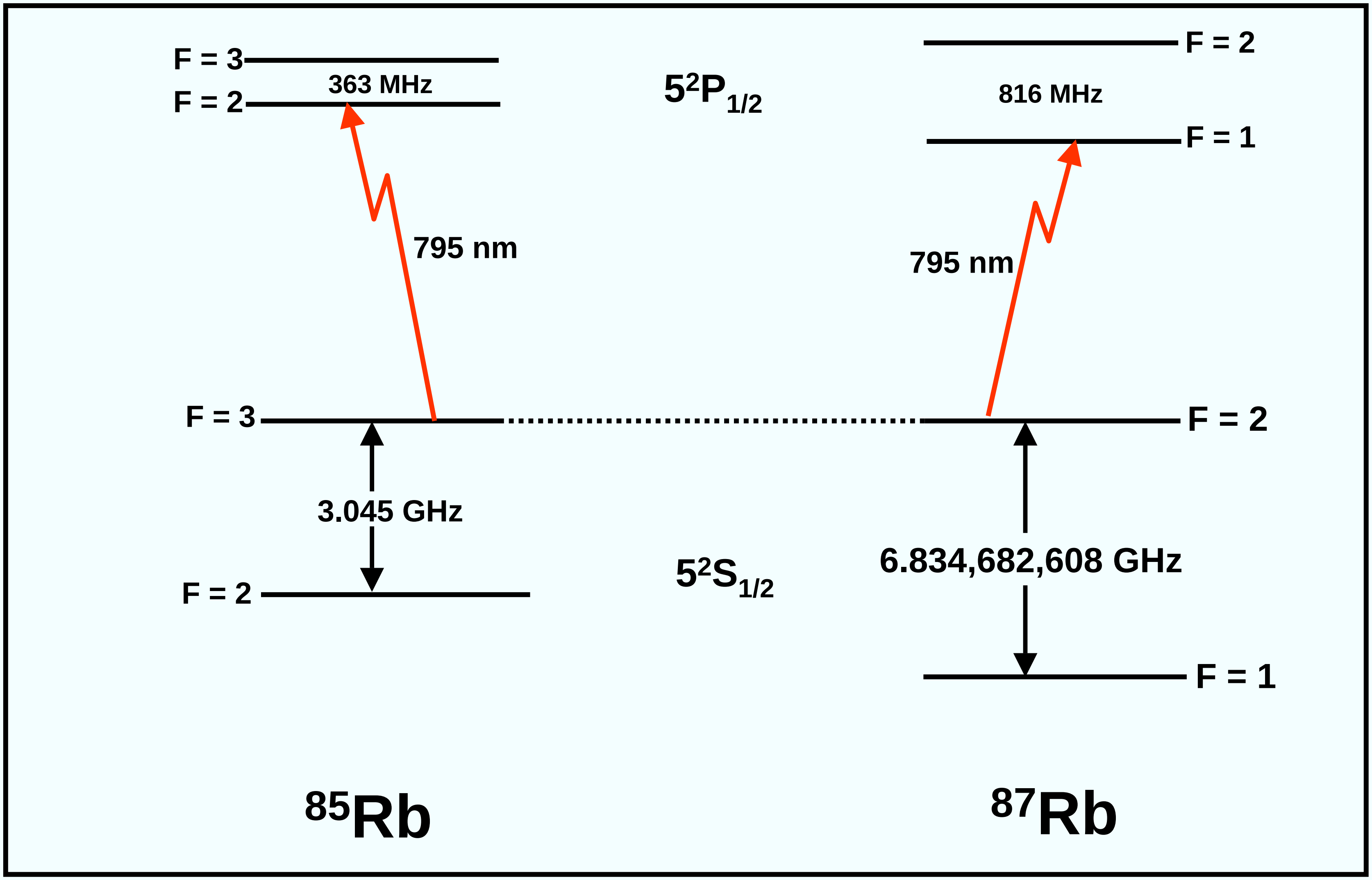


Atomic Resonator Concepts

- The energy levels used are due to the spin-spin interaction between the atomic nucleus and the outer electron in the ground state ($^2S_{1/2}$) of the atom; i.e., the ground state hyperfine transitions.
- Nearly all atomic standards use Rb or Cs atoms; nuclear spins I = 3/2 and 7/2, respectively.
- Energy levels split into $2(I \pm 1/2)+1$ sublevels in a magnetic field; the "clock transition" is the transition between the least magnetic-field-sensitive sublevels. A constant magnetic field, the "C-field," is applied to minimize the probability of the more magnetic-field-sensitive transitions.
- Magnetic shielding is used to reduce external magnetic fields (e.g., the earth's) at least 100-fold.
- The Heisenberg uncertainty principle limits the achievable accuracy: $\Delta E \Delta t \geq h/2\pi$, $E = h\nu$, therefore, $\Delta\nu\Delta t \geq 1$, and, long observation time $\rightarrow$ small frequency uncertainty.
- Resonance linewidth (i.e., 1/Q) is inversely proportional to coherent observation time Δt; Δt is limited by: 1.) when atom enters and leaves the apparatus, and 2.) when the atom stops oscillating due to collisions with other atoms or with container walls (collisions disturb atom's electronic structure).
- Since atoms move with respect to the microwave source, resonance frequency is shifted due to the Doppler effect (**k•v**); velocity distribution results in "Doppler broadening"; the second-order Doppler shift ($1/2\ v^2/c^2$) is due to relativistic time dilation.

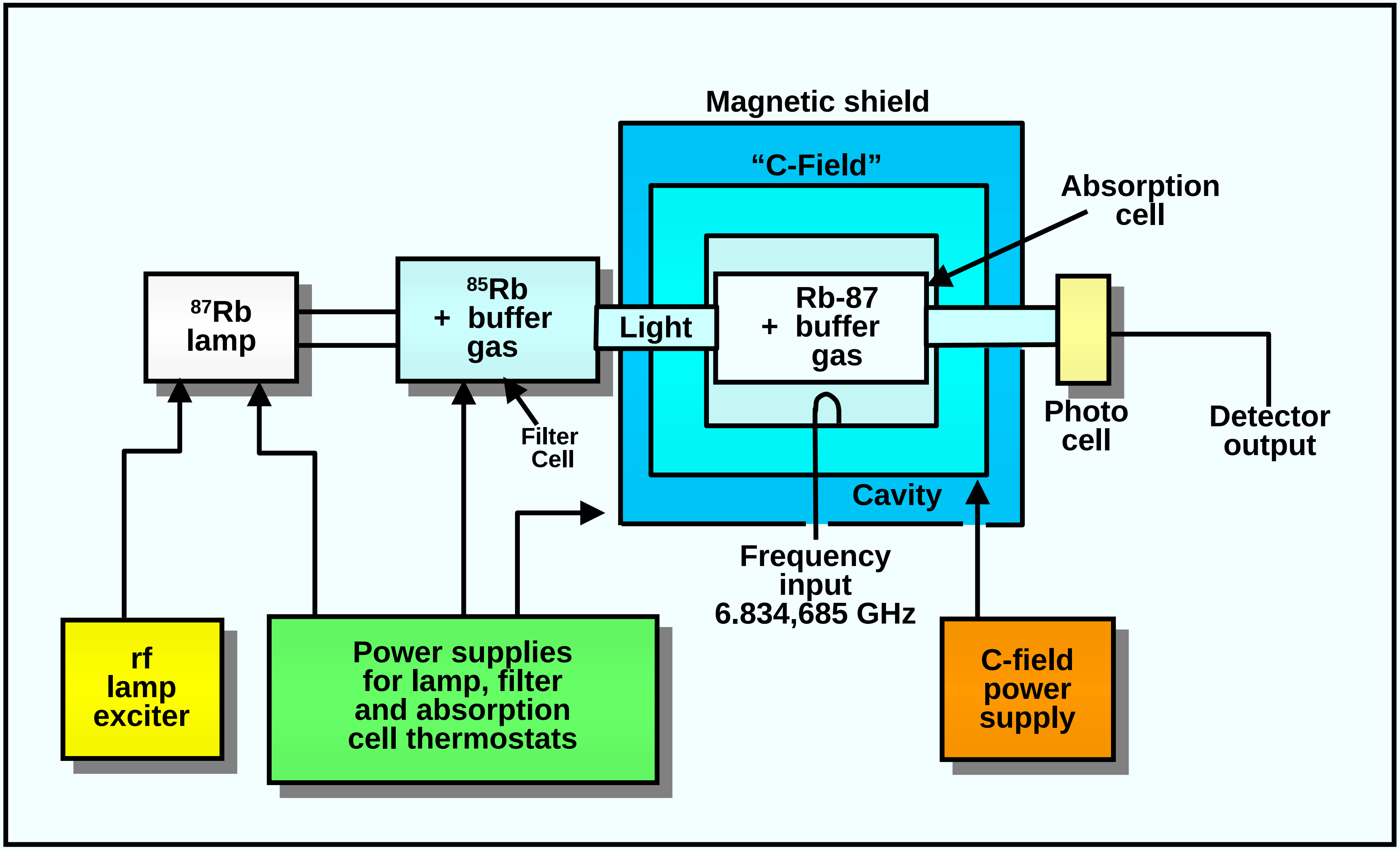
Rubidium Cell Frequency Standard

Energy level diagrams of ^{85}Rb and ^{87}Rb

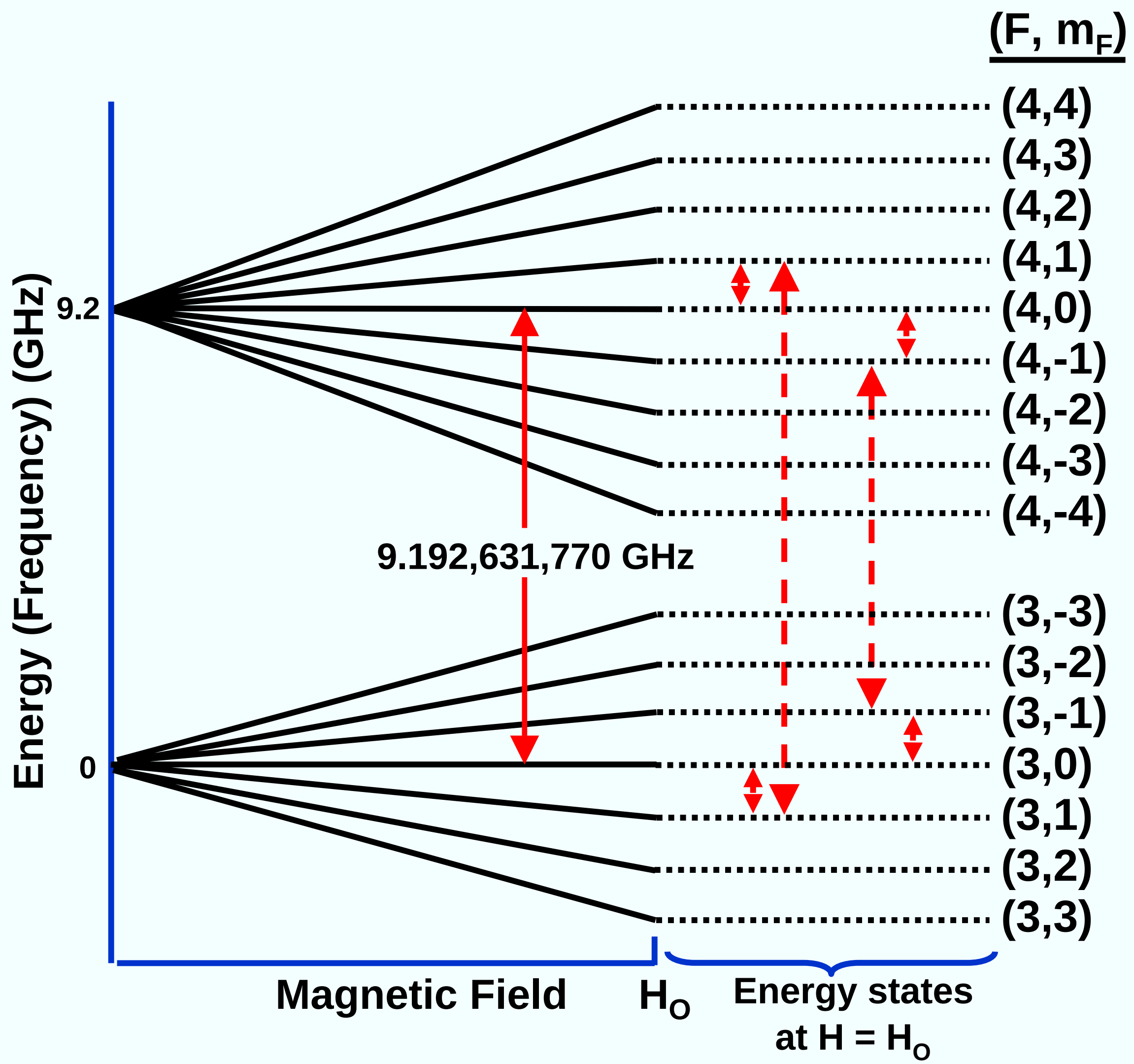


Rubidium Cell Frequency Standard

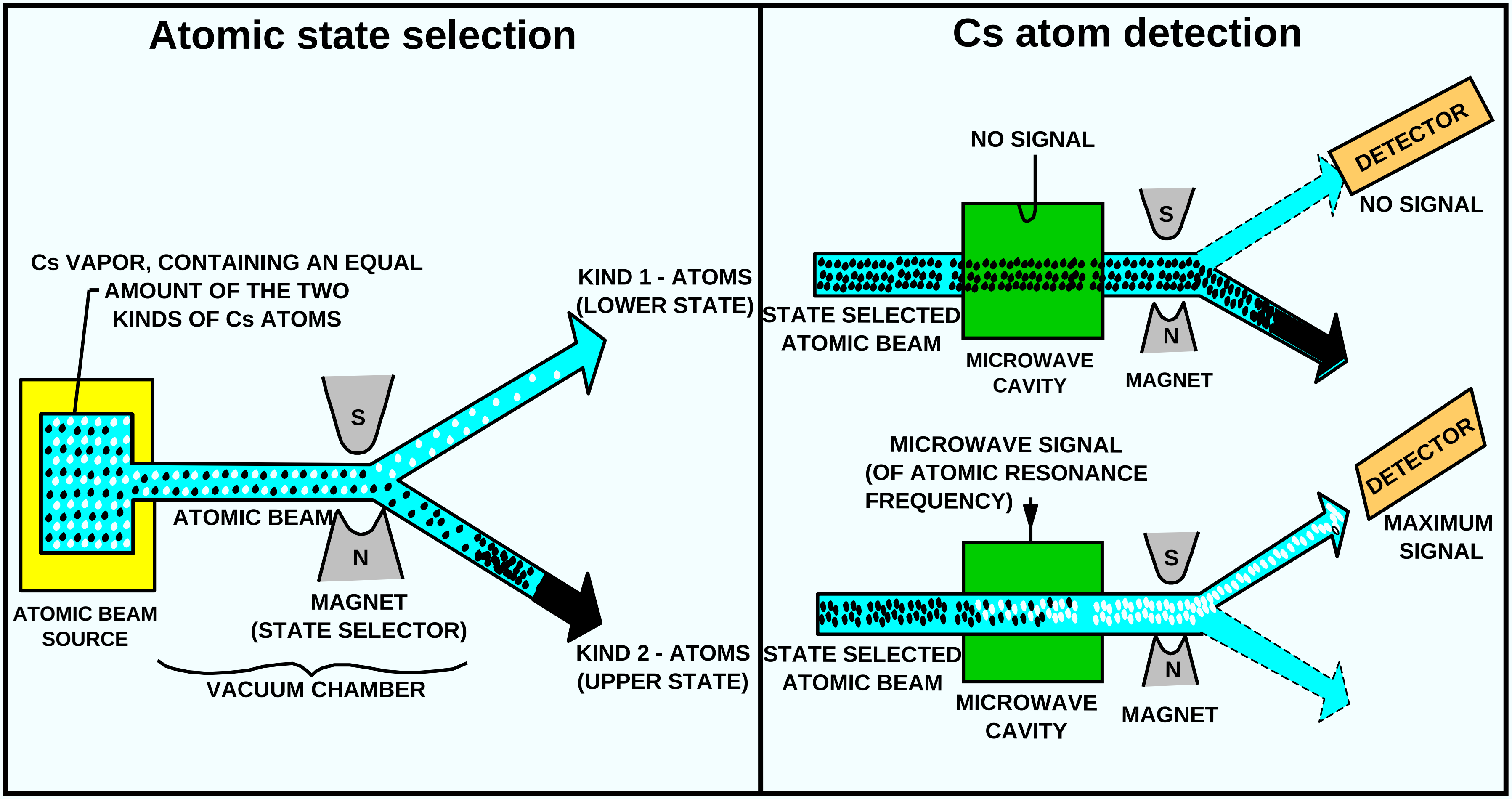
Atomic resonator schematic diagram



Cs Hyperfine Energy Levels

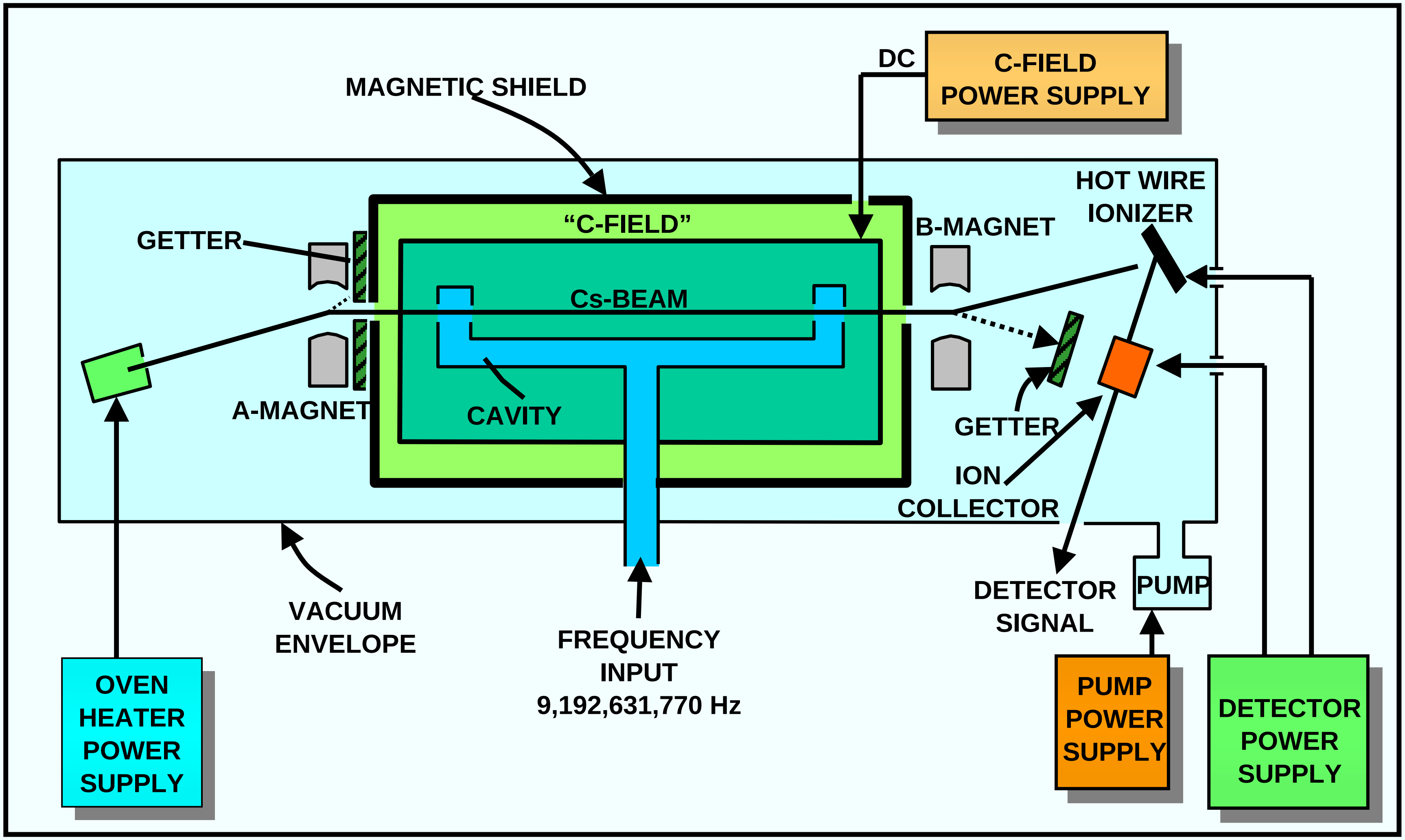


Cesium-Beam Frequency Standard

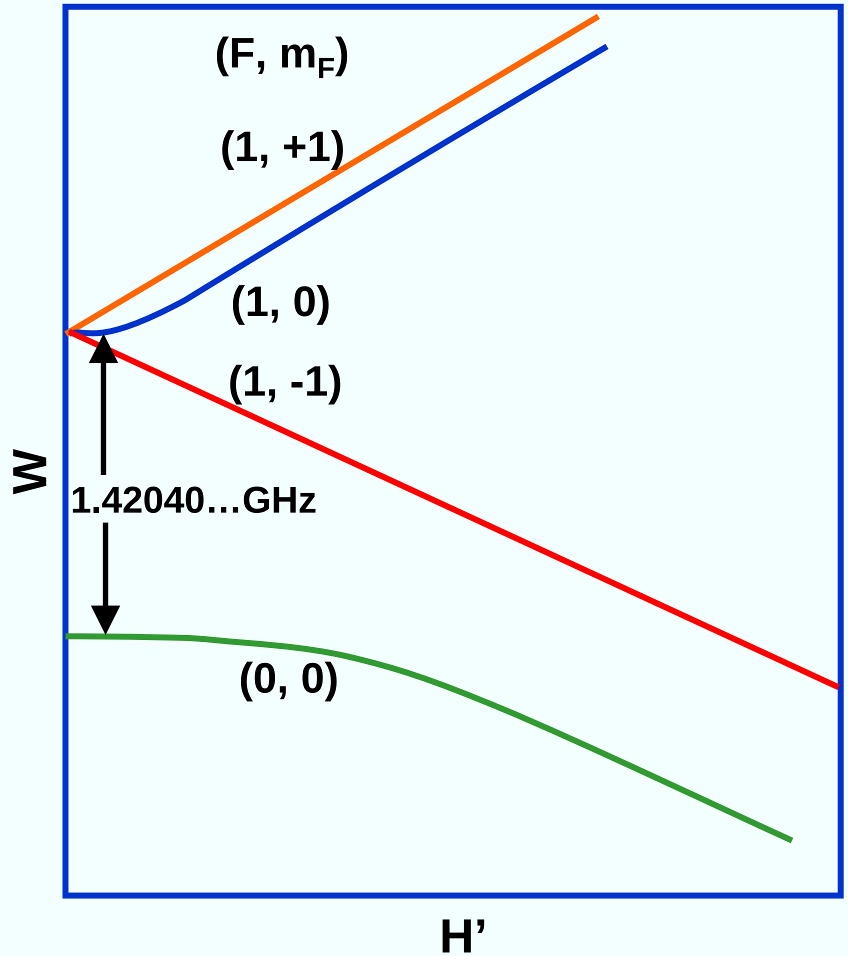


Cesium-Beam Frequency Standard

Cs atomic resonator schematic diagram

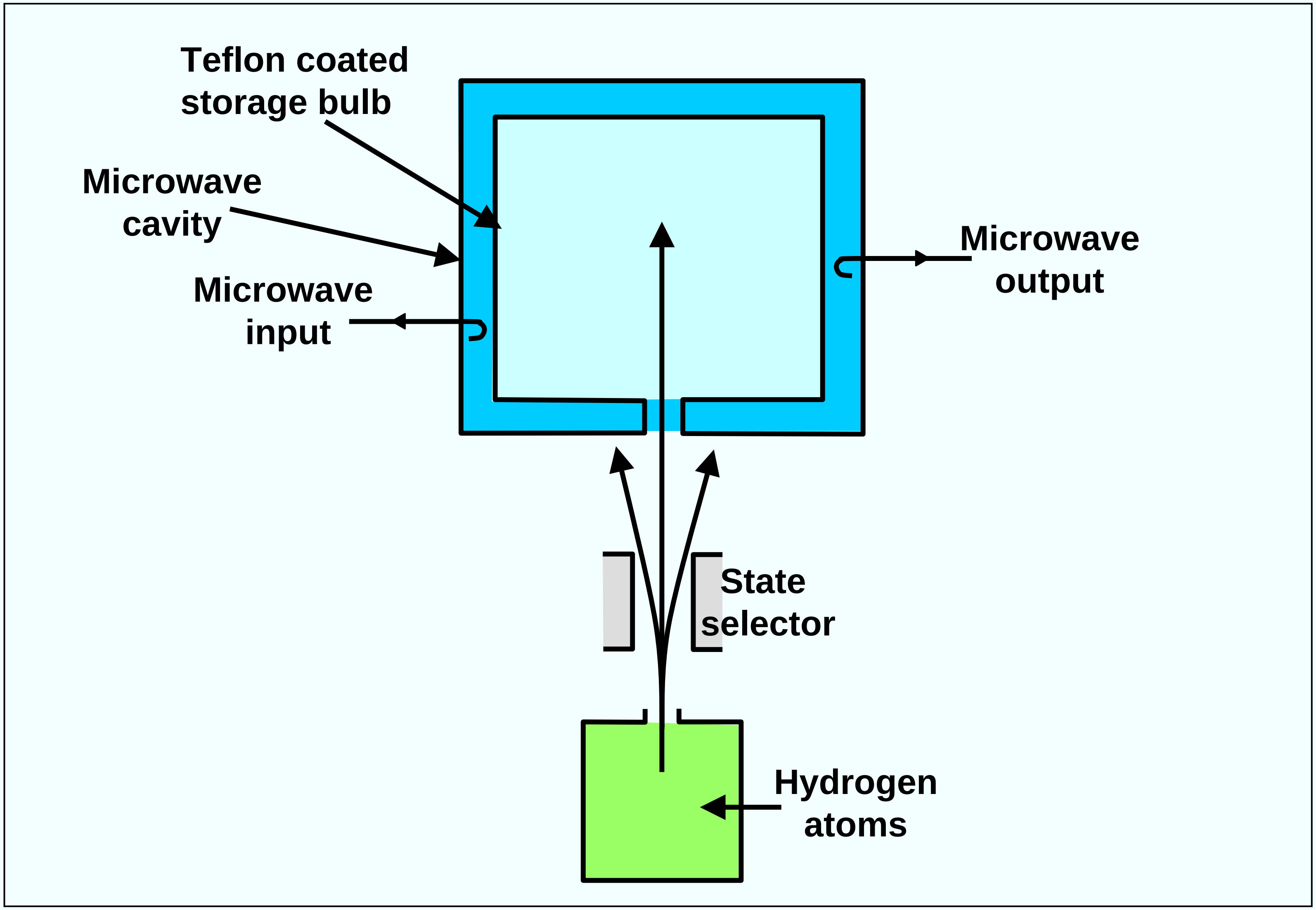


Atomic Hydrogen Energy Levels



Ground state energy levels of atomic hydrogen as a function of magnetic field H'.

Passive H-Maser Schematic Diagram



Atomic Resonator Instabilities

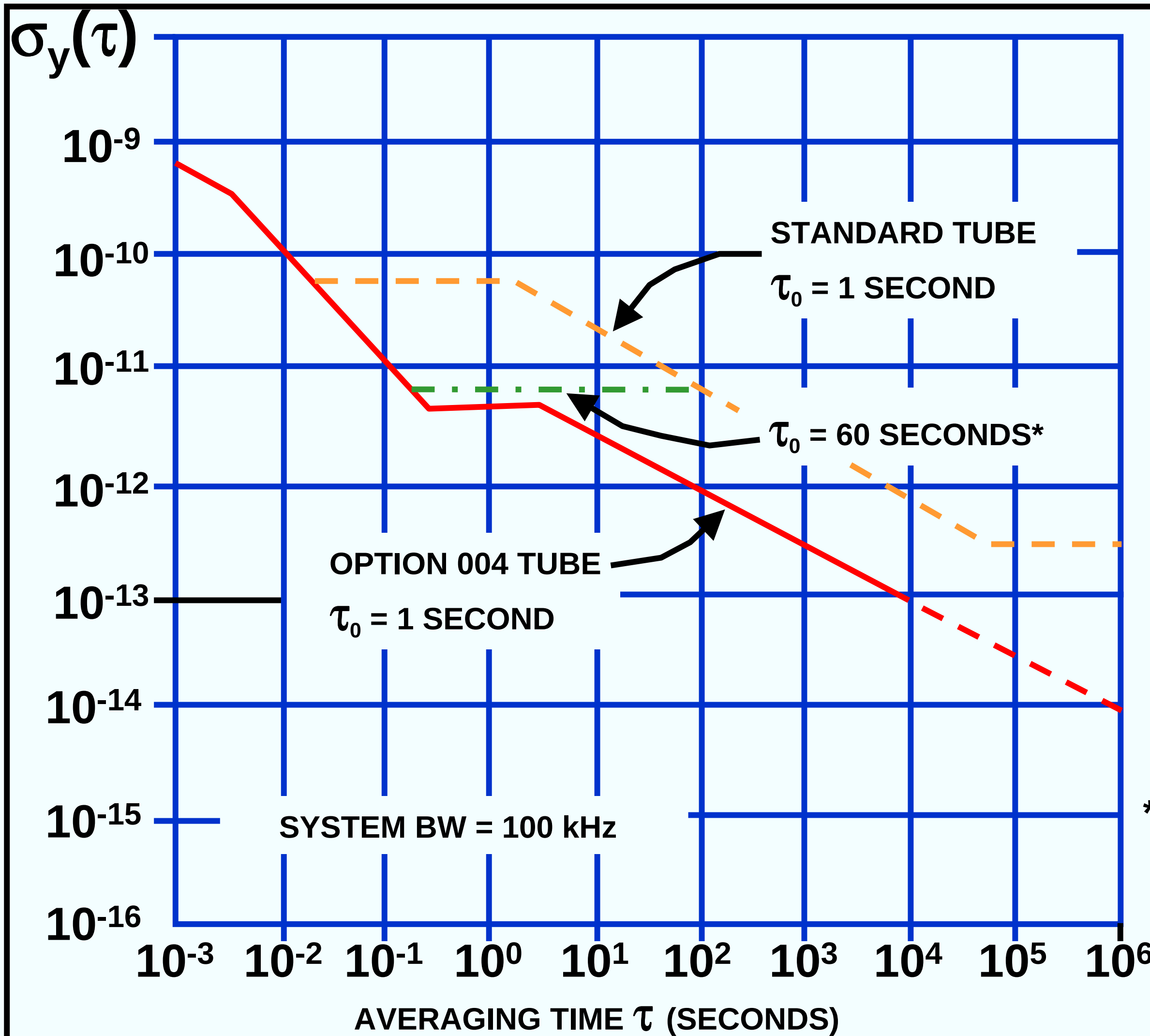
- **Noise** - due to the circuitry, crystal resonator, and atomic resonator. (See next page.)
- **Cavity pulling** - microwave cavity is also a resonator; atoms and cavity behave as two coupled oscillators; effect can be minimized by tuning the cavity to the atomic resonance frequency, and by maximizing the atomic resonance Q to cavity Q ratio.
- **Collisions** - cause frequency shifts and shortening of oscillation duration.
- **Doppler effects** - 1st order is classical, can be minimized by design; 2nd order is relativistic; can be minimized by slowing the atoms via laser cooling - see "Laser Cooling of Atoms" later in this chapter.
- **Magnetic field** - this is the only influence that directly affects the atomic resonance frequency.
- **Microwave spectrum** - asymmetric frequency distribution causes frequency pulling; can be made negligible through proper design.
- **Environmental effects** - magnetic field changes, temperature changes, vibration, shock, radiation, atmospheric pressure changes, and He permeation into Rb bulbs.

Noise in Atomic Frequency Standards

If the time constant for the atomic-to-crystal servo-loop is t_o, then at $\tau < t_o$, the crystal oscillator determines $\sigma_y(\tau)$, i.e., $\sigma_y(\tau) \sim \tau^{-1}$. From $\tau > t_o$ to the τ where the "flicker floor" begins, variations in the atomic beam intensity (shot-noise) determine $\sigma_y(\tau)$, and $\sigma_y(\tau) \sim (i\tau)^{-1/2}$, where i = number of signal events per second. Shot noise within the feedback loop shows up as white frequency noise (random walk of phase). Shot noise is generally present in any electronic device (vacuum tube, transistor, photodetector, etc.) where discrete particles (electrons, atoms) move across a potential barrier in a random way.

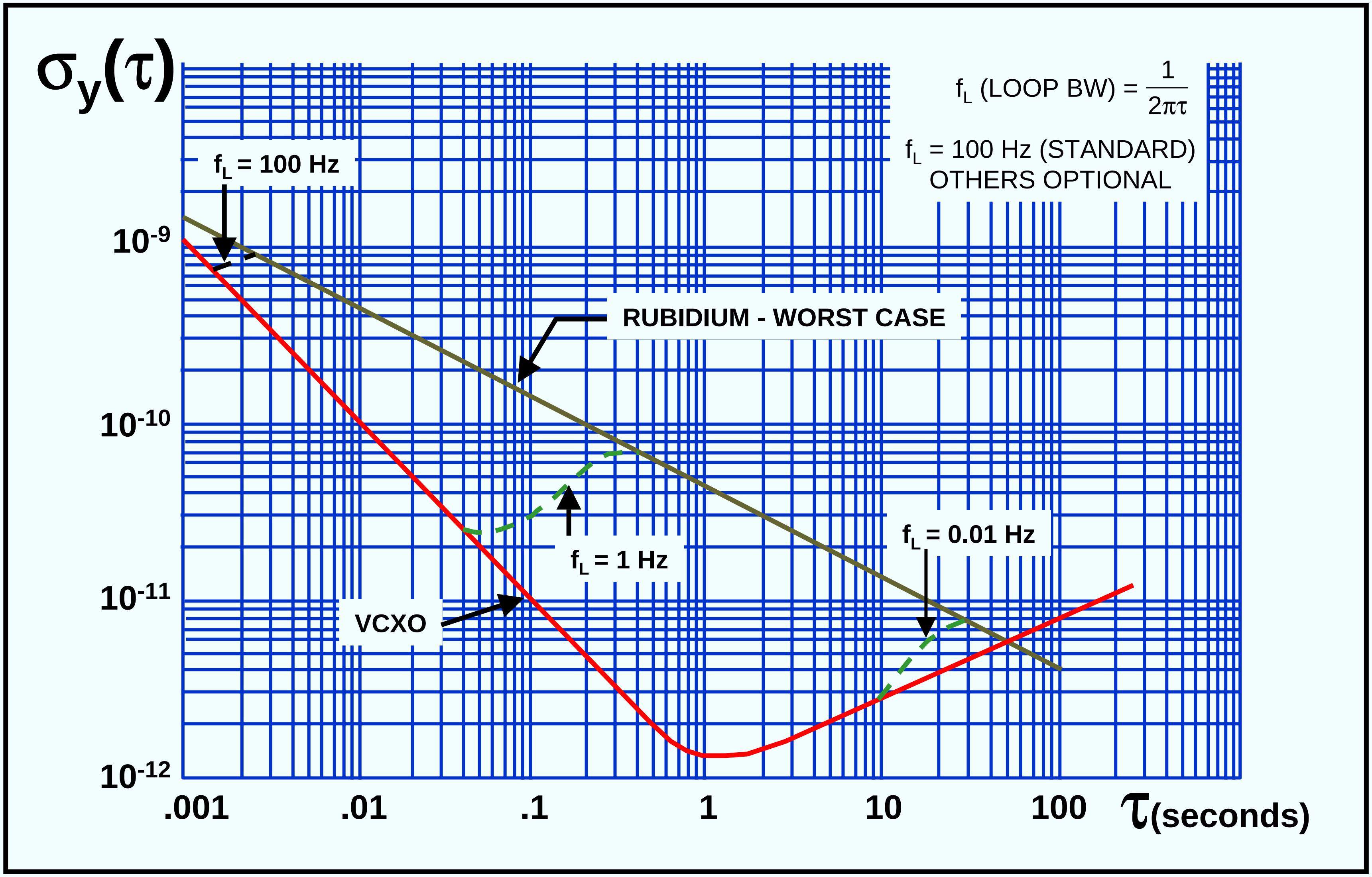
In commercial standards, t_o ranges from 0.01 s for a small Rb standard to 60 s for a high-performance Cs standard. In the regions where $\sigma_y(\tau)$ varies as τ^{-1} and $\tau^{-1/2}$, $\sigma_y(\tau) \propto (QS_R)^{-1}$, where S_R is the signal-to-noise ratio, i.e., the higher the Q and the signal-to-noise ratio, the better the short term stability (and the phase noise far from the carrier, in the frequency domain).

Short-Term Stability of a Cs Standard

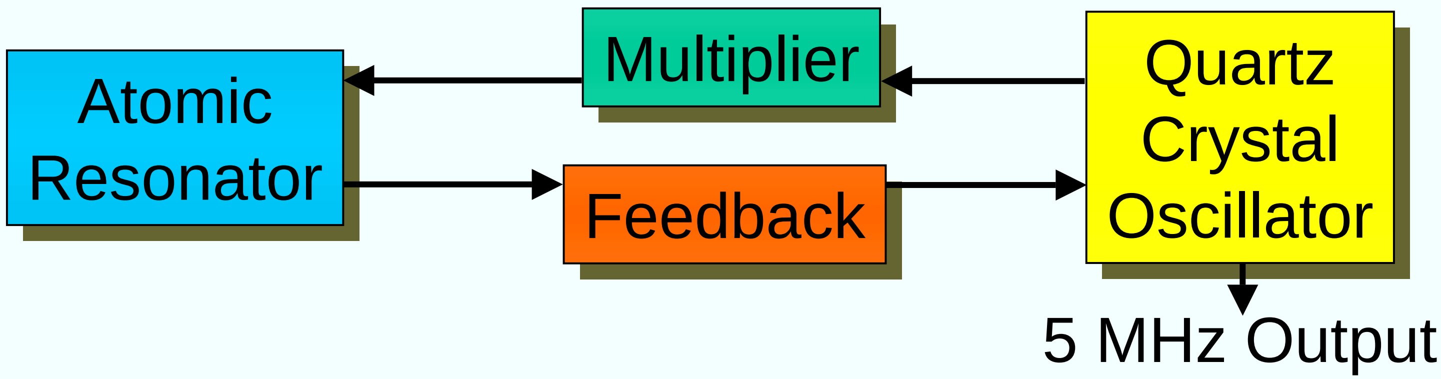


*** The 60 s time constant provides better short-term stability, but it is usable only in benign environments.**

Short-Term Stability of a Rb Standard



Acceleration Sensitivity of Atomic Standards



Let the servo loop time constant = t_0, let the atomic standard's $\Gamma = \Gamma_A$, and the crystal oscillator's (VCXO's) $\Gamma = \Gamma_O$. Then,

- For fast acceleration changes ($f_{vib} >> 1/2\pi t_0$), $\Gamma_A = \Gamma_O$
- For slow acceleration changes, ($f_{vib} << 1/2\pi t_0$), $\Gamma_A << \Gamma_O$
- For $f_{vib} \approx f_{mod}$, $2f_{mod}$, servo is confused, $\Gamma_A \approx \Gamma_O$, plus a frequency offset
- For small f_{vib}, (at Bessel function null), loss of lock, $\Gamma_A \approx \Gamma_O$

Atomic Standard Acceleration Effects

<u>In Rb cell standards</u>, high acceleration can cause Δf due to light shift, power shift, and servo effects:

- Location of molten Rb in the Rb lamp can shift
- Mechanical changes can deflect light beam
- Mechanical changes can cause rf power changes

<u>In Cs beam standards</u>, high acceleration can cause Δf due to changes in the atomic trajectory with respect to the tube & microwave cavity structures:

- Vibration modulates the amplitude of the detected signal. Worst when $f_{\mathbf{vib}} = f_{\mathbf{mod}}$.
- Beam to cavity position change causes cavity phase shift effects
- Velocity distribution of Cs atoms can change
- Rocking effect can cause Δf even when $f_{\mathbf{vib}} < f_{\mathbf{mod}}$

<u>In H-masers</u>, cavity deformation causes Δf due to cavity pulling effect

Magnetic Field Sensitivities of Atomic Clocks

Clock transition frequency $\nu = \nu_o + C_H H_o^2$, where C_H is the quadratic Zeeman effect coefficient (which varies as $1/\nu_o$).

Atom	Transition Frequency	C-field* (milligauss)**	Shielding Factor*	Sensitivity per gauss**
Rb	ν=6.8 GHz + (574 Hz/G^2) B_o^2	250	5k	10^{-11}
Cs	ν=9.2 GHz + (427 Hz/G^2) B_o^2	60	50k	10^{-13}
H	ν=1.4 GHz + (2750 Hz/G^2) B_o^2	0.5	50k	10^{-13}

* Typical values

** 1 gauss = 10^{-4} Tesla; Tesla is the SI unit of magnetic flux density.

Crystal's Influences on Atomic Standard

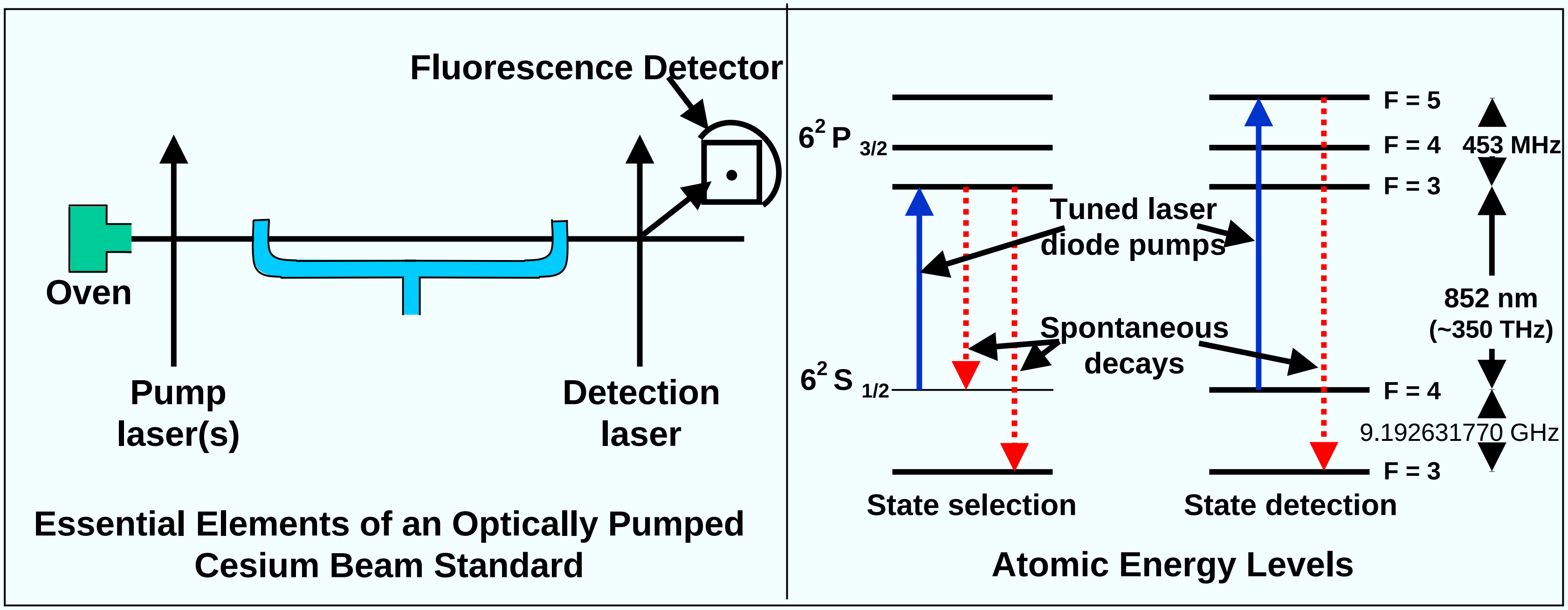
- **Short term stability** - for averaging times less than the atomic-to-crystal servo loop time constant, τ_L, the crystal oscillator determines $\sigma_y(\tau)$.

- **Loss of lock** - caused by large phase excursions in $t < \tau_L$ (due to shock, attitude change, vibration, thermal transient, radiation pulse). At a Rb standard's 6.8 GHz, for a $\Delta f = 1 \times 10^{-9}$ in 1s, as in a 2g tipover in 1s, $\Delta\phi \sim 7\pi$. Control voltage sweeping during reacquisition attempt can cause the phase and frequency to change wildly.

- **Maintenance or end of life** - when crystal oscillator frequency offset due to aging approaches EFC range (typically ~ 1 to 2×10^{-7}).

- **Long term stability** - noise at second harmonic of modulation f causes time varying Δf's; this effect is significant only in the highest stability (e.g., H and Hg) standards.

Optically Pumped Cs Standard

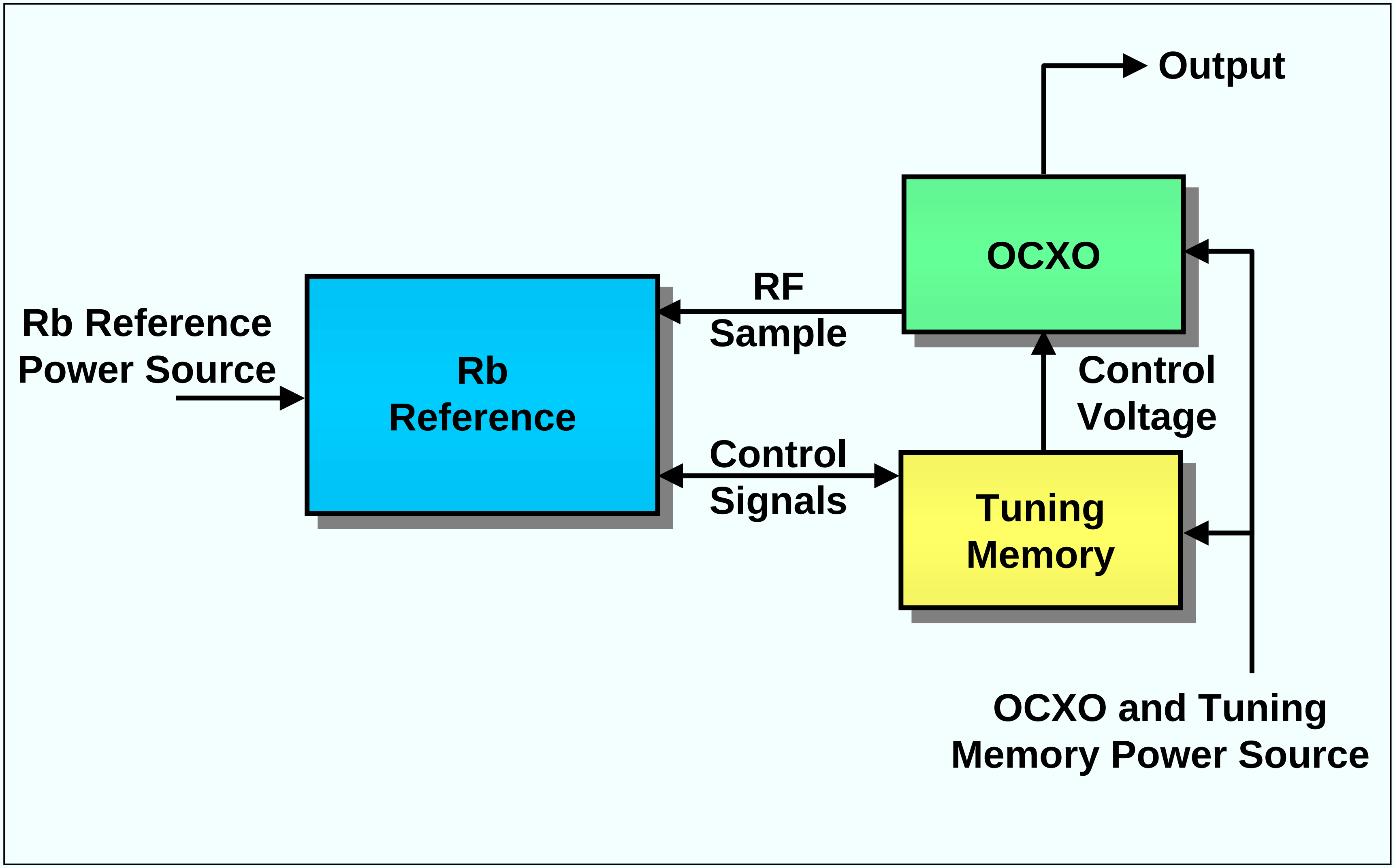


The proper atomic energy levels are populated by optical pumping with a laser diode. This method provides superior utilization of Cs atoms, and provides the potential advantages of: higher S/N, longer life, lower weight, and the possibility of trading off size for accuracy. A miniature Cs standard of 1 x 10^{-11} accuracy, and <<1 liter volume, i.e., about 100x higher accuracy than a Rb standard, in a smaller volume (but not necessarily the same shape factor) seems possible.

Rubidium - Crystal Oscillator (RbXO)

Rubidium Frequency Standard ($\approx$25W @ -55^{0}C)	**RbXO Interface**	**Low-power Crystal Oscillator**

RbXO Principle of Operation



CHAPTER 7
Oscillator Comparisons and Specifications

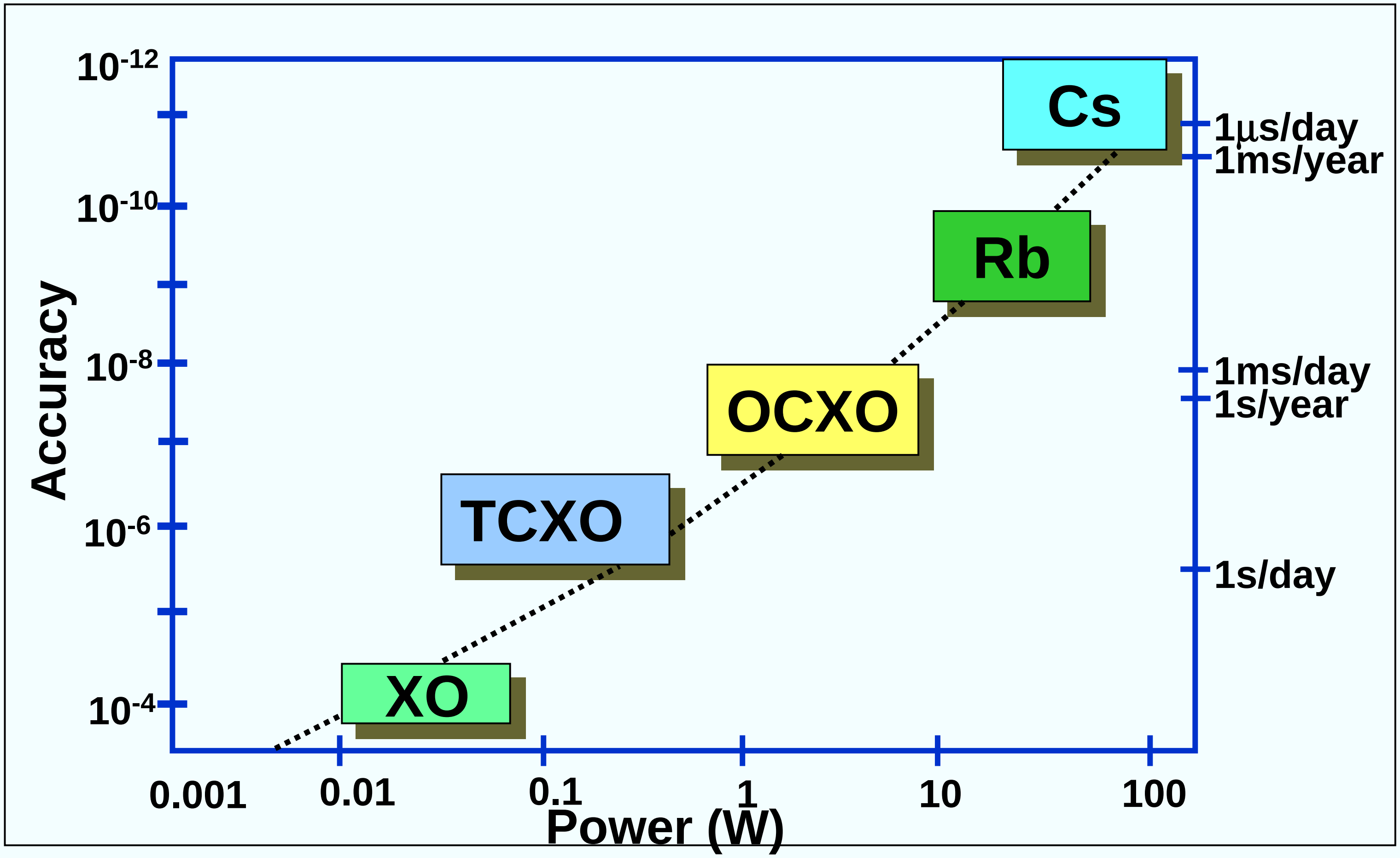
Oscillator Comparison

	Quartz Oscillators			Atomic Oscillators		
	TCXO	MCXO	OCXO	Rubidium	RbXO	Cesium
Accuracy * (per year)	2×10^{-6}	5×10^{-8}	1×10^{-8}	5×10^{-10}	7×10^{-10}	2×10^{-11}
Aging/Year	5×10^{-7}	2×10^{-8}	5×10^{-9}	2×10^{-10}	2×10^{-10}	0
Temp. Stab. (range, ^{0}C)	5×10^{-7} (-55 to +85)	3×10^{-8} (-55 to +85)	1×10^{-9} (-55 to +85)	3×10^{-10} (-55 to +68)	5×10^{-10} (-55 to +85)	2×10^{-11} (-28 to +65)
Stability, $\sigma_y(\tau)$ (τ = 1s)	1×10^{-9}	3×10^{-10}	1×10^{-12}	3×10^{-12}	5×10^{-12}	5×10^{-11}
Size (cm^3)	10	30	20-200	200-800	1,000	6,000
Warmup Time (min)	0.03 (to 1×10^{-6})	0.03 (to 2×10^{-8})	4 (to 1×10^{-8})	3 (to 5×10^{-10})	3 (to 5×10^{-10})	20 (to 2×10^{-11})
Power (W) (at lowest temp.)	0.04	0.04	0.6	20	0.65	30
Price (~$)	10 - 100	<1,000	200-2,000	2,000-8,000	<10,000	50,000

* Including environmental effects (note that the temperature ranges for Rb and Cs are narrower than for quartz).

Clock Accuracy vs. Power Requirement*

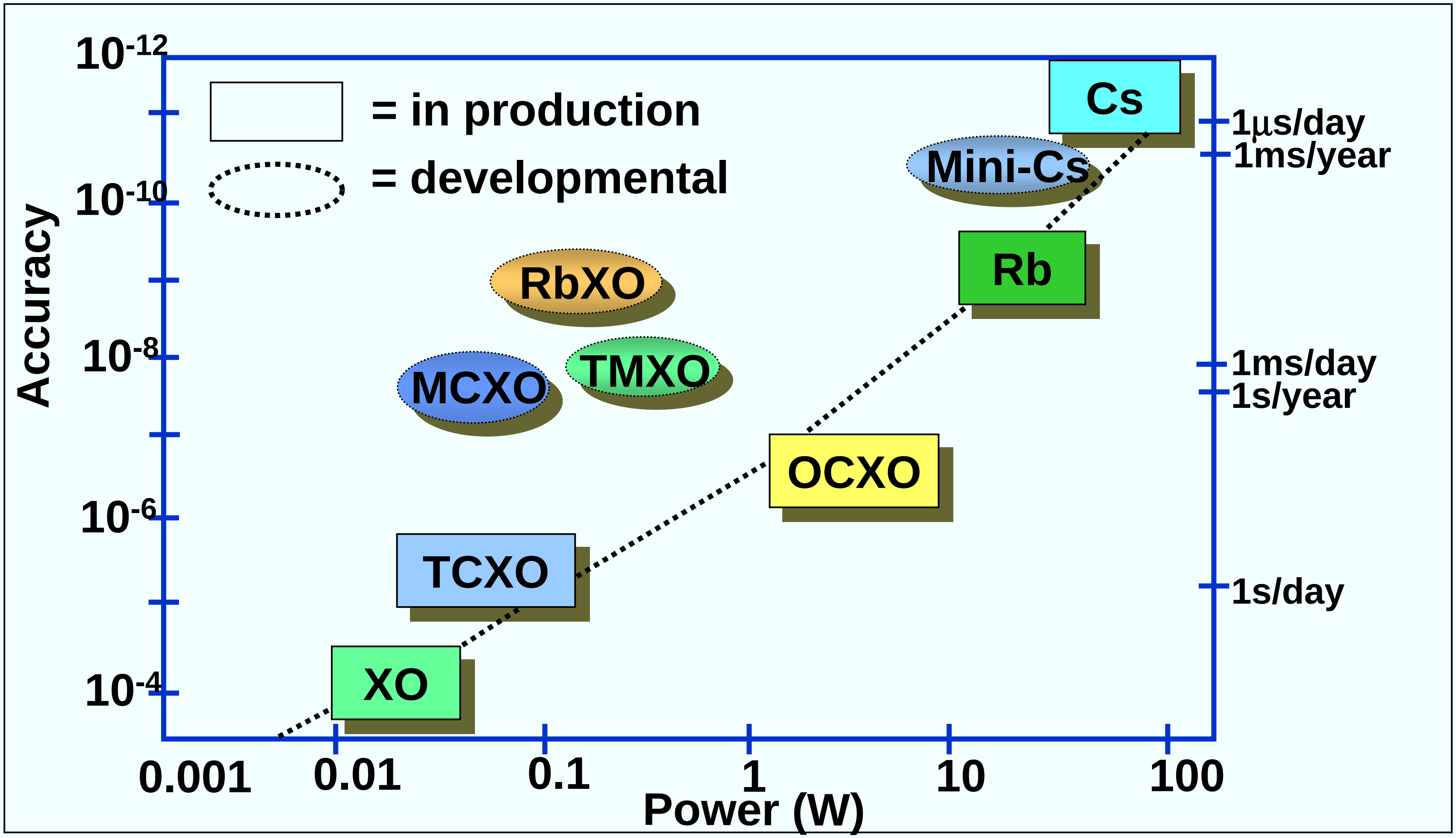
(Goal of R&D is to move technologies toward the upper left)



* Accuracy vs, size, and accuracy vs. cost have similar relationships

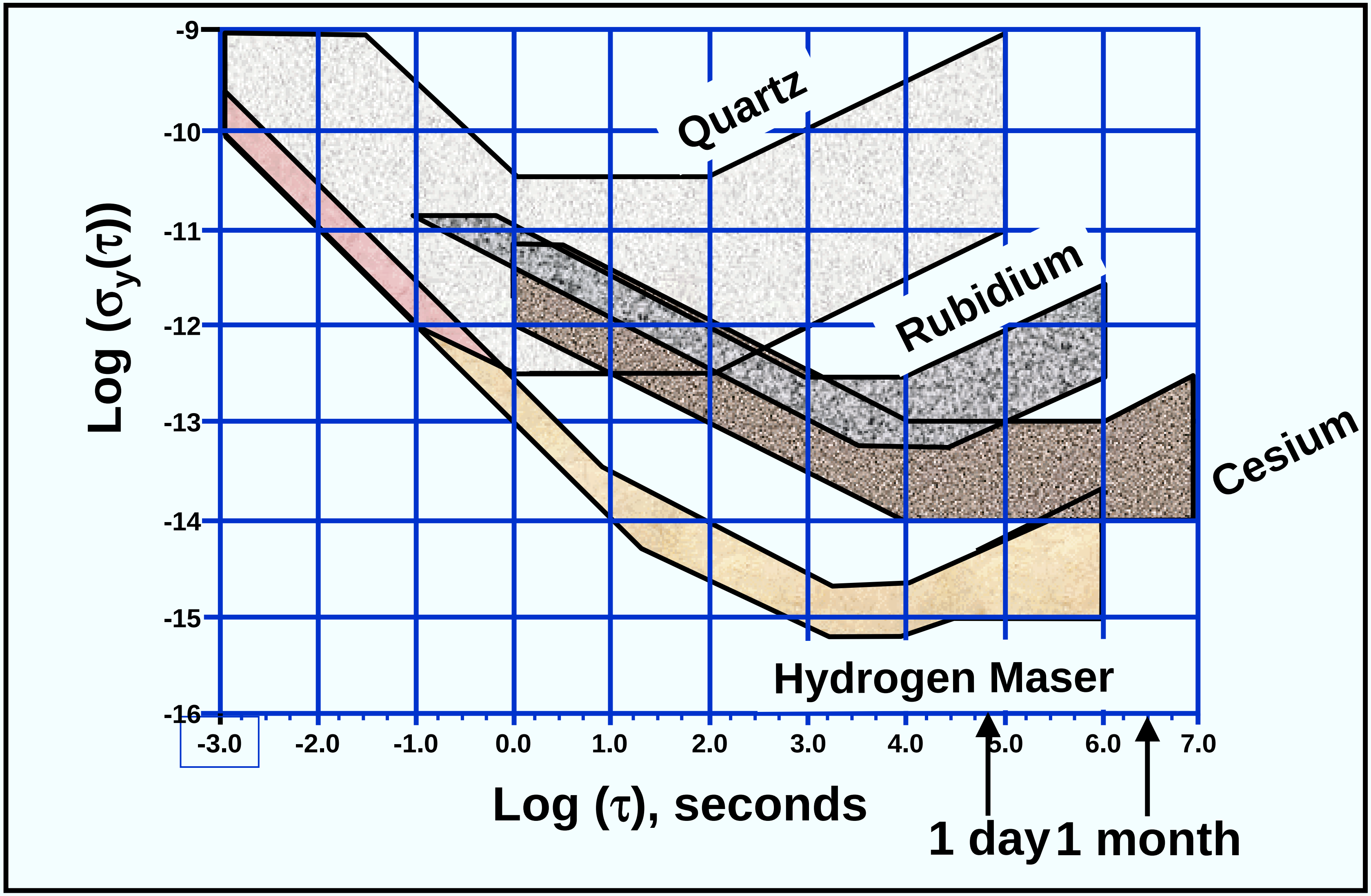
Clock Accuracy vs. Power Requirement*

(Goal of R&D is to move technologies toward the upper left)

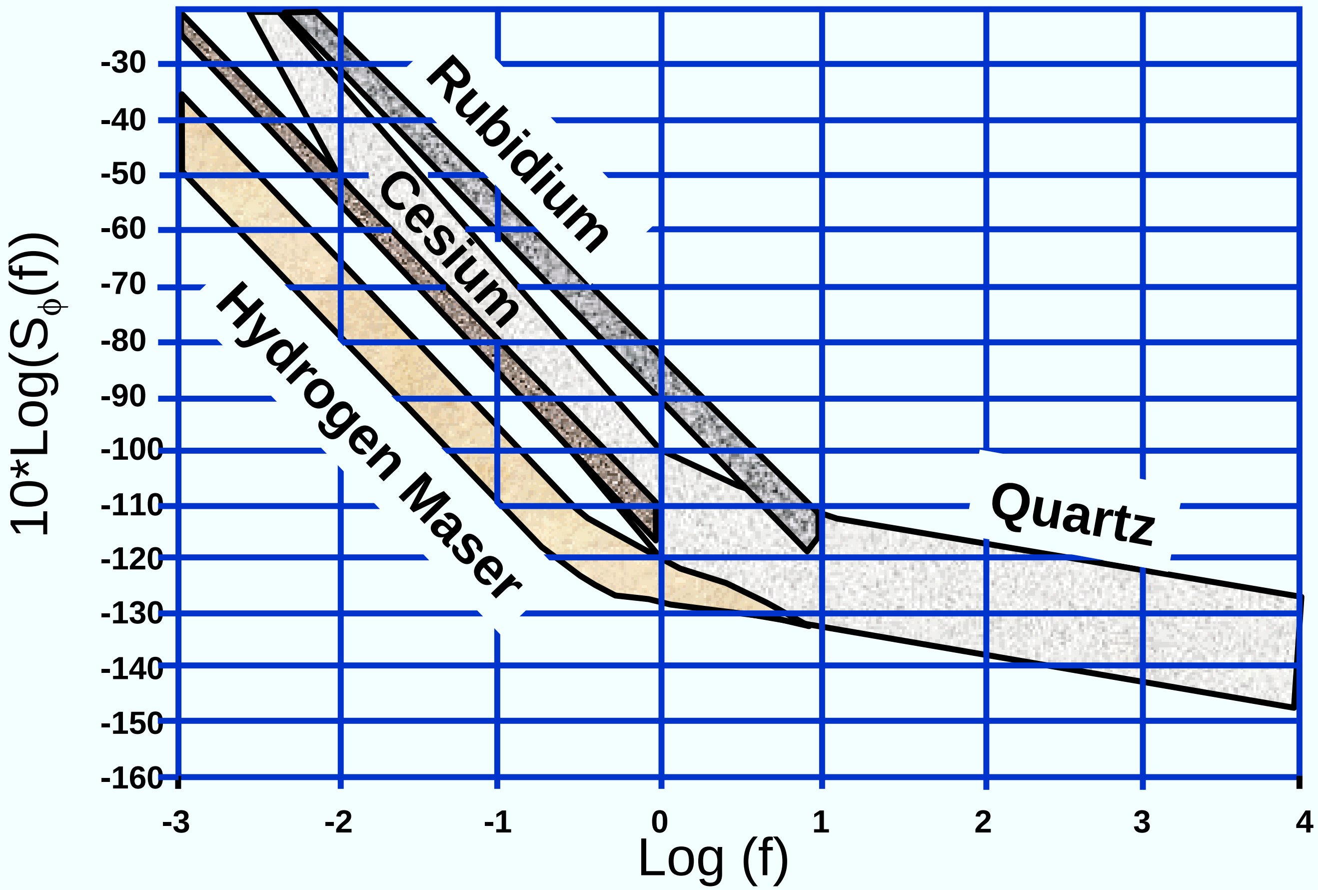


* Accuracy vs, size, and accuracy vs. cost have similar relationships

Short Term Stability Ranges of Various Frequency Standards



Phase Instabilities of Various Frequency Standards



Typical one-sided spectral density of phase deviation vs. offset frequency, for various standards, calculated at 5 MHz. L(f) = ½ S_ϕ

Weaknesses and Wearout Mechanisms

	Weaknesses	Wearout Mechanisms
Quartz	Aging Rad hardness	None
Rubidium	Life Power Weight	Rubidium depletion Buffer gas depletion Glass contaminants
Cesium	Life Power Weight Cost Temp. range	Cesium supply depletion Spent cesium gettering Ion pump capacity Electron multiplier

Why Do Crystal Oscillators Fail?

Crystal oscillators have no **inherent** failure mechanisms. Some have operated for decades without failure. Oscillators do fail (go out of spec.) occasionally for reasons such as:

- Poor workmanship & quality control - e.g., wires come loose at poor quality solder joints, leaks into the enclosure, and random failure of components
- Frequency ages to outside the calibration range due to high aging plus insufficient tuning range
- TCXO frequency vs. temperature characteristic degrades due to aging and the "trim effect".
- OCXO frequency vs. temperature characteristic degrades due to shift of oven set point.
- Oscillation stops, or frequency shifts out of range or becomes noisy at certain temperatures, due to activity dips
- Oscillation stops or frequency shifts out of range when exposed to ionizing radiation - due to use of unswept quartz or poor choice of circuit components
- Oscillator noise exceeds specifications due to vibration induced noise
- Crystal breaks under shock due to insufficient surface finish

Oscillator Selection Considerations

- Frequency accuracy or reproducibility requirement
- Recalibration interval
- Environmental extremes
- Power availability - must it operate from batteries?
- Allowable warmup time
- Short term stability (phase noise) requirements
- Size and weight constraints
- Cost to be minimized - acquisition or life cycle cost

Crystal Oscillator Specification: MIL-PRF-55310

MIL-PRF-55310D
15 March 1998

SUPERSEDING
MIL-0-55310C
15 Mar 1994

PERFORMANCE SPECIFICATION

OSCILLATOR, CRYSTAL CONTROLLED

GENERAL SPECIFICATION FOR

This specification is approved for use by all Departments and Agencies of the Department of Defense.

1. SCOPE

1.1 Statement of scope. This specification covers the general requirements for quartz crystal oscillators used in electronic equipment.

Full text version is available via a link from <http:\\www.ieee.org/uffc/fc>

CHAPTER 8
Time and Timekeeping

What Is Time?

- "What, then, is time? If no one asks me, I know; if I wish to explain to him who asks, I know not." --- Saint Augustine, circa 400 A.D.
- The question, both a philosophical and a scientific one, has no entirely satisfactory answer. "Time is what a clock measures." "It defines the temporal order of events." "It is an element in the four-dimensional geometry of space-time." “It is nature’s way of making sure that everything doesn't happen at once.”
- Why are there "arrows" of time? The arrows are: entropy, electromagnetic waves, expansion of the universe, k-meson decay, and psychological. Does time have a beginning and an end? (Big bang; no more "events", eventually.)
- The unit of time, the second, is one of the seven base units in the International System of Units (SI units)*. Since time is the quantity that can be measured with the highest accuracy, it plays a central role in metrology.

Dictionary Definition of “Time”

(From The Random House Dictionary of the English Language ©1987)

time (tim), n., *adj., u,* timed, tim~ing. —n. **1**. the system of those sequential relations that any event has to any other, as past, present, or future, indefinite and continuous duration regarded as that in which events succeed one another. **2.** duration regarded as belonging to the present life as distinct from the life to come or from eternity; finite duration. **3**. (*sometimes cap.)* a system or method of measuring or reckoning the passage of time: *mean time; apparent* time; Greenwich Time. **4.** a limited period or interval, as between two successive events: a *long time.* **5.** a particular period considered as distinct from other periods: *Youth is the best time of life.* **6.** Often, times. a. a period in the history of the world, or contemporary with the life or activities of a notable person: prehistoric times; *in Lincoln's* time. **b.** the period or era now or previously present: *a sign of the times*; How times have changed! **c.** a period considered with reference to it's events or prevailing conditions, tendencies Ideas, etc.: *hard times; a time of war*. **7.** a prescribed or allotted period, as of one's life, for payment of a debt, etc. 8. the end of a prescribed or allotted period, as of one's life or a pregnancy: *His time had come, but there was no one left to mourn over him. When her time came, her husband accompanied her to the delivery room*. **9.** a period with reference to personal experience of a specified kind: to have a good time; a hot time in the *old* town tonight. **10**. a period of work of an employee, or the pay for it; working hours or days or an hourly or daily pay rate. **11**. *Informal.* a term of enforced duty or imprisonment: to serve time *in* the army, do time *in prison.* **12**. the period necessary for or occupied by something: *The time of the baseball game was two hours and two minutes. The bus takes too much time, so I'll take a plane.* **13**. *leisure time; sufficient or spare time: to have time for a vacation; I* have no time to stop now. **14.** a particular or definite point in time, as indicated by a clock: What time is it? **15.** a particular part of a year, day, etc.; season or period: *It's time for lunch***. 16**. an appointed, fit, due, or proper instant or period: *A time for sowing; the time when the sun crosses the meridian There is A time for everything.* **17.** the particular point in time when an event Is scheduled to take place: *train time; curtain time*. **18**. an indefinite, frequently prolonged period or duration in the future: *Time will tell if what we have done here today was right*. **19.** the right occasion or opportunity: to watch one's time. **20.** each occasion of a recurring action or event: to *do* a thing *five times*, *It's* the *pitcher's* time at bat. **21.** times, used as a multiplicative word in phrasal combinations expressing how many instances of a quantity or factor are taken together: *Two goes into six three times, five times faster* **22.** Drama. *one of the three unities. Cf.* unity (def. 8) **23**. Pros. a unit or a group of units in the measurement of meter. **24.** *Music* **a**. tempo, relative rapidity of movement. **b.** the metrical duration of a note or rest. **c.** proper or characteristic tempo. **d**. the general movement of a particular kind of musical composition with reference to its rhythm, metrical structure, and tempo. **e.** the movement of a dance or the like to music so arranged: waltz time. **25**. *Mil.* rate of marching, calculated on the number of paces taken per minute: *double* time; *quick* time. **26.** *Manege.* each completed action or movement of the horse. **27. against time,** in an effort to finish something within a limited period: *We worked against time to get out the newspaper.* **28**. **ahead of time,** before the time due; early: *The building was completed ahead of time.* **29**. **at one time, a.** once; in a former time: *At one time they owned a restaurant.* **b**. *at the same time*; at once: *They all tried to talk at one time*. **30**. **at the same time,** *nevertheless; yet: I'd like to try it but at the same time I'm a little afraid* **31. at times** at intervals; occasionally: *At times the city becomes intolerable.* **32. beat someone's time.** Slang. to compete for or win a person being dated or courted by another; prevail over a rival: *He accused me, his own brother, of trying to beat his time.* **33. behind the times,** old-fashioned; dated: These attitudes are *behind the* times. **34. for the time being, temporarily;** for the present: *Let's forget about it for the time being.* **35**. **from time to time,** on occasion; occasionally; at intervals: *She comes to see us from time to time.* **36. gain time,** to postpone in order to make preparations or gain an advantage delay the outcome of: He *hoped* to gain time by putting *off* signing the papers *for* a few days more. **37. in good time, a.** at the right time; on time, punctually. **b.** in advance of the right time; early: *We arrived at the appointed* spot in good time. **38. in no time,** in a very brief time; almost at once: *Working together, they cleaned the entire house in no time.* **39. in time, a.** early enough: to come in time for dinner. **b**. in the future; eventually: *In time he'll see what is right.* **c.** in the correct rhythm or tempo: *There would always be at least one child who couldn't play in time with the music* **40**. **keep time, a.** to record time, as a watch or clock does h to mark or observe the tempo. **c.**to perform rhythmic movements in unison. **41**. **kill time,** to occupy oneself with some activity to make time pass quickly: *While I was waiting, I killed time counting the cars on the freight trains*. **42. make time, a. to move** quickly, esp. in an attempt to recover lost time. **b.** to travel at a particular speed. **43. make time with,** Slang. to pursue or take as a sexual partner. **44. many a time,** again and again; frequently: *Many a time they didn't have enough to eat and went to bed hungry.* **45. mark time, a. to** suspend progress temporarily, as to await developments; fail to advance. **b.** *Mil.* to move the feet alternately as in marching, but without advancing. **46**. **on one's own time,** during one's free time; without payment: *He worked out more efficient production methods on his own time.* **47. on time, a.** at the specified time; punctuaBy. **b.** to be paid for within a designated period of time, as in installments: *Many people are never out of debt because they buy everything on time.* **48. out of time,** not in the proper rhythm: *His singing was out of time with the music* **49. Pass the time of day.** to converse briefly with or greet someone: *The women would stop in the market to pass the time of day.* **50. take one's time,** to be slow or leisurely; dawdle: *Speed was important here, but he just took his time.* **51. time after time.** again and again; repeatedly; often: *I've told him time after time not to slam the door.* **52**. **time and time again,** repeatedly; often: *Time and time again I warned her to stop smoking.* Also, **time and again. 53**. **time of llfe,** (one's) age: *At your time of life you must be careful not to overdo things.* **54. time of one's life,** I*nformal.* an extremely enjoyable experience: *They had* the time of their *lives* on their trip to Europe.. **55.** of. pertaining to, or showing the passage of time **56.** (of an explosive device) containing a clock so that it will detonate at the desired moment a time bomb **57**. Com. payable at a stated period of time after presentment: *time drafts* or *notes*. **58.** of or pertaining to purchases on the installment plan, or with payment postponed. v.t. **59.** to measure or record the speed, duration, or rate of: to time a race. **60.** to fix the duration of: *The proctor timed the test at 16 minutes.* **61.** to fix the interval between (actions, events, etc.): *They timed their strokes at six per minute. **62.** to* regulate (a train, clock etc.) as to time. **63**. to appoint or choose the moment or occasIon for; schedule: *He timed the attack perfectly.* —v.i. **64.** to keep time; sound or move in unison. [bef. 900; (n.)

The Second

- The SI unit of time is the second (symbol s).
- The second was defined, by international agreement, in October, 1967, at the XIII General Conference of Weights and Measures.
- **The second is "the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium atom 133."**
- Prior to 1967, the unit of time was based on astronomical observations; the second was defined in terms of ephemeris time, i.e., as "1/31,556,925.9747 of the tropical year..."
- The unit of frequency is defined as the hertz (symbol Hz). One hertz equals the repetitive occurrence of one "event" per second.

Frequency and Time

$$f = \frac{1}{\tau}$$

where f = frequency (= number of "events" per unit time), and
τ = period (= time between "events")

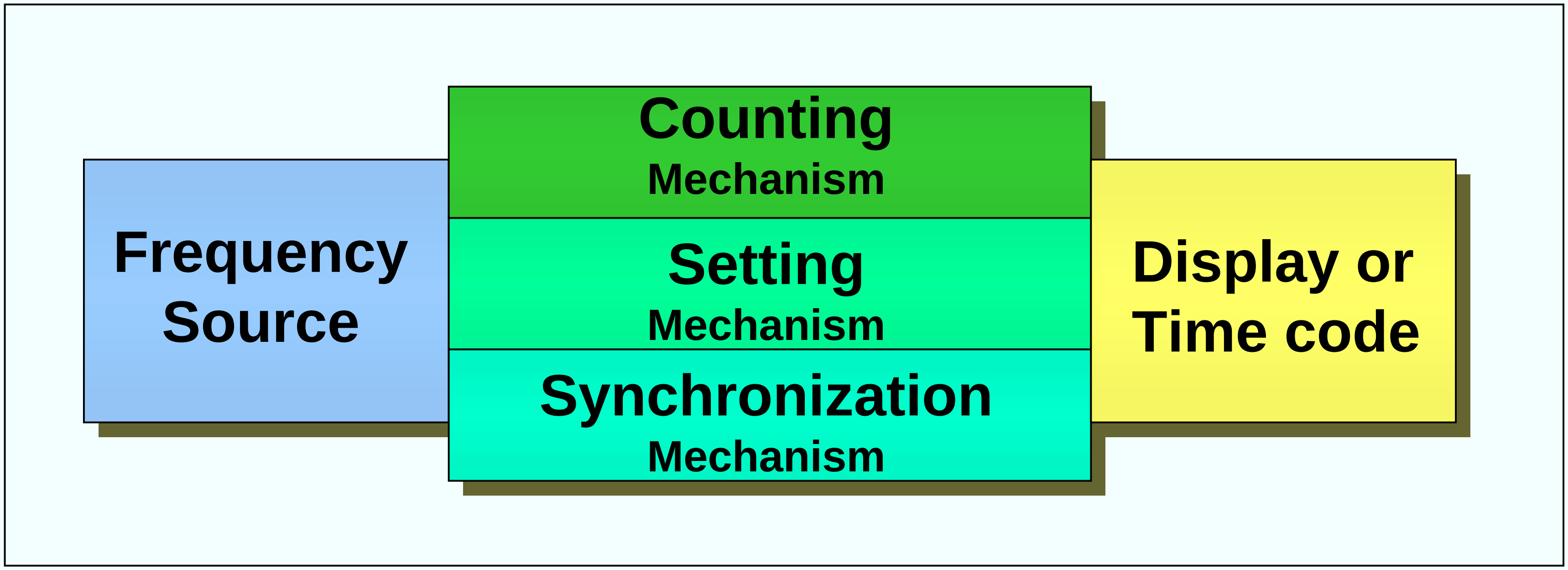
$$\text{Accumulated clock time} = \frac{\text{Total number of events}}{\text{Number of events per unit of time}}$$

$$\text{Example}: \frac{3 \text{ rotations of the earth}}{1 \text{ rotation/day}} = 3 \text{ days}$$

Frequency source + counting mechanism $\rightarrow$ clock

Examples of frequency sources: the rotating earth, pendulum, quartz crystal oscillator, and atomic frequency standard.

Typical Clock System



$$t = t_0 + \Sigma\Delta\tau$$

Where t is the time output, t_0 is the initial setting, and $\Delta\tau$ is the time interval being counted.

Evolution of Clock Technologies

- Sundials, and **continuous flow** of:
  - Water (clepsydra)
  - Sand (hour glass)
  - Falling weights, with frictional control of rate

- Vibrating, but **non-resonant motion** - escapement mechanisms: falling weight applies torque through train of wheels; rate control depends on moments of inertia, friction and torque; period is the time it takes to move from one angular position to another.

- **Resonant control**
  - Mechanical: pendulum, hairspring and balance wheel
  - Mechanical, electrically driven: tuning fork, quartz resonator
  - Atomic and molecular

Progress in Timekeeping

Time Period	Clock/Milestone	Accuracy Per Day
4th millennium B.C.	Day & night divided into 12 equal hours	
Up to 1280 A.D.	Sundials, water clocks (clepsydrae)	~1 h
~1280 A.D.	Mechanical clock invented- assembly time for prayer was first regular use	~30 to 60 min
14th century	Invention of the escapement; clockmaking becomes a major industry	~15 to 30 min
~1345	Hour divided into minutes and seconds	
15th century	Clock time used to regulate people's lives (work hours)	~2 min
16th century	Time's impact on science becomes significant (Galileo times physical events, e.g., free-fall)	~1 min
1656	First pendulum clock (Huygens)	~100 s
18th century	Temperature compensated pendulum clocks	1 to 10 s
19th century	Electrically driven free-pendulum clocks	10^{-2} to 10^{-1} s
~1910 to 1920	Wrist watches become widely available	
1920 to 1934	Electrically driven tuning forks	10^{-3} to 10^{-2} s
1921 to present	Quartz crystal clocks (and watches. Since ~1971)	10^{-5} to 10^{-1} s
1949 to present	Atomic clocks	10^{-9} to 10^{-4} s

Clock Errors

$$\mathbf{T(t) = T_0 + \int_0^t R(t)dt + \varepsilon(t) = T_0 + (R_0 t + 1/2At^2 + \ldots) + \int_0^t E_i(t)dt + \varepsilon(t)}$$

Where,

$T(t)$ = time difference between two clocks at time **t** after synchronization

$T_{\mathbf{0}}$ = synchronization error at t = 0

$R(t)$ = the rate (i.e., fractional frequency) difference between the two clocks under comparison; $R(t) = R_{\mathbf{0}} + At + \ldots E_{\mathbf{i}}(t)$

$\varepsilon(t)$ = error due to random fluctuations = $\tau\sigma_y(\tau)$

R_0 = R(t) at t = 0

A = aging term (higher order terms are included if the aging is not linear)

$E_i(t)$ = rate difference due to environmental effects (temperature, etc.)

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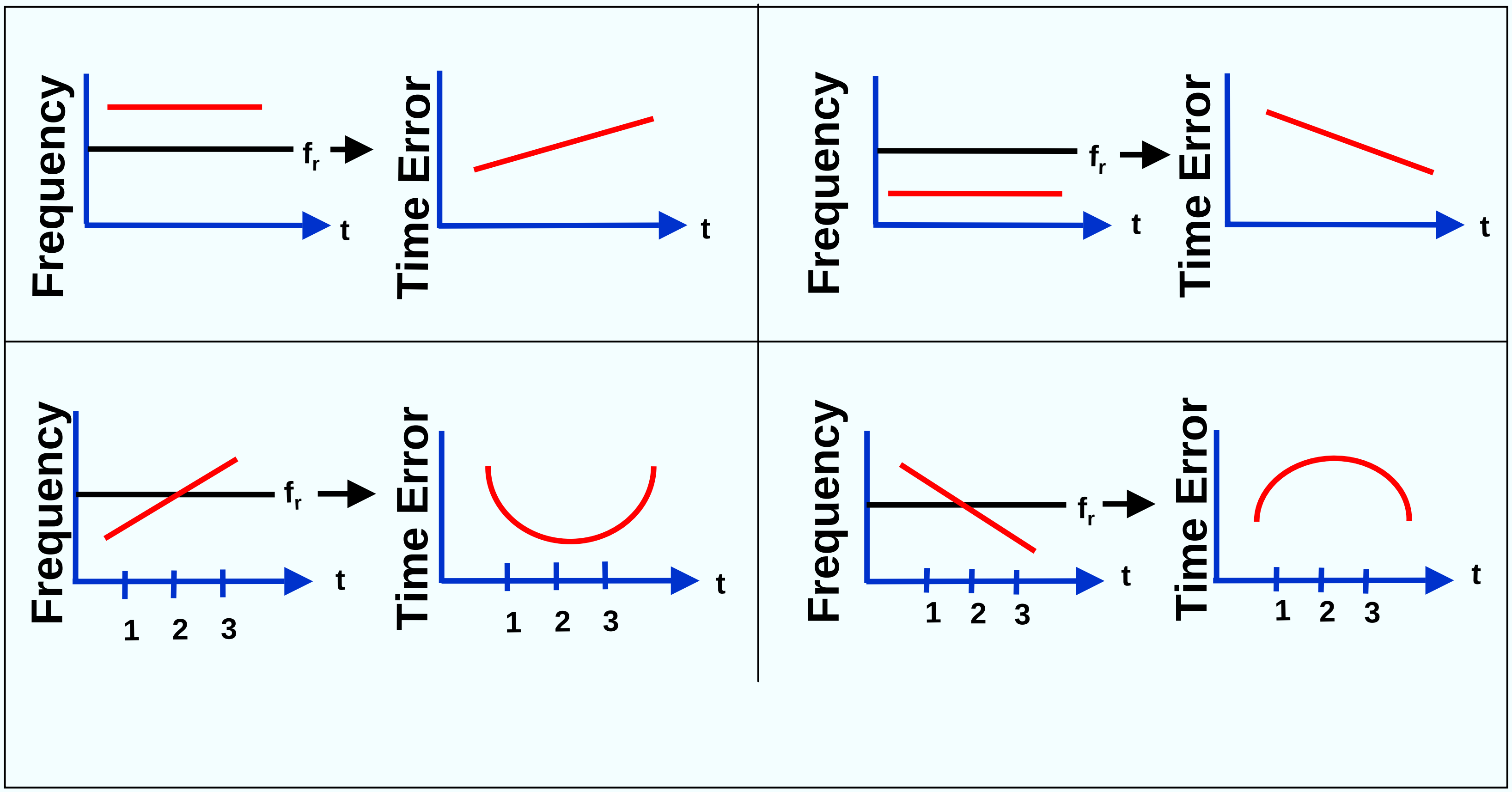
Example: If a watch is set to within 0.5 seconds of a time tone (T_0 = 0.5 s), and the watch initially gains 2 s/week (R_0 = 2 s/week), and the watch rate ages -0.1 s per week^2, (A = -0.1 s/week^2), then after 10 weeks (and assuming $E_i(t) = 0$):

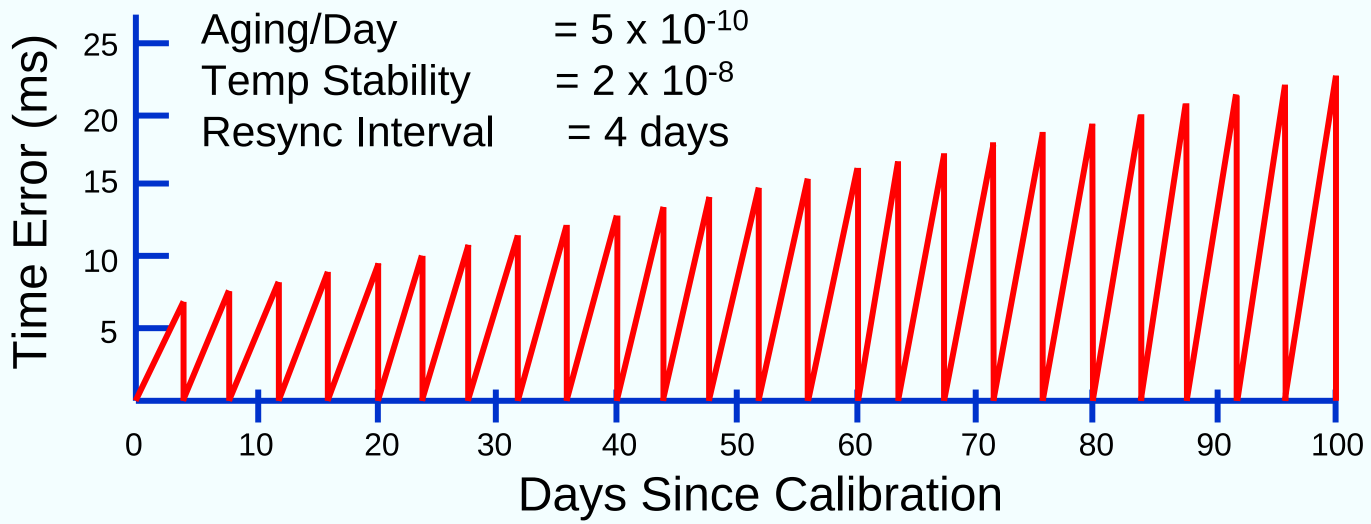
T (10 weeks) = 0.5 (2 x 10) + 1/2(-0.1 x (10)^2) = 15.5 seconds.

Frequency Error vs. Time Error



f_r = reference (i.e., the “correct”) frequency

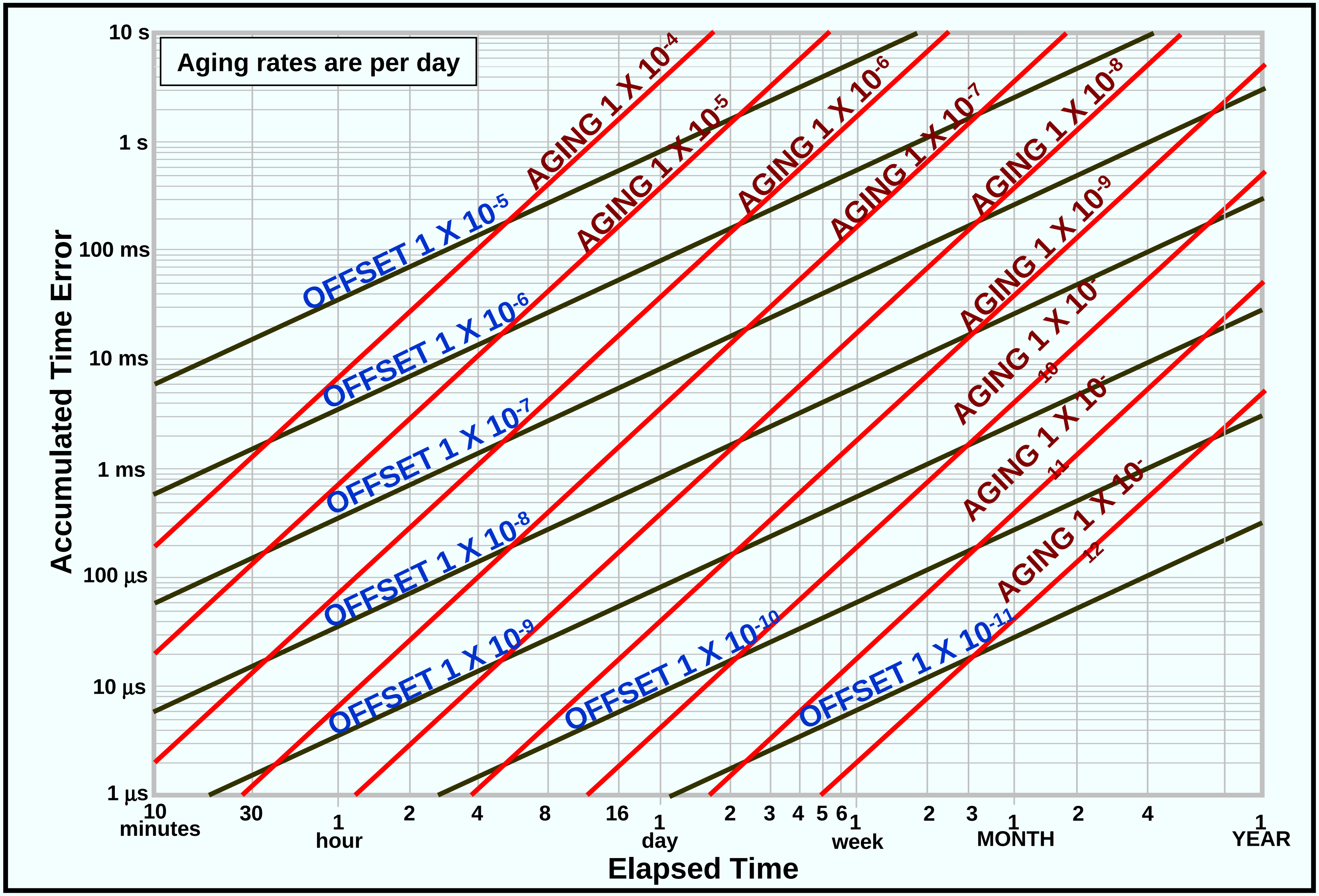
Clock Error vs. Resynchronization Interval



	TCXO		OCXO		MCXO		RbXO	
Avg. Temp. Stab.	1×10^{-6}		2×10^{-8}		2×10^{-8}		2×10^{-8}	
Aging/Day	1×10^{-8}		1×10^{-10}		5×10^{-11}		5×10^{-13}	
Resynch Interval* (A/J & security)	10 min	4 hrs	6 hours	4 days	6 hours	4 days	6 hours	4 days
Recalibr. Interval * (Maintenance cost)	10 yrs	80 days	50 years	1.5 yrs	94 yrs	3 yrs	None needed	300 yrs

* Calculated for an accuracy requirement of 25 milliseconds. Many modern systems need much better.

Time Error vs. Elapsed Time



On Using Time for Clock Rate Calibration

It takes time to measure the clock rate (i.e., frequency) difference between two clocks. The smaller the rate difference between a clock to be calibrated and a reference clock, the longer it takes to measure the difference ($\Delta t/t \approx \Delta f/f$).

For example, assume that a reference timing source (e.g., Loran or GPS) with a time uncertainty of 100 ns is used to calibrate the rate of a clock to 1×10^{-11} accuracy. A frequency offset of 1×10^{-11} will produce1 $\times 10^{-11} \times$ 3600 s/hour = 36 ns time error per hour. Then, to have a high certainty that the measured time difference is due to the frequency offset rather than the reference clock uncertainty, one must accumulate a sufficient amount ($\geq$100 ns) of time error. It will take hours to perform the calibration. (See the next page for a different example.) If one wishes to know the frequency offset to a $\pm 1 \times 10^{-12}$ precision, then the calibration will take more than a day.

Of course, if one has a cesium standard for frequency reference, then, for example, with a high resolution frequency counter, one can make frequency comparisons of the same precision much faster.

Calibration With a 1 pps Reference

Let A = desired clock rate accuracy after calibration

A' = actual clock rate accuracy

$\Delta\tau$ = jitter in the 1 pps of the reference clock, rms

$\Delta\tau'$ = jitter in the 1 pps of the clock being calibrated, rms

$\mathbf{t}$ = calibration duration

Δt = accumulated time error during calibration

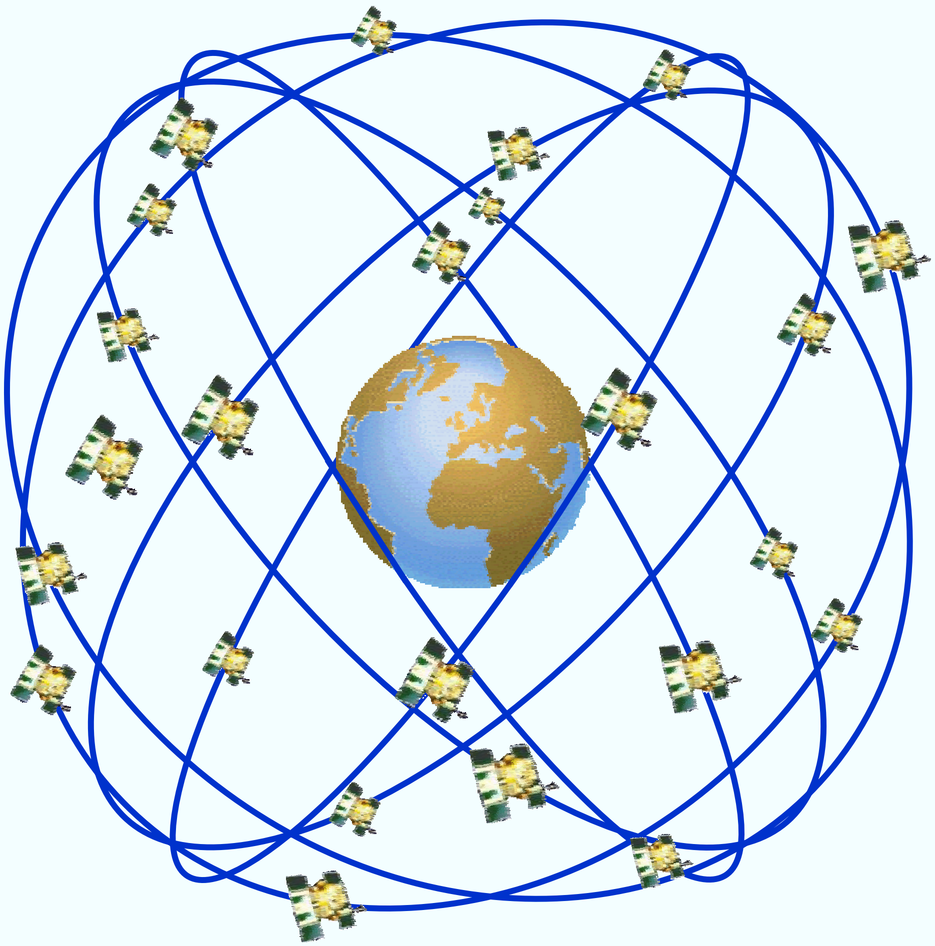
Then, what should be the **t** for a given set of A, Δt, and $\Delta t'$?

Example: The crystal oscillator in a clock is to be calibrated by comparing the 1 pps output from the clock with the 1 pps output from a standard. If $A = 1 \times 10^{-9}$; $\Delta\tau = 0.1\ \mu s$, and $\Delta\tau' = 1.2\ \mu s$, then, $[(\Delta\tau)^2 + (\Delta\tau')^2]^{1/2} \approx 1.2\ \mu s$, and when $A = A'$, $\Delta t = (1 \times 10^{-9})\mathbf{t} \equiv (1.2\ \mu s)N$, and $\mathbf{t} = (1200N)$ s. The value of N to be chosen depends on the statistics of the noise processes, on the confidence level desired for A' to be $\leq$ A, and on whether one makes measurements every second or only at the end points. If one measures at the end points only, and the noise is white phase noise, and the measurement errors are normally distributed, then, with N = 1, 68% of the calibrations will be within A; with N = 2, and 3, 95% and 99.7%, respectively, will be within A. One can reduce **t** by about a factor $2/N^{3/2}$ by making measurements every second; e.g., from 1200 s to $2 \times (1200)^{2/3} = 225$ s.

Time Transfer Methods

Method	Accuracy	~ Cost ('95)
Portable Cs clock	10 - 100 ns	\$45K - 70K
GPS time dissemination GPS common view	20 - 100 ns 5 - 20 ns	\$100 - 5K
Two-way via satellite	~1 ns	\$60k
Loran-C	100 ns	\$1K - 5K
HF (WWV)	2 ms	\$100 - 5K
Portable quartz & Rb clocks	Calibration interval dependent	\$200 - 2K

Global Positioning System (GPS)



<u>GPS Nominal Constellation:</u>
24 satelites in 6 orbital planes,
4 satelites in each plane,
20,200 km altitude, 55 degree inclinations

GPS

GPS can provide global, all-weather, 24-hour, real-time, accurate navigation and time reference to an unlimited number of users.

- **GPS Accuracies (2σ)**

Position:	120 m for Standard Positioning Service, SPS
	40 m for Precise Positioning Service, PPS
	1 cm + 1ppm for differential, static land survey
Velocity:	0.3 m/s (SPS), 0.1 m/s (PPS).
Time:	350 ns to < 10 ns

- 24 satellites in 6 orbital planes; 6 to 10 visible at all times; ~12 h period 20,200 km orbits.

- Pseudorandom noise (PRN) navigation signals are broadcast at L1 = 1.575 GHz (19 cm) and L2 = 1.228 GHz (24 cm); two codes, C/A and P are sent; messages provide satellite position, time, and atmospheric propagation data; receivers select the optimum 4 (or more) satellites to track. PPS (for DoD users) uses L1 and L2, SPS uses L1 only.

Oscillator's Impact on GPS

- Satellite oscillator's (clock's) inaccuracy & noise are major sources of navigational inaccuracy.
- Receiver oscillator affects GPS performance, as follows:

Oscillator Parameter	GPS Performance Parameter
Warmup time	**Time to first fix**
Power	**Mission duration, logistics costs (batteries)**
Size and weight	**Manpack size and weight**
Short term stability (0.1 s to 100 s)	**Δ range measurement accuracy, acceleration performance, jamming resistance**
Short term stability (~15 minute)	**Time to subsequent fix**
Phase noise	**Jamming margin, data demodulation, tracking**
Acceleration sensitivity	**See short term stability and phase noise effects**

Time Scales

- A "**time scale**" is a system of assigning dates, i.e., a "time," to events; e.g., 6 January 1989, 13 h, 32 m, 46.382912 s, UTC, is a date.

- A "**time interval**" is a "length" of time between two events; e.g., five seconds.

- **Universal time** scales, UT0, UT1, and UT2, are based on the earth's spin on its axis, with corrections.

- Celestial navigation: clock (UT1) + sextant $\longrightarrow$ position.

- **International Atomic Time (TAI)** is maintained by the International Bureau of Weights and Measures (BIPM; in France), and is derived from an ensemble of more than 200 atomic clocks, from more than 60 laboratories around the world.

- **Coordinated Universal Time (UTC)** is **<u>the</u>** time scale today, by international agreement. The rate of UTC is determined by TAI, but, in order to not let the time vs. the earth's position change indefinitely, UTC is adjusted by means of leap seconds so as to keep UTC within 0.9 s of UT1.

Clock Ensembles and Time Scales

- An ensemble of clocks is a group of clocks in which the time outputs of individual clocks are combined, via a “time-scale algorithm,” to form a time scale.

- Ensembles are often used in mission critical applications where a clock’s failure (or degraded performance) presents an unacceptable risk.

- Advantages of an ensemble**:**
  - system time & frequency are maintained even if a clock in the ensemble fails
  - ensemble average can be used to estimate the characteristics of each clock; outliers can be detected
  - performance of ensemble can (sometimes) be better than any of the contributors
  - a proper algorithm can combine clocks of different characteristics, and different duty cycles, in an optimum way

Relativistic Time

- Time is not absolute. The "time" at which a distant event takes place depends on the observer. For example, if two events, **A** and **B**, are so close in time or so widely separated in space that no signal traveling at the speed of light can get from one to the other before the latter takes place, then, even after correcting for propagation delays, it is possible for one observer to find that **A** took place before **B**, for a second to find that **B** took place before **A**, and for a third to find that **A** and **B** occurred simultaneously. Although it seems bizarre, all three can be right.

- Rapidly moving objects exhibit a "time dilation" effect. (“Twin paradox”: Twin on a spaceship moving at 0.87c will age 6 months while twin on earth ages 1 year. There is no "paradox" because spaceship twin must accelerate; i.e., there is no symmetry to the problem.)

- A clock's rate also depends on its position in a gravitational field. A high clock runs faster than a low clock.

Relativistic Time Effects

- Transporting "perfect" clocks slowly around the surface of the earth along the equator yields Δt = -207 ns eastward and Δt = +207 ns westward (portable clock is late eastward). The effect is due to the earth's rotation.

- At latitude 40º, for example, the rate of a clock will change by 1.091 x 10^{-13} per kilometer above sea level. Moving a clock from sea level to 1km elevation makes it gain 9.4 nsec/day at that latitude.

- In 1971, atomic clocks flown eastward then westward around the world in airlines demonstrated relativistic time effects; eastward Δt = -59 ns, westward Δt = +273 ns; both values agreed with prediction to within the experimental uncertainties.

- Spacecraft Examples:
  - For a space shuttle in a 325 km orbit, $\Delta t = t_{space} - t_{ground}$ = -25 µsec/day
  - For GPS satellites (12 hr period circular orbits), Δt = +44 µsec/day

- In precise time and frequency comparisons, relativistic effects must be included in the comparison procedures.

Relativistic Time Corrections

The following expression accounts for relativistic effects, provides for clock rate accuracies of better than 1 part in 10^{14}, and allows for global-scale clock comparisons of nanosecond accuracy, via satellites:

$$\Delta t = -\frac{1}{c^2}\int_0^T \left[\frac{1}{2}\left(v_s^2 - v_g^2\right) - \left(\Phi_S - \Phi_g\right)\right] dt + \frac{2\omega}{c^2} A_E$$

Where Δt = time difference between spacecraft clock and ground clock, $t_S - T_g$

V_S = spacecraft velocity (<<c), V_g = velocity of ground station

Φ_S = gravitational potential at the spacecraft

Φ_g = gravitational potential at the ground station

ω = angular velocity of rotation of the earth

A_E = the projected area on the earth's equatorial plane swept out by the vector whose tail is at the center of the earth and whose head is at the position of the portable clock or the electromagnetic signal pulse. The A_E is taken positive if the head of the vector moves in the eastward direction.

Within 24 km of sea level, $\Phi = gh$ is accurate to 1×10^{-14} where $g = (9.780 + 0.052 \sin^2\Psi)$ m/s^2, Ψ = the latitude, h = the distance above sea level, and where the $\sin^2\Psi$ term accounts for the centrifugal potential due to the earth's rotation. The "Sagnac effect," $(2\omega/c^2)A_E = (1.6227 \times 10^{-21} \text{s/m}^2)A_E$, accounts for the earth-fixed coordinate system being a rotating, noninertial reference frame.

Some Useful Relationships

- Propagation delay = 1 ns/30 cm = 1 ns/ft = 3.3 μs/km ≈ 5 μs/mile
- 1 day = 86,400 seconds; 1 year = 31.5 million seconds
- Clock accuracy: 1 ms/day ≈ 1 x 10^{-8}
- At 10 MHz: period = 100 ns; phase deviation of 1° = 0.3 ns of time deviation
- Doppler shift* = $\Delta f/f = 2v/c$

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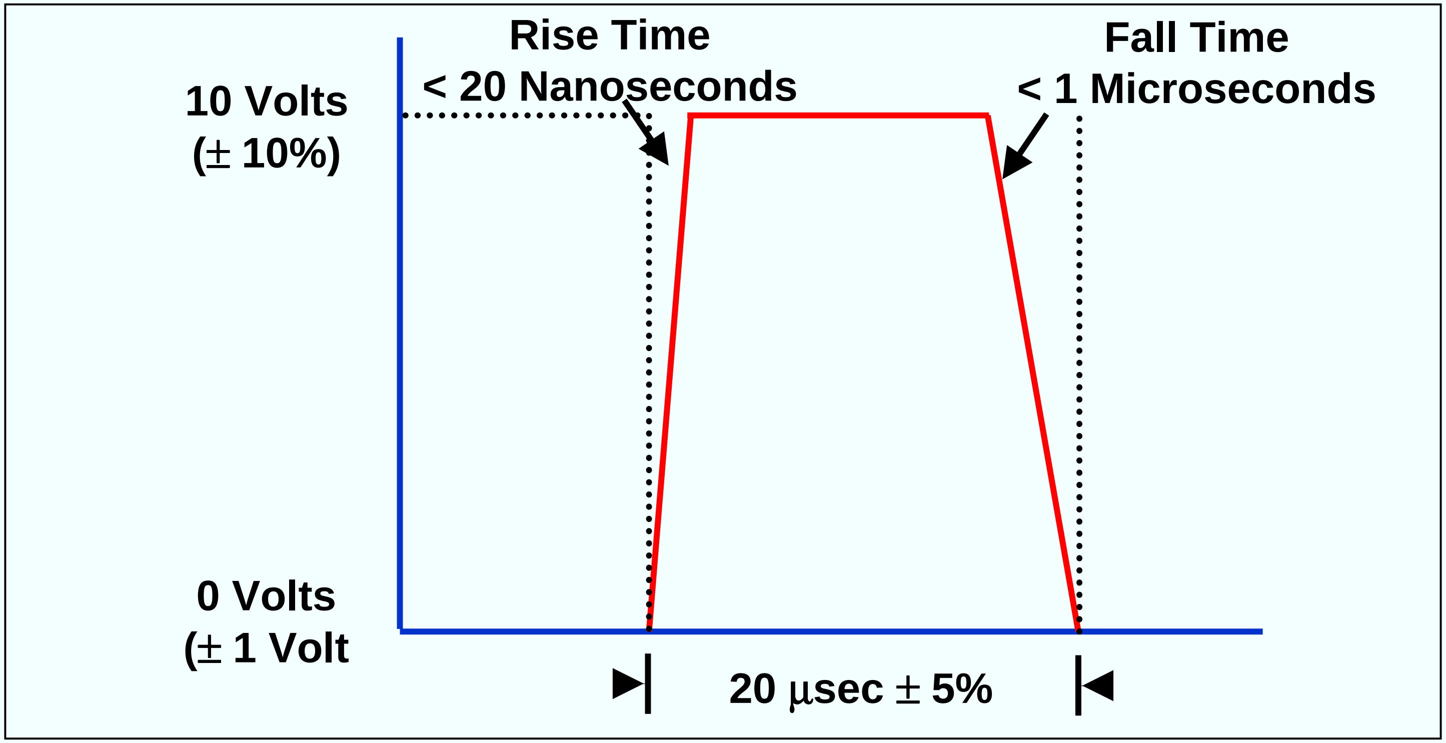
* **Doppler shift example**: if v = 4 km/h and f = 10 GHz (e.g., a slow-moving vehicle approaching an X-band radar), then Δf = 74 Hz, i.e., an oscillator with low phase noise at 74Hz from the carrier is necessary in order to "see" the vehicle.

One Pulse-Per-Second Timing Signal

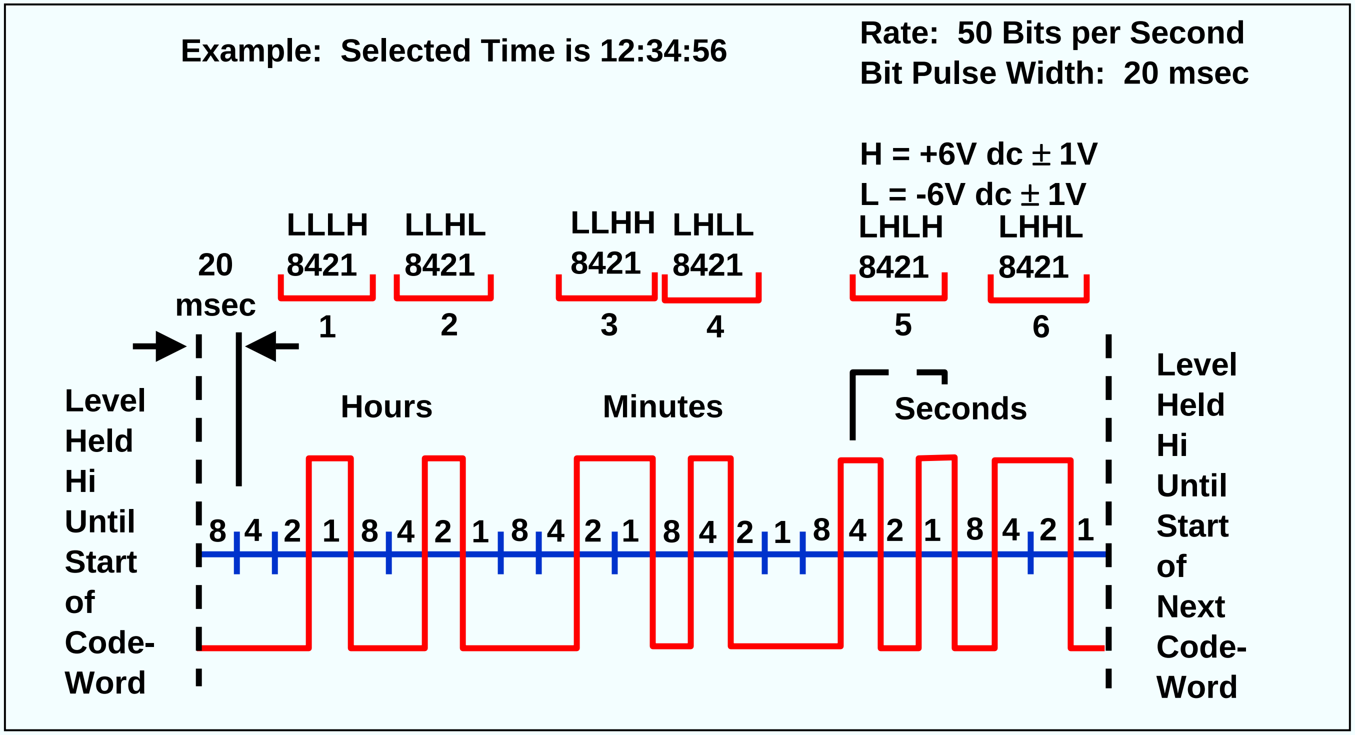
(MIL-STD-188-115)



"The leading edge of the BCD code (negative going transitions after extended high level) shall coincide with the on-time (positive going transition) edge of the one pulse-per-second signal to within ±1 millisecond." See next page for the MIL-STD BCD code.

BCD Time Code

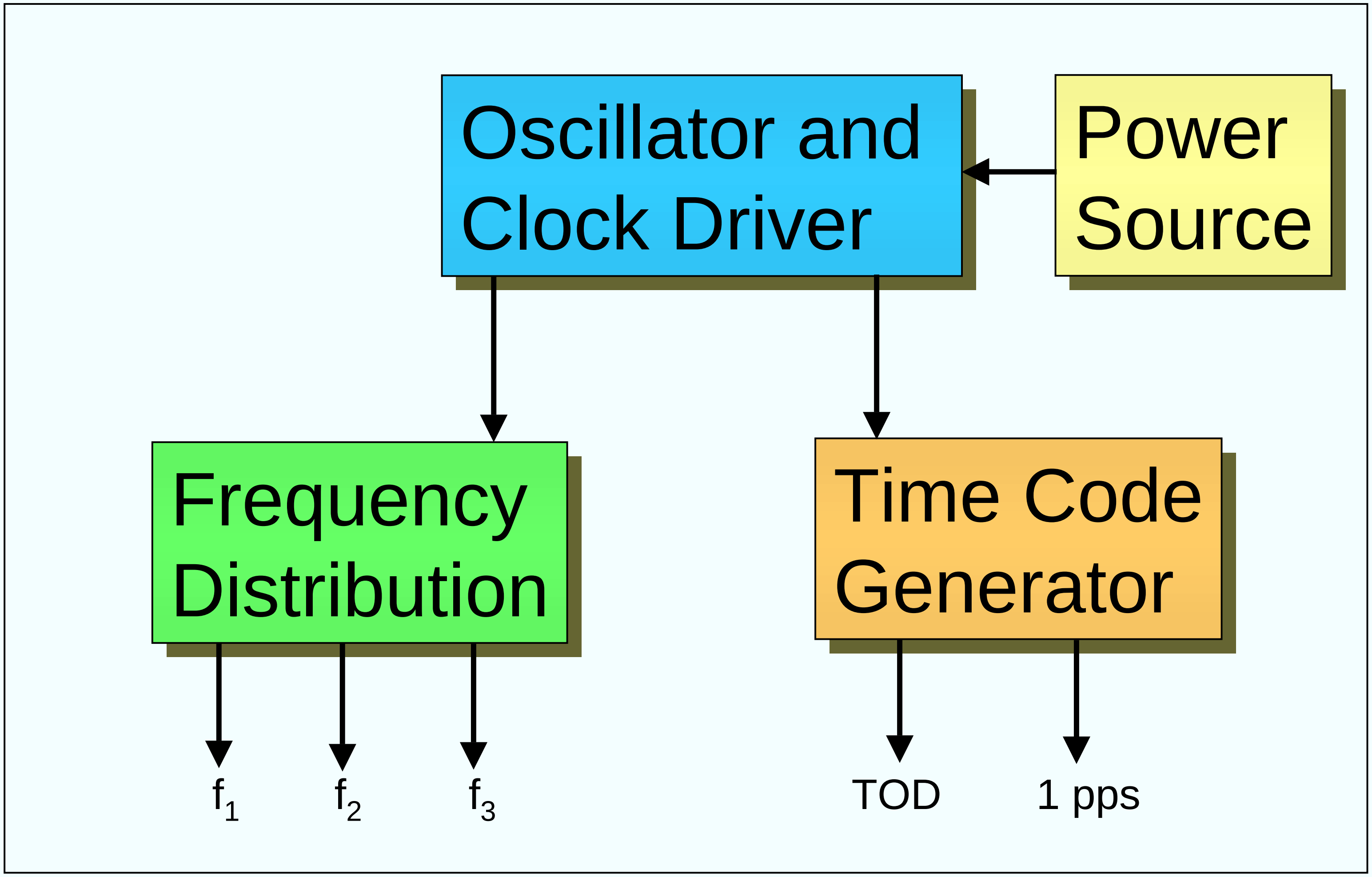
(MIL-STD-188-115)



24 Bit BCD Time Code*

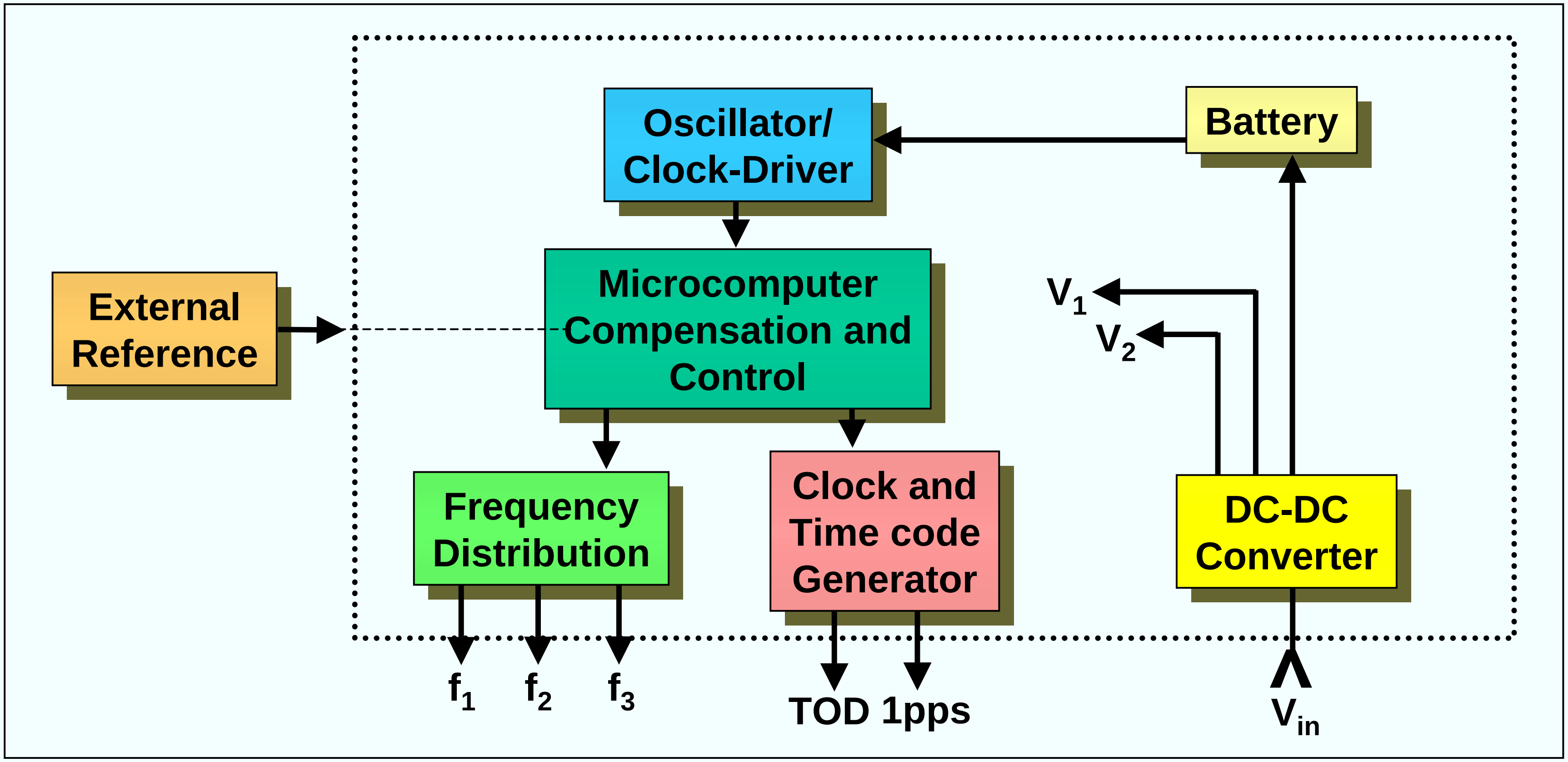
* May be followed by 12 bits for day-of-year and/or 4 bits for figure-of-merit (FOM). The FOM ranges from better than 1 ns (BCD character 1) to greater than 10 ms (BCD character 9).

Time and Frequency Subsystem



The MIFTTI Subsystem

MIFTTI = Modular Intelligent Frequency, Time and Time Interval

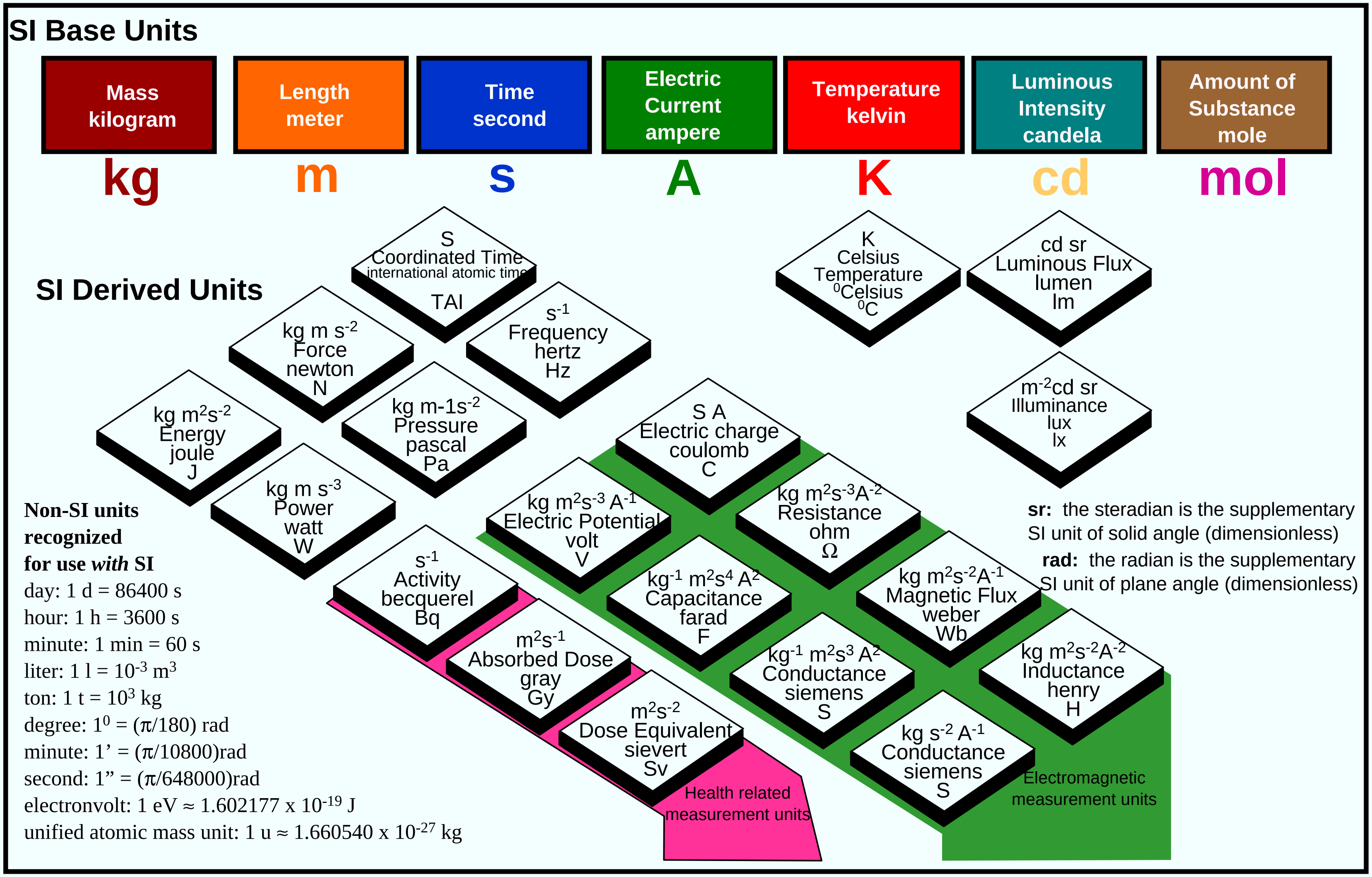


* The microcomputer compensates for systematic effects (after filtering random effects), and performs: automatic synchronization and calibration when an external reference is available, and built-in-testing.

"Time" Quotations

- 3 o'clock is always too late or too early for anything you want to do.................Jean-Paul Sartre
- Time ripens all things. No man's born wise...........Cervantes.
- Time is the rider that breaks youth...........George Herbert
- Time wounds all heels.................Jane Ace
- Time heals all wounds.................Proverb
- The hardest time to tell: when to stop.....Malcolm Forbes
- Time is on our side.................William E. Gladstone
- It takes time to save time............Joe Taylor
- Time, whose tooth gnaws away everything else, is powerless against truth.................Thomas H. Huxley
- Time has a wonderful way of weeding out the trivial.................Richard Ben Sapir
- Time is a file that wears and makes no noise..........English proverb
- The trouble with life is that there are so many beautiful women - and so little time.................John Barrymore
- Life is too short, and the time we waste yawning can never be regained.................Stendahl
- Time goes by: reputation increases, ability declines.................Dag Hammarskjöld
- Remember that time is money..............Benjamin Franklin
- The butterfly counts not months but moments, and has time enough.................Rabindranath Tagore
- Everywhere is walking distance if you have the time.................Steven Wright
- The only true time which a man can properly call his own, is that which he has all to himself; the rest, though in some sense he may be said to live it, is other people's time, not his.................Charles Lamb
- It is familiarity with life that makes time speed quickly. When every day is a step in the unknown, as for children, the days are long with gathering of experience.................George Gissing
- Time is a great teacher, but unfortunately it kills all its pupils.................Hector Berlioz
- To everything there is a season, and a time to every purpose under the heaven.................Ecclesiastes 3:1
- Time goes, you say? Ah no! Time stays, we go.................Henry Austin Dobson
- Time is money - says the vulgarest saw known to any age or people. Turn it around, and you get a precious truth - Money is time.................George (Robert) Gissing

Units of Measurement Having Special Names in the International System of Units (SI)



Non-SI units recognized for use *with* SI

day: 1 d = 86400 s
hour: 1 h = 3600 s
minute: 1 min = 60 s
liter: 1 l = 10^{-3} m^3
ton: 1 t = 10^3 kg
degree: $1^0 = (\pi/180)$ rad
minute: 1’ = $(\pi/10800)$rad
second: 1” = $(\pi/648000)$rad
electronvolt: 1 eV ≈ 1.602177 x 10^{-19} J
unified atomic mass unit: 1 u ≈ 1.660540 x 10^{-27} kg

Units of Measurement Having Special Names in the SI Units, NOT Needing Standard Uncertainty in SI Average Frequency

SI Base Units

Mass
kilogram
kg

Temperature
kelvin
K

Amount of Substance
mole
mol

SI Derived Units

K
Celsius
Temperature
^{0}Celsius
^{0}C

Non-SI units
recognized
for use *with* SI

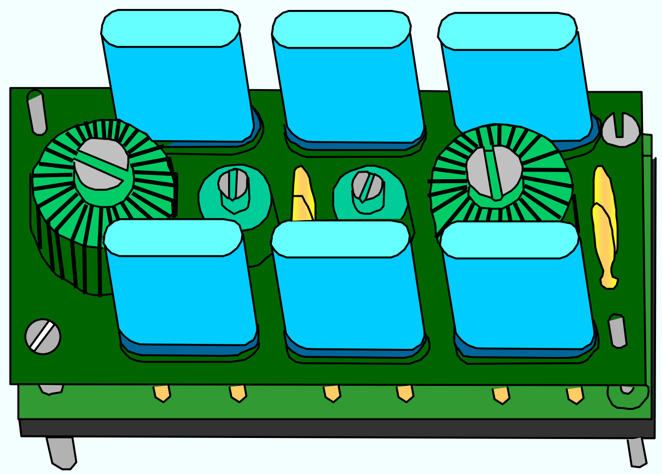
ton: 1 t = 10^3 kg
degree: $1^0 = (\pi/180)$ rad
minute: 1' = $(\pi/10800)$rad
second: 1" = $(\pi/648000)$rad
unified atomic mass unit: 1 u $\approx 1.660540 \times 10^{-27}$ kg

CHAPTER 9
Related Devices and Application

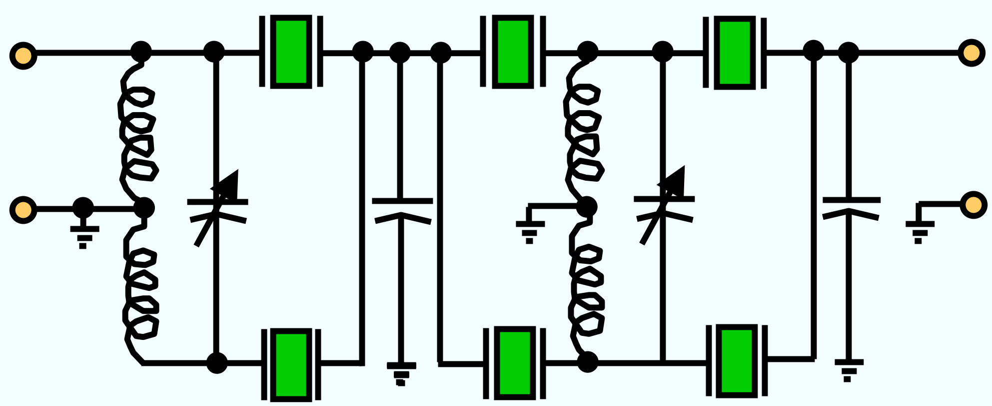
Discrete-Resonator Crystal Filter

A Typical Six-pole Narrow-band Filter

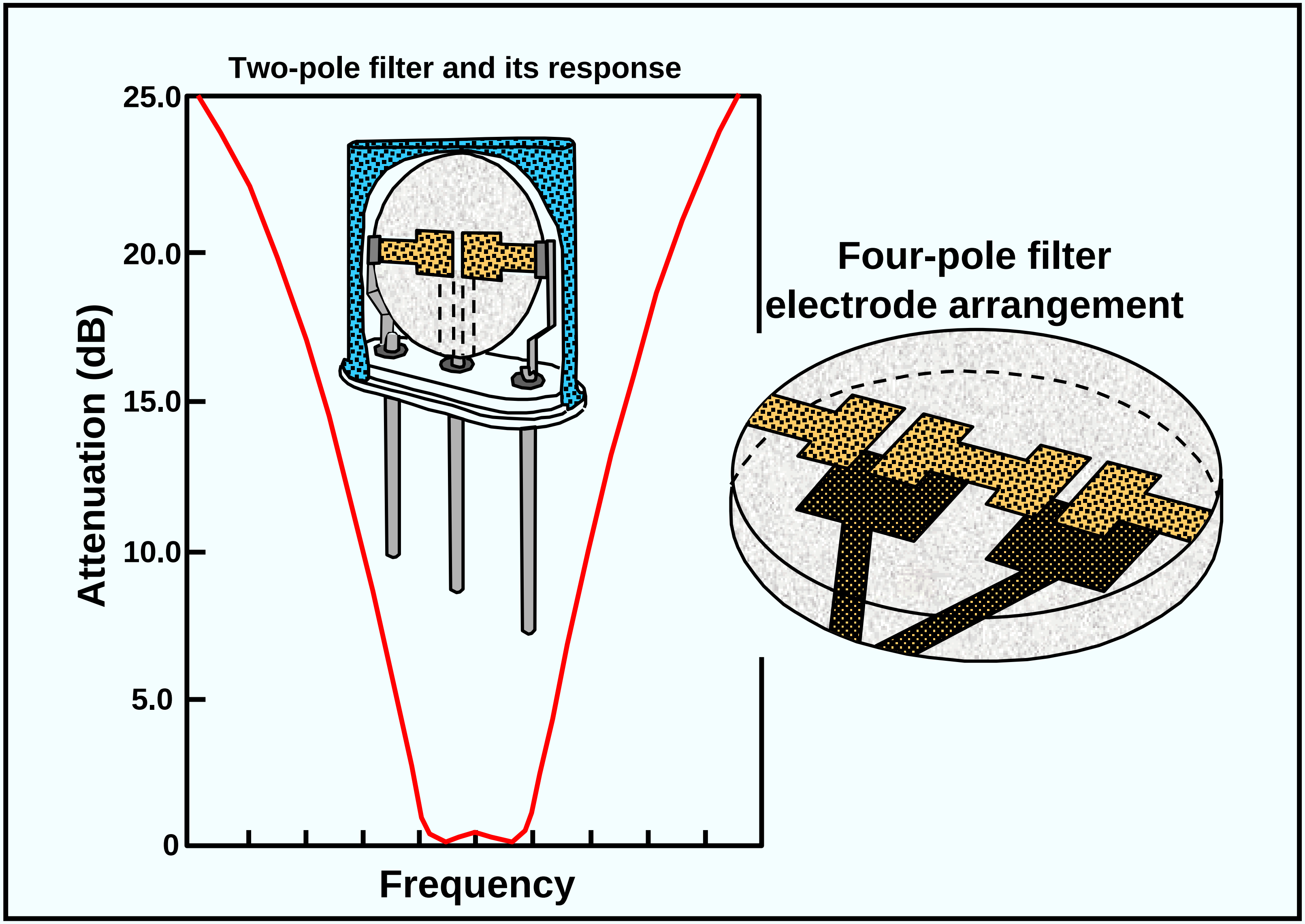
Layout



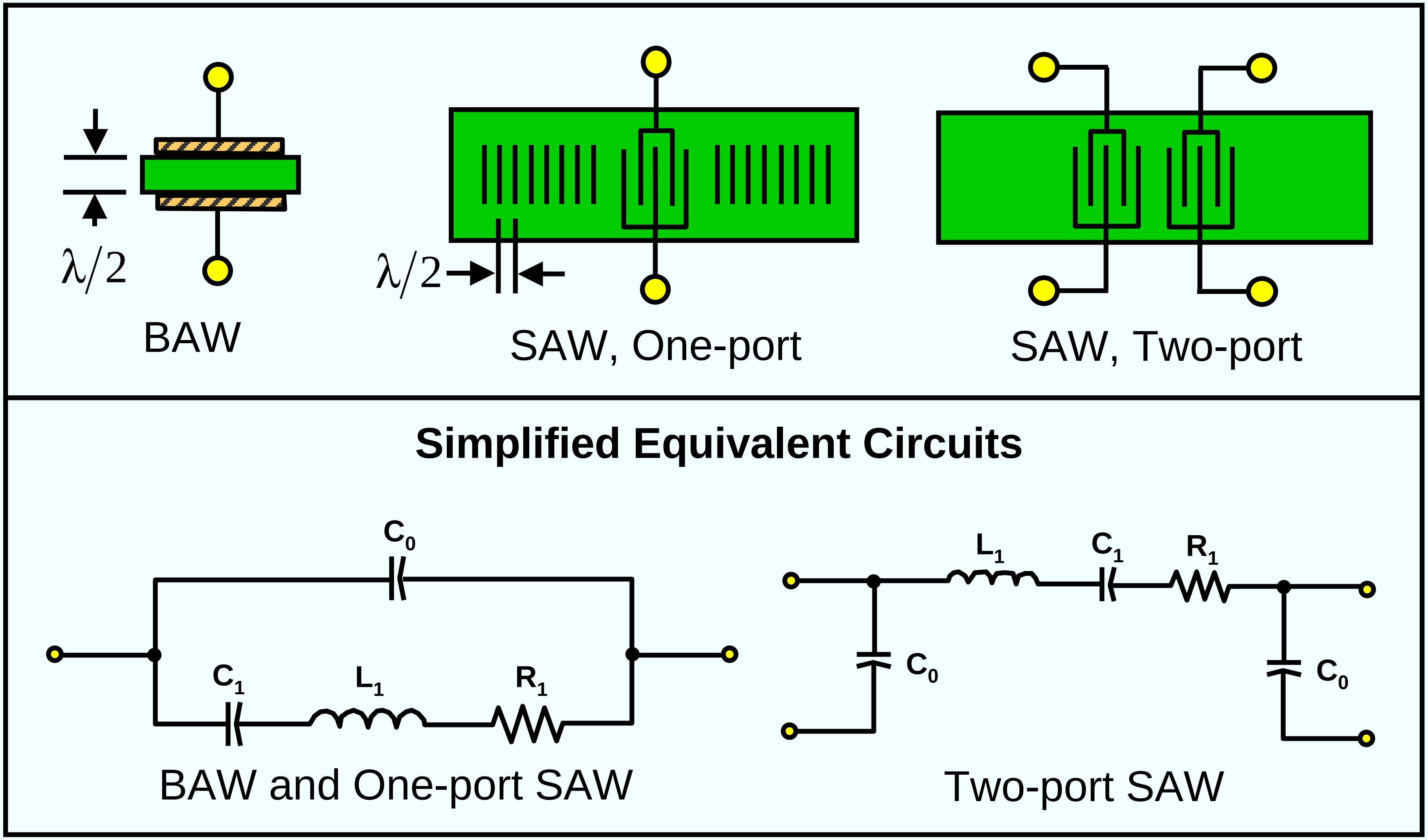
Circuit



Monolithic Crystal Filters



Surface Acoustic Wave (SAW) Devices



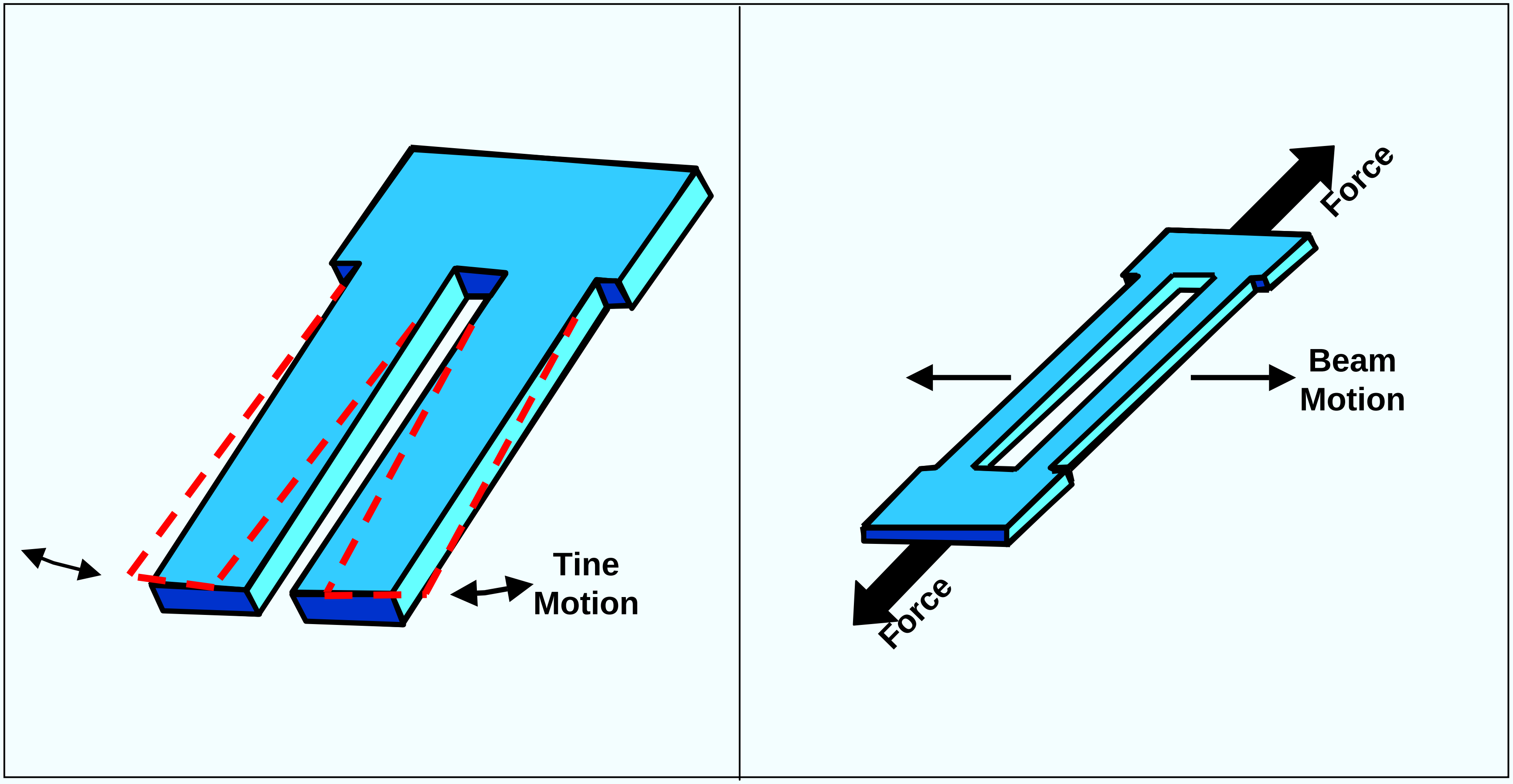
Quartz Bulk-Wave Resonator Sensors

In frequency control and timekeeping applications, resonators are designed to have minimum sensitivity to environmental parameters. In sensor applications, the resonator is designed to have a high sensitivity to an environmental parameter, such as temperature, adsorbed mass, force, pressure and acceleration.

Quartz resonators' advantages over other sensor technologies are:

- High resolution and wide dynamic range (due to excellent short-term stability); e.g., one part in 10^{7} (10^{-6} g out of 20 g) accelerometers are available, and quartz sorption detectors are capable of sensing 10^{-12} grams.
- High long-term accuracy and stability, and
- Frequency counting is inherently digital.

Tuning Fork Resonator Sensors



Photolithographically produced tuning forks, single- and double-ended (flexural-mode or torsional-mode), can provide low-cost, high-resolution sensors for measuring temperature, pressure, force, and acceleration. Shown are flexural-mode tuning forks.

Dual Mode SC-cut Sensors

•Advantages

- Self temperature sensing by dual mode operation allows separation/compensation of temp. effects
- Thermal transient compensated
- Isotropic stress compensated
- Fewer activity dips than AT-cut
- Less sensitive to circuit reactance changes
- Less sensitive to drive level changes

•Disadvantage

- Severe attenuation in a liquid
- Fewer SC-cut suppliers than AT-cut suppliers

Separation of Mass and Temperature Effects

- Frequency changes

$$\frac{\Delta f(m,T,x)}{f_o} = \frac{\Delta f(m)}{f_o} + \frac{\Delta f(T)}{f_o} + \frac{\Delta f(x)}{f_o}$$

total **mass** **temperature** **other effects**

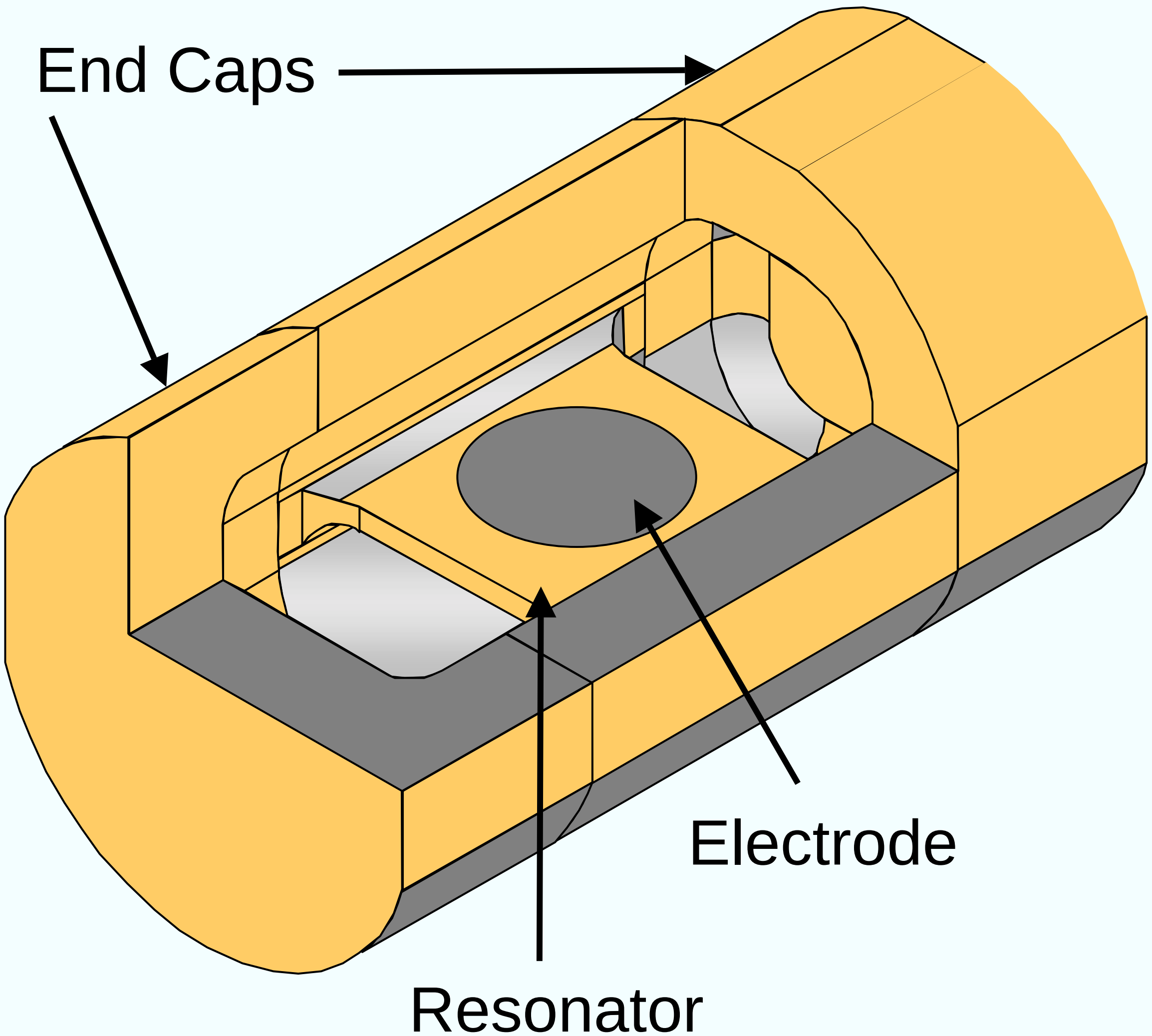
- Mass: adsorption and desorption

$$\frac{\Delta f(m)}{f_o} \cong -\frac{\Delta m}{m_o}$$

- Temperature/beat frequency

$$\frac{\Delta f(T)}{f_o} = \frac{\sum_i c_i \cdot \Delta f_\beta^i}{f_o} \qquad f_\beta \equiv 3f_{c1}(T) - f_{c3}(T)$$

Dual-Mode Pressure Sensor



CHAPTER 10

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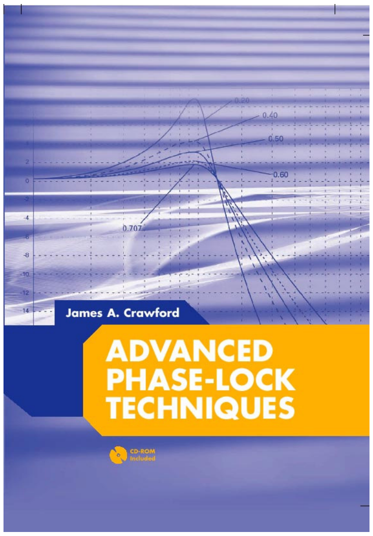

Advanced Phase-Lock Techniques

James A. Crawford

2008

Artech House

510 pages, 480 figures, 1200 equations
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Chapter	Brief Description	*Pages*
1	*Phase-Locked Systems—A High-Level Perspective* An expansive, multi-disciplined view of the PLL, its history, and its wide application.	26
2	*Design Notes* A compilation of design notes and formulas that are developed in details separately in the text. Includes an exhaustive list of closed-form results for the classic type-2 PLL, many of which have not been published before.	44
3	*Fundamental Limits* A detailed discussion of the many fundamental limits that PLL designers may have to be attentive to or else never achieve their lofty performance objectives, e.g., Paley-Wiener Criterion, Poisson Sum, Time-Bandwidth Product.	38
4	*Noise in PLL-Based Systems* An extensive look at noise, its sources, and its modeling in PLL systems. Includes special attention to $1/f$ noise, and the creation of custom noise sources that exhibit specific power spectral densities.	66
5	*System Performance* A detailed look at phase noise and clock-jitter, and their effects on system performance. Attention given to transmitters, receivers, and specific signaling waveforms like OFDM, M-QAM, M-PSK. Relationships between EVM and image suppression are presented for the first time. The effect of phase noise on channel capacity and channel cutoff rate are also developed.	48
6	*Fundamental Concepts for Continuous-Time Systems* A thorough examination of the classical continuous-time PLL up through 4^{th}-order. The powerful Haggai constant phase-margin architecture is presented along with the type-3 PLL. Pseudo-continuous PLL systems (the most common PLL type in use today) are examined rigorously. Transient response calculation methods, 9 in total, are discussed in detail.	71
7	*Fundamental Concepts for Sampled-Data Control Systems* A thorough discussion of sampling effects in continuous-time systems is developed in terms of the z-transform, and closed-form results given through 4^{th}-order.	32
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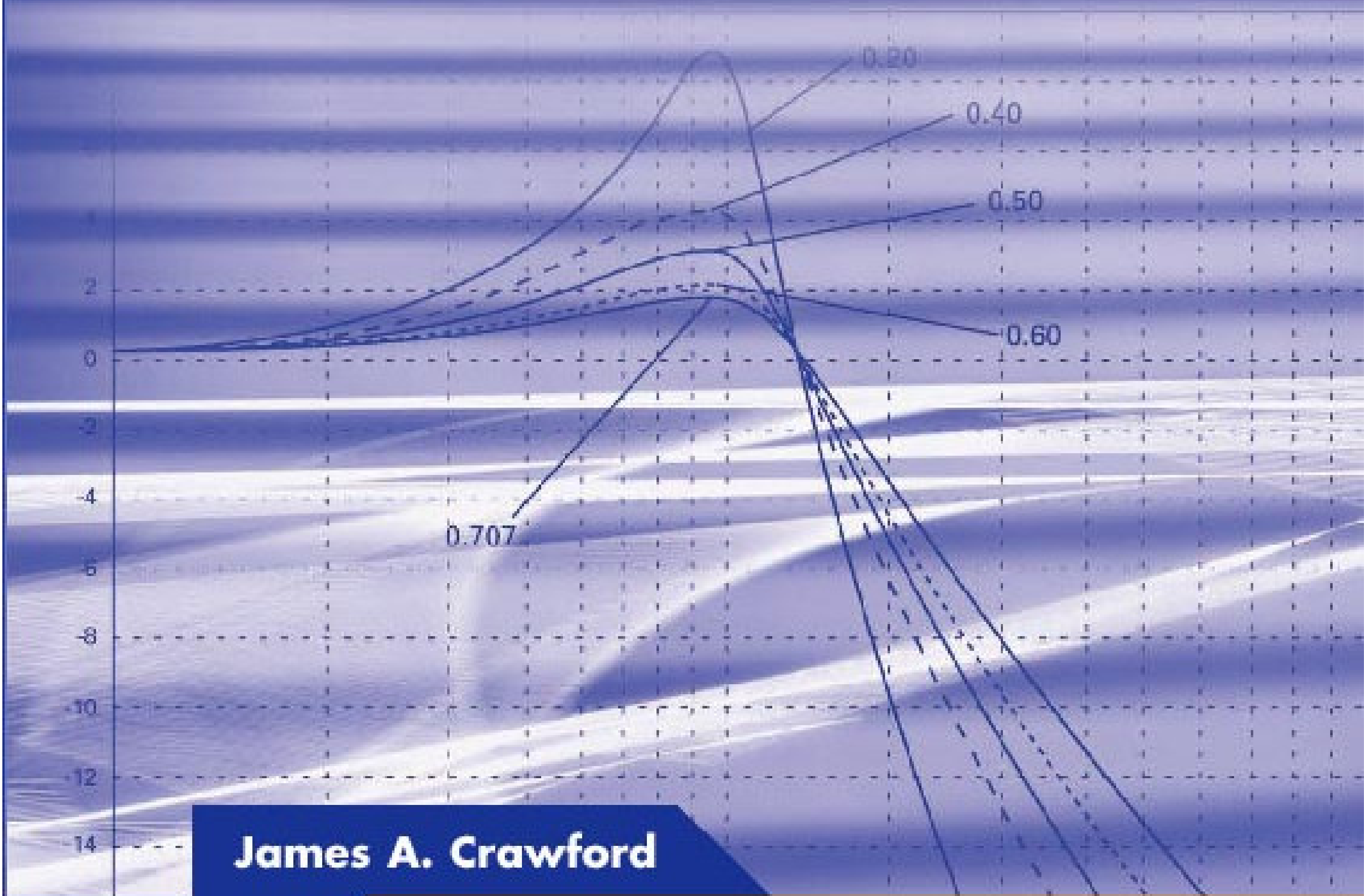

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are described further for the ideal type-2 PLL in Table 1-1. The feedback divider is normally present only in frequency synthesis applications, and is therefore shown as an optional element in this figure.

PLLs are most frequently discussed in the context of continuous-time and Laplace transforms. A clear distinction is made in this text between continuous-time and discrete-time (i.e., sampled) PLLs because the analysis methods are, rigorously speaking, related but different. A brief introduction to continuous-time PLLs is provided in this section with more extensive details provided in Chapter 6.

PLL type and *PLL order* are two technical terms that are frequently used interchangeably even though they represent distinctly different quantities. PLL *type* refers to the number of ideal poles (or integrators) within the linear system. A voltage-controlled oscillator (VCO) is an ideal integrator of phase, for example. PLL *order* refers to the order of the characteristic equation polynomial for the linear system (e.g., denominator portion of (1.4)). The loop-order must always be greater than or equal to the loop-type. Type-2 third- and fourth-order PLLs are discussed in Chapter 6, as well as a type-3 PLL, for example.

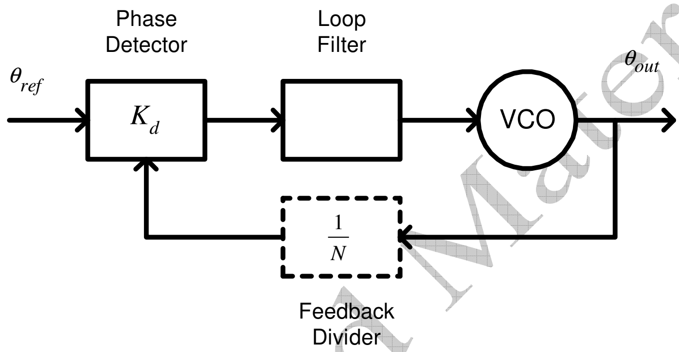

Figure 1-2 Basic PLL structure exhibiting the basic functional ingredients.

Table 1-1
Basic Constitutive Elements for a Type-2 Second-Order PLL

Block Name	Laplace Transfer Function	Description
Phase Detector	K_d, V/rad	Phase error metric that outputs a voltage that is proportional to the phase error existing between its input θ_{ref} and the feedback phase θ_{out}/N. Charge-pump phase detectors output a current rather than a voltage, in which case K_d has units of A/rad.
Loop Filter	$\frac{1+s\tau_2}{s\tau_1}$	Also called the lead-lag network, it contains one ideal pole and one finite zero.
VCO	$\frac{K_v}{s}$	The voltage-controlled oscillator (VCO) is an ideal integrator of phase. K_v normally has units of rad/s/V.
Feedback Divider	$1/N$	A digital divider that is represented by a continuous divider of phase in the continuous-time description.

The type-2 second-order PLL is arguably the workhorse even for modern PLL designs. This PLL is characterized by (i) its natural frequency ω_n (rad/s) and (ii) its damping factor ζ. These terms are used extensively throughout the text, including the examples used in this chapter. These terms are separately discussed later in Sections 6.3.1 and 6.3.2. The role of these parameters in shaping the time- and frequency-domain behavior of this PLL is captured in the extensive list of formula provided in Section 2.1. In the continuous-time-domain, the type-2 second-order PLL[3] open-loop gain function is given by

[3] See Section 6.2.

$$G_{OL}(s)=\left(\frac{\omega_n}{s}\right)^2\frac{1+s\tau_2}{s\tau_1} \tag{1.1}$$

and the key loop parameters are given by

$$\omega_n=\sqrt{\frac{K_d K_v}{N\tau_1}} \tag{1.2}$$

$$\zeta=\frac{1}{2}\omega_n\tau_2 \tag{1.3}$$

The time constants τ_1 and τ_2 are associated with the loop filter's R and C values as developed in Chapter 6. The closed-loop transfer function associated with this PLL is given by the classical result

$$H_1(s)=\frac{1}{N}\frac{\theta_{out}(s)}{\theta_{ref}(s)}=\frac{\omega_n^2\left(1+\dfrac{2\zeta}{\omega_n}s\right)}{s^2+2\zeta\omega_n s+\omega_n^2} \tag{1.4}$$

The transfer function between the synthesizer output phase noise and the VCO self-noise is given by $H_2(s)$ where

$$H_2(s)=1-H_1(s) \tag{1.5}$$

A convenient frequency-domain description of the open-loop gain function is provided in Figure 1-3. The frequency break-points called out in this figure and the next two appear frequently in PLL work and are worth committing to memory. The unity-gain radian frequency is denoted by ω_u in this figure and is given by

$$\omega_u=\omega_n\sqrt{2\zeta^2+\sqrt{4\zeta^4+1}} \tag{1.6}$$

A convenient approximation for the unity-gain frequency (1.6) is given by $\omega_u \cong 2\zeta\omega_n$. This result is accurate to within 10% for $\zeta \geq 0.704$.

The $H_1(s)$ transfer function determines how phase noise sources appearing at the PLL input are conveyed to the PLL output and a number of other important quantities. Normally, the input phase noise spectrum is assumed to be spectrally flat resulting in the output spectrum due to the reference noise being shaped entirely by $|H_1(s)|^2$. A representative plot of $|H_1|^2$ is shown in Figure 1-4. The key frequencies in the figure are the frequency of maximum gain, the zero dB gain frequency, and the –3 dB gain frequency which are given respectively by

$$F_{Pk}=\frac{1}{2\pi}\frac{\omega_n}{2\zeta}\sqrt{\sqrt{1+8\zeta^2}-1}\ \text{Hz} \tag{1.7}$$

$$F_{0dB}=\frac{1}{2\pi}\sqrt{2}\omega_n\ \text{Hz} \tag{1.8}$$

$$F_{3dB}=\frac{\omega_n}{2\pi}\sqrt{1+2\zeta^2+2\sqrt{\zeta^4+\zeta^2+\frac{1}{2}}}\ \text{Hz} \tag{1.9}$$

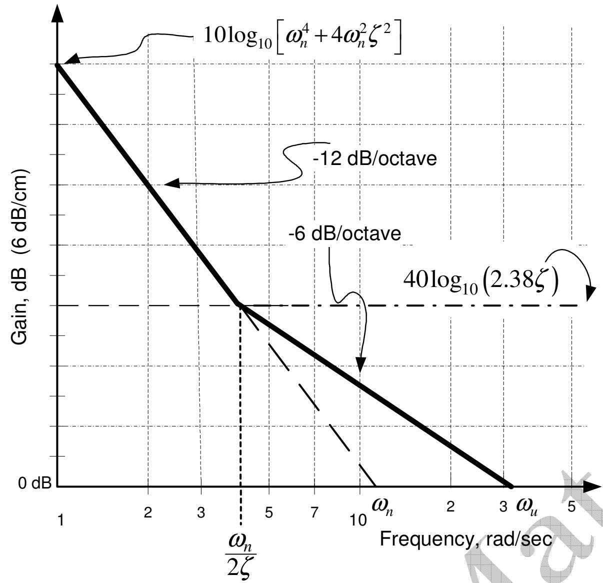

Figure 1-3 Open-loop gain approximations for classic continuous-time type-2 PLL.

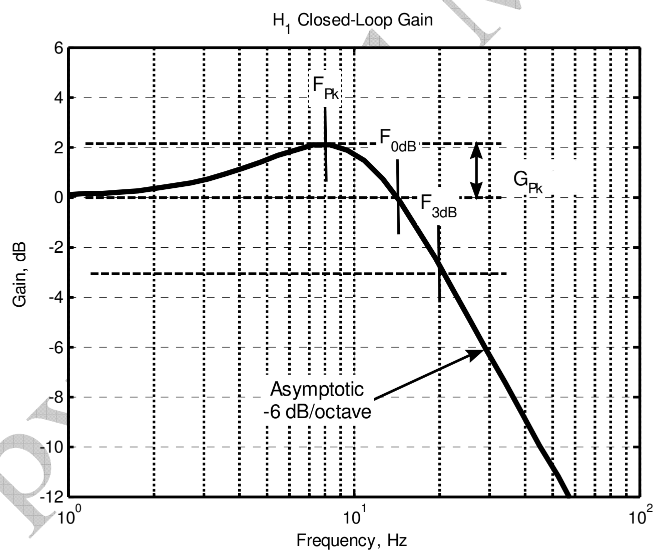

Figure 1-4 Closed-loop gain $H_1(f)$ for type-2 second-order PLL[4] from (1.4).

The amount of *gain-peaking* that occurs at frequency F_{pk} is given by

$$G_{Pk} = 10\log_{10}\left(\frac{8\zeta^4}{8\zeta^4 - 4\zeta^2 - 1 + \sqrt{1+8\zeta^2}}\right)\text{dB} \tag{1.10}$$

For situations where the close-in phase noise spectrum is dominated by reference-related phase noise, the amount of gain-peaking can be directly used to infer the loop's damping factor from (1.10), and the

[4] Book CD:\Ch1\u14033_figequs.m, $\zeta = 0.707$, $\omega_n = 2\pi\, 10$ Hz.

loop's natural frequency from (1.7). Normally, the close-in (i.e., radian offset frequencies less than $\omega_n / 2\zeta$) phase noise performance of a frequency synthesizer is entirely dominated by reference-related phase noise since the VCO phase noise generally increases 6 dB/octave with decreasing offset frequency[5] whereas the open-loop gain function exhibits a 12 dB/octave increase in this same frequency range.

VCO-related phase noise is attenuated by the H_2(s) transfer function (1.5) at the PLL's output for offset frequencies less than approximately ω_n. At larger offset frequencies, $H_2(s)$ is insufficient to suppress VCO-related phase noise at the PLL's output. Consequently, the PLL's output phase noise spectrum is normally dominated by the VCO self-noise phase noise spectrum for the larger frequency offsets. The key frequency offsets and relevant $H_2(s)$ gains are shown in Figure 1-5 and given in Table 1-2.

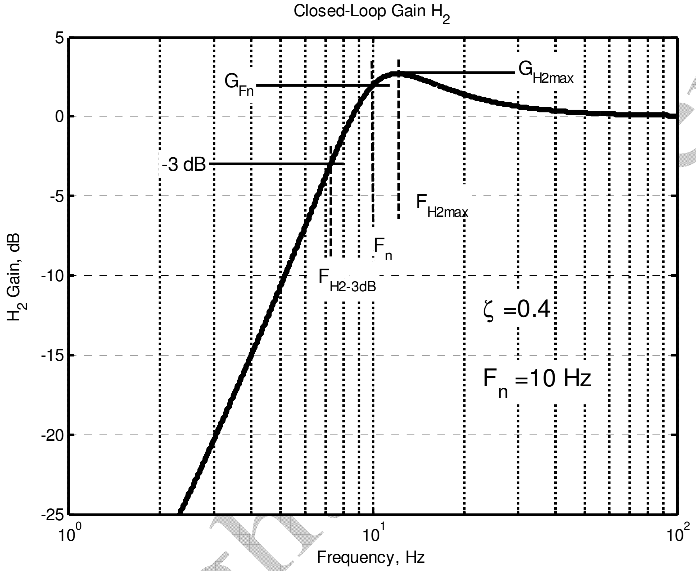

Figure 1-5 Closed-loop gain[6] H_2 and key frequencies for the classic continuous-time type-2 PLL.

Table 1-2
Key Frequencies Associated with $H_2(s)$ for the Ideal Type-2 PLL

Frequency, Hz	Associated H_2 Gain, dB	Constraints on ζ
$1/2\pi$	$G_{H_2\_1rad/s} = -10\log_{10}\left[\omega_n^4 + \omega_n^2\left(4\zeta^2 - 2\right) + 1\right]$	—
$F_{H_2 -3dB} = \dfrac{\omega_n}{2\pi}\left[\begin{matrix} 2\zeta^2 - 1 + \\ \sqrt{2 - 4\zeta^2 + 4\zeta^4} \end{matrix}\right]^{1/2}$	–3	—
$F_{H_2\_0dB} = \dfrac{1}{2\pi}\dfrac{\omega_n}{\sqrt{2 - 4\zeta^2}}$	0	$\zeta < \dfrac{\sqrt{2}}{2}$
$F_n = \omega_n / 2\pi$	$G_{H_2\_\omega_n} = -10\log_{10}\left(4\zeta^2\right)$	—
$F_{H_2\_\max} = \dfrac{1}{2\pi}\dfrac{\omega_n}{\sqrt{1 - 2\zeta^2}}$	$G_{H_2\_\max} = -10\log_{10}\left(4\zeta^2 - 4\zeta^4\right)$	$\zeta < \dfrac{\sqrt{2}}{2}$

5 Leeson's model in Section 9.5.1; Haggai oscillator model in Section 9.5.2.
6 Book CD:\Ch1\u14035_h2.m.

Assuming that the noise samples have equal variances and are uncorrelated, $R = \sigma_n^2 I$ where I is the $K \times K$ identity matrix. In order to maximize (1.43) with respect to θ, a necessary condition is that the derivative of (1.43) with respect to θ be zero, or equivalently

$$\begin{aligned}\frac{\partial L}{\partial \theta} &= \frac{\partial}{\partial \theta}\sum_k \left[r_k - A\cos\left(\omega_o t_k + \theta\right)\right]^2 = 0 \\ &= \sum_k 2\left[r_k - A\cos\left(\omega_o t_k + \theta\right)\right] A\sin\left(\omega_o t_k + \theta\right) = 0\end{aligned} \tag{1.44}$$

Simplifying this result further and discarding the double-frequency terms that appear, the maximum-likelihood estimate for θ is that value that satisfies the constraint

$$\overline{\sum_k r_k \sin\left(\omega_o t_k + \hat{\theta}\right)} = 0 \tag{1.45}$$

The top line indicates that double-frequency terms are to be filtered out and discarded. This result is equivalent to the minimum-variance estimator just derived in (1.40).

Under the assumed linear Gaussian conditions, the minimum-variance (MV) and maximum-likelihood (ML) estimators take the same form when implemented with a PLL. Both algorithms seek to reduce any quadrature error between the estimate and the observation data to zero.

1.4.3 PLL as a Maximum A Posteriori (MAP)-Based Estimator

The *MAP estimator* is used for the estimation of random parameters whereas the maximum-likelihood (ML) form is generally associated with the estimation of deterministic parameters. From *Bayes rule* for an observation z, the a posteriori probability density is given by

$$p\left(\theta | z\right) = \frac{p\left(z|\theta\right) p\left(\theta\right)}{p\left(z\right)} \tag{1.46}$$

and this can be re-written in the logarithmic form as

$$\log_e\left[p\left(\theta|z\right)\right] = \log_e\left[p\left(z|\theta\right)\right] + \log_e\left[p\left(\theta\right)\right] - \log_e\left[p\left(z\right)\right] \tag{1.47}$$

This log-probability may be maximized by setting the derivative with respect to θ to zero thereby creating the necessary condition that[27]

$$\frac{d}{d\theta}\left\{\log_e\left[p\left(z|\theta\right)\right] + \log_e\left[p\left(\theta\right)\right]\right\}_{\theta=\hat{\theta}_{MAP}} = 0 \tag{1.48}$$

If the density $p(\theta)$ is not known, the second term in (1.48) is normally discarded (set to zero) which degenerates naturally to the maximum-likelihood form as

$$\frac{d}{d\theta}\left\{\log_e\left[p\left(z|\theta\right)\right]\right\}_{\theta=\hat{\theta}_{ML}} = 0 \tag{1.49}$$

---

[27] [15] Section 6.2.1, [17] Section 2.4.1, [18] Section 5.4, and [22].

Time of Peak Phase-Error with Frequency-Step Applied

$$T_{fstep} = \frac{1}{\omega_n\sqrt{1-\zeta^2}}\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right) \tag{2.29}$$

Note.[1] See Figure 2-19 and Figure 2-20.

Time of Peak Phase-Error with Phase-Step Applied

$$T_{\theta step} = \frac{1}{\omega_n\sqrt{1-\zeta^2}}\tan^{-1}\left(2\zeta\sqrt{1-\zeta^2}, 2\zeta^2-1\right) = \frac{2}{\omega_n\sqrt{1-\zeta^2}}\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right) \tag{2.30}$$

See Figure 2-19 and Figure 2-20.

Time of Peak Frequency-Error with Phase-Step Applied

$$T_{pk} = \frac{1}{\omega_n\sqrt{1-\zeta^2}}\begin{cases}\zeta \le \frac{1}{2}: & \theta_u \\ \zeta > \frac{1}{2}: & \theta_u + \pi\end{cases} \tag{2.31}$$

$$\text{with } \theta_u = \tan^{-1}\left[\left(1-4\zeta^2\right)\sqrt{1-\zeta^2}, 3\zeta - 4\zeta^3\right] \tag{2.32}$$

See Figure 2-21 and Figure 2-22.
T_{pk} corresponds to the *first point* in time where $df_o/dt = 0$.

Maximum Frequency-Error with Phase-Step Applied

Use (2.31) in (2.28). (2.33)

Time of Peak Frequency-Error with Frequency-Step Applied

$$T_{pk} = \frac{2}{\omega_n\sqrt{1-\zeta^2}}\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right) \tag{2.34}$$

% Transient Frequency Overshoot for Frequency-Step Applied

$$OS_{\%} = \left[\cos\left(\sqrt{1-\zeta^2}\,\omega_n T_{pk}\right) - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\sqrt{1-\zeta^2}\,\omega_n T_{pk}\right)\right]e^{-\zeta\omega_n T_{pk}} \times 100\% \tag{2.35}$$

$$T_{pk} = \frac{2}{\omega_n\sqrt{1-\zeta^2}}\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right) \tag{2.36}$$

Note.[2] See Figure 2-23 and Figure 2-24.

Linear Hold-In Range with Frequency-Step Applied (Without Cycle-Slip)

$$\Delta F_{\max} = \omega_n \exp\left[\frac{\zeta}{\sqrt{1-\zeta^2}}\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right] \text{ Hz} \tag{2.37}$$

See Figure 2-25.

Linear Settling Time with Frequency-Step Applied (Without Cycle-Slip) (Approx.)

$$T_{Lock} \le \frac{1}{\zeta\omega_n}\log_e\left(\frac{\Delta F}{\delta F}\frac{1}{\sqrt{1-\zeta^2}}\right) \text{ sec} \tag{2.38}$$

for applied frequency-step of ΔF and residual δF remaining at lock
See Figure 2-26.

[1] The peak occurrence time is precisely one-half that given by (2.34).
[2] See Figure 2-24 for time of occurrence T_{pk} for peak overshoot/undershoot with $\omega_n = 2\pi$. Amount of overshoot/undershoot in percent provided in Figure 2-23.

2.3.2.2 Second-Order Gear Result for $H_1(z)$ for Ideal Type-2 PLL

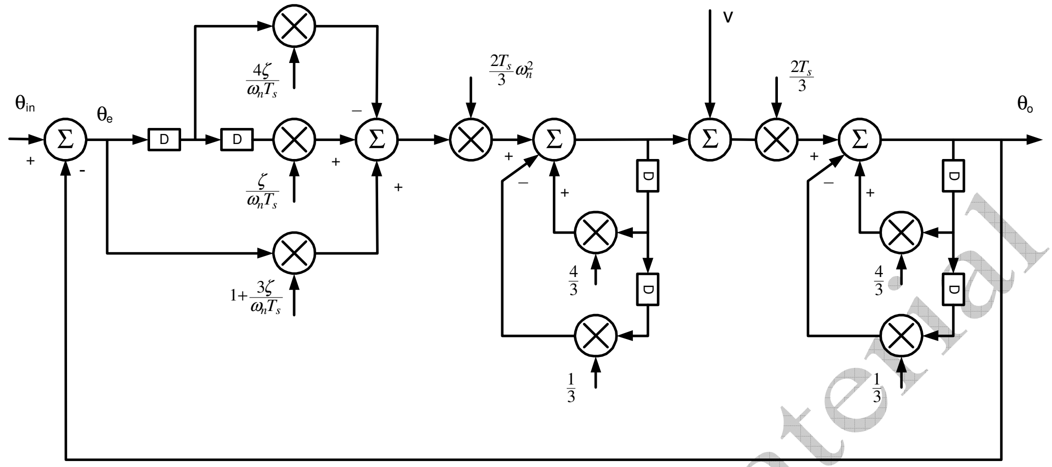

Figure 2-32 Second-order Gear redesign of $H_1(s)$ (2.4).

$$G_{OL}(z) = \left(\frac{2\omega_n T_s}{3}\right)^2 \frac{1+\dfrac{3\zeta}{\omega_n T_s}\left(1-\dfrac{4}{3}z^{-1}+\dfrac{1}{3}z^{-2}\right)}{\left(1-\dfrac{4}{3}z^{-1}+\dfrac{1}{3}z^{-2}\right)^2} \tag{2.52}$$

$$\theta_o(k) = \frac{1}{D}\left[\sum_{n=0}^{2} a_n \theta_{in}(k-n) + \sum_{n=0}^{2} b_n v(k-n) + \sum_{n=1}^{4} c_n \theta_o(k-n)\right] \tag{2.53}$$

$$a_0 = 1+\frac{3\zeta}{\omega_n T_s} \qquad a_1 = -\frac{4\zeta}{\omega_n T_s} \qquad a_2 = \frac{\zeta}{\omega_n T_s} \tag{2.54}$$

$$b_0 = \frac{3}{2\omega_n^2 T_s} \qquad b_1 = -\frac{2}{\omega_n^2 T_s} \qquad b_2 = \frac{1}{2\omega_n^2 T_s} \tag{2.55}$$

$$c_1 = \frac{6}{(\omega_n T_s)^2} + \frac{4\zeta}{\omega_n T_s} \qquad c_2 = -\frac{11}{2(\omega_n T_s)^2} - \frac{\zeta}{\omega_n T_s} \qquad c_3 = \frac{2}{(\omega_n T_s)^2} \qquad c_4 = -\frac{1}{(2\omega_n T_s)^2} \tag{2.56}$$

$$D = 1+\frac{3\zeta}{\omega_n T_s} + \left(\frac{3}{2\omega_n T_s}\right)^2 \tag{2.57}$$

2.3.3 Higher-Order Differentiation Formulas

In cases where a precision first-order time-derivative $f(x_{n+1})$ must be computed from an equally spaced sample sequence, higher-order formulas may be helpful.[8] Several of these are provided here in Table 2-2. The uniform time between samples is represented by T_s.

[8] Precisions compared in Book CD:\Ch2\u14028_diff_forms.m.

2.5.5 64-QAM Symbol Error Rate

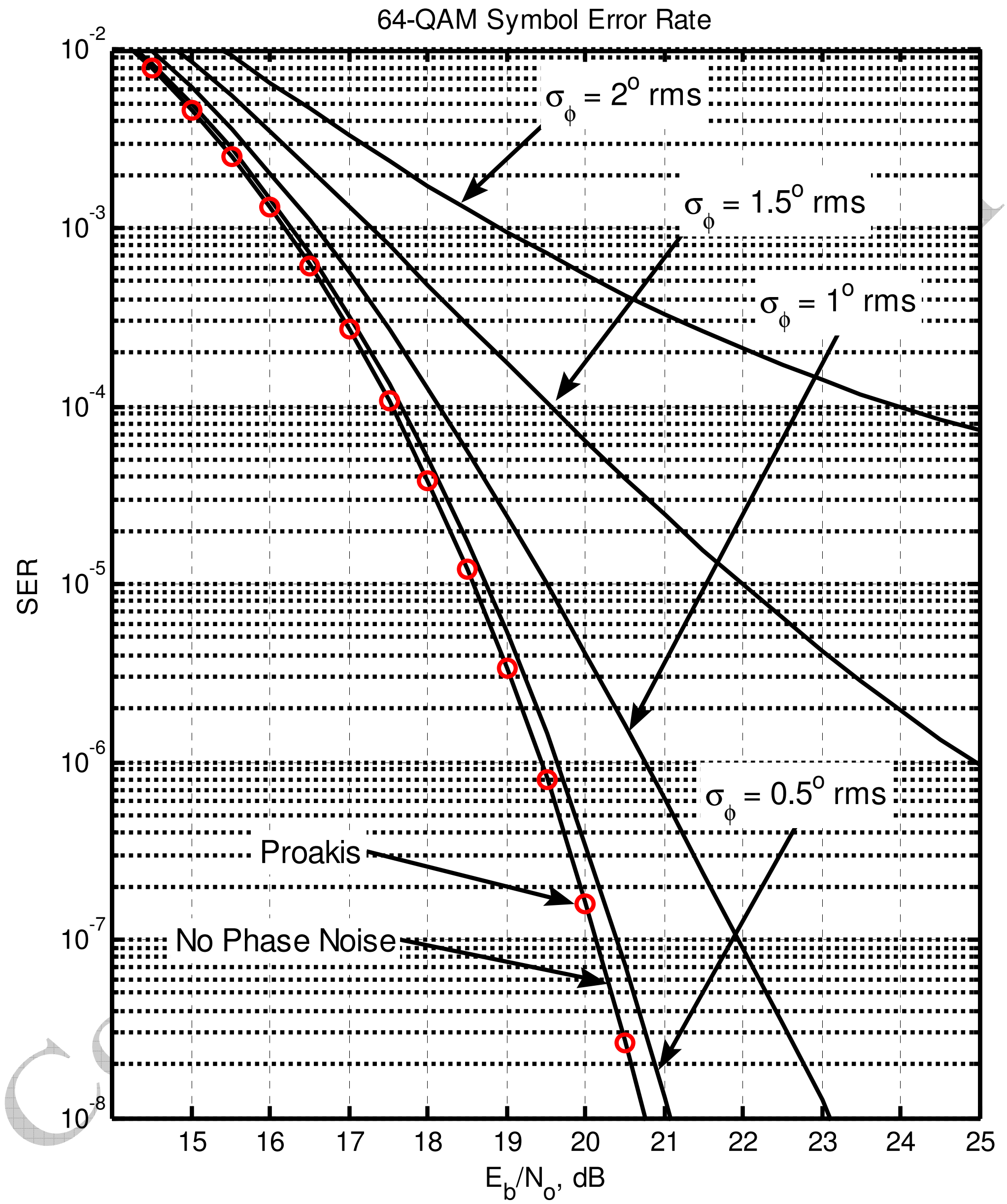

Figure 2-37 64-QAM uncoded symbol error rate with noisy local oscillator.[13] Circled datapoints are from (2.87).

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[13] Book CD:\Ch5\u13159_qam_ser.m. See Section 5.5.3 for additional information. Circled datapoints are based on Proakis [3] page 282, equation (4.2.144), included in this text as (2.87).

A more detailed discussion of the Chernoff bound and its applications is available in [9].

Key Point: The Chernoff bound can be used to provide a tight upper-bound for the tail-probability of a one-sided probability density. It is a much tighter bound than the Chebyschev inequality given in Section 3.5. The bound given by (3.43) for the complementary error function can be helpful in bounding other performance measures.

3.7 CRAMER-RAO BOUND

The Cramer-Rao bound[16] (CRB) was first introduced in Section 1.4.4, and frequently appears in phase- and frequency-related estimation work when low SNR conditions prevail. Systems that asymptotically achieve the CRB are called *efficient* in estimation theory terminology. In this text, the CRB is used to quantify system performance limits pertaining to important quantities such as phase and frequency estimation, signal amplitude estimation, bit error rate, etc.

The CRB is used in Chapter 10 to assess the performance of several synchronization algorithms with respect to theory. Owing to the much larger signal SNRs involved with frequency synthesis, however, the CRB is rarely used in PLL-related synthesis work. The CRB is developed in considerable detail in the sections that follow because of its general importance, and its widespread applicability to the analysis of many communication system problems.

The CR bound provides a lower limit for the error covariance of any unbiased estimator of a deterministic parameter θ based on the probability density function of the data observations. The data observations are represented here by z_k for $k = 1, \ldots, N$, and the probability density of the observations is represented by $p(z_1, z_2, \ldots, z_N) = p(z)$. When θ represents a single parameter and θ-hat represents the estimate of the parameter based on the observed data z, the CRB is given by three equivalent forms as

$$\begin{aligned}
\operatorname{var}\left[\left(\hat{\theta}-\theta\right)\right] &= \mathbf{E}\left[\left(\hat{\theta}-\theta\right)^2\right] \\
&\geq \left\{\mathbf{E}\left\{\left[\frac{\partial}{\partial\theta}\log_e p\left(\mathbf{z}\,|\,\theta\right)\right]^2\right\}\right\}^{-1} \\
&\geq -\left\{\mathbf{E}\left[\frac{\partial^2}{\partial\theta^2}\log_e p\left(\mathbf{z}\,|\,\theta\right)\right]\right\}^{-1} \\
&\geq \left\{\int_{-\infty}^{+\infty}\left[\frac{\partial}{\partial\theta}p\left(\mathbf{z}\right)\right]^2 \frac{1}{p\left(\mathbf{z}\right)}d\mathbf{z}\right\}^{-1}
\end{aligned} \tag{3.46}$$

The first form of the CR bound in (3.46) can be derived as follows. Since θ-hat is an unbiased (zero-mean) estimator of the deterministic parameter θ, it must be true that

$$\mathbf{E}\left(\tilde{\theta}\right) = \int_{-\infty}^{+\infty}\left[\hat{\theta}-\theta\right]p\left(\mathbf{z}\right)d\mathbf{z} = 0 \tag{3.47}$$

in which $d\mathbf{z} = dz_1 dz_2 \ldots dz_N$. Differentiating (3.47) with respect to θ produces the equality

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[16] See [10]–[14].

$$\operatorname{var}\left\{\hat{b}_o\right\} \geq \frac{\sigma^2}{M} \quad \text{for all cases} \tag{3.62}$$

$$\operatorname{var}\left\{\hat{\omega}_o T_s\right\} \geq \begin{cases} \dfrac{\sigma^2}{b_0^2 Q} & \text{Phase known, amplitude known or unknown} \\ \dfrac{12\sigma^2}{\mathrm{b}_0^2 M\left(M^2-1\right)} & \text{Phase unknown, amplitude known or unknown} \end{cases} \tag{3.63}$$

$$\operatorname{var}\left\{\hat{\theta}_o\right\} \geq \begin{cases} \dfrac{\sigma^2}{b_0^2 M} & \text{Frequency known, amplitude known or unknown} \\ \dfrac{12\sigma^2 Q}{b_0^2 M^2\left(M^2-1\right)} & \text{Frequency unknown, amplitude known or unknown} \end{cases} \tag{3.64}$$

In the formulation presented by (3.55), the signal-to-noise ratio ρ is given by $\rho = b_0^2 / (2\sigma^2)$.

For the present example, the CR bound is given by the top equation in (3.63) and is as shown in Figure 3-9 when the initial signal phase θ_o is known a priori. Usually, the carrier phase θ_o is not known a priori when estimating the signal frequency, however, and the additional unknown parameter causes the estimation error variance to be increased, making the variance asymptotically 4-times larger than when the phase is known a priori. This CR variance bound for this more typical unknown signal phase situation is shown in Figure 3-10.

Beginning with (3.57), a maximum-likelihood[17] frequency estimator can be formulated as described in Appendix 3A. It is insightful to compare this estimator's performance with its respective CR bound. For simplicity, the initial phase θ_o is assumed to be random but known a priori. The results for $M = 80$ are shown in Figure 3-11 where the onset of thresholding is apparent for $\rho \cong -2$ dB. Similar results are shown in Figure 3-12 for $M = 160$ where the threshold onset has been improved to about $\rho \cong -5$ dB.

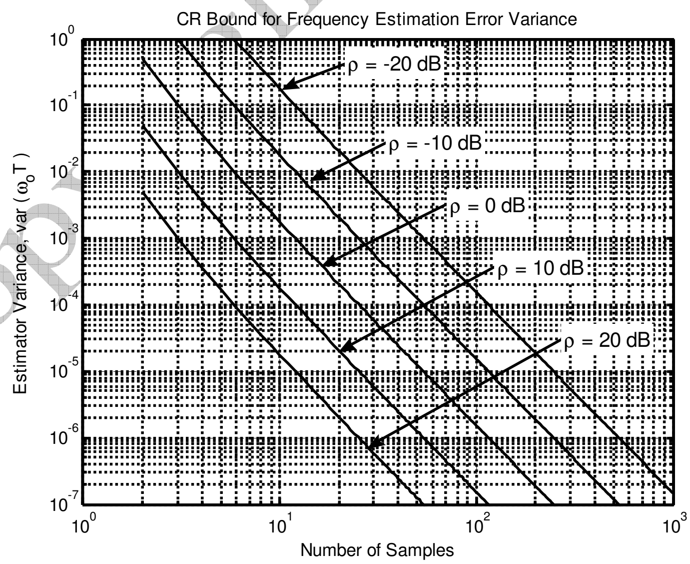

Figure 3-9 CR bound[18] for frequency estimation error with phase θ_o known a priori (3.63).

[17] See Section 1.4.2.
[18] Book CD:\Ch3\u13000_crb.m. Amplitude known or unknown, frequency unknown, initial phase known.

would be measured and displayed on a spectrum analyzer. Having recognized the carrier and continuous spectrum portions within (4.65), it is possible to equate[29]

$$\mathscr{L}(f) \cong P_\theta(f) \quad \text{rad}^2/\text{Hz} \tag{4.66}$$

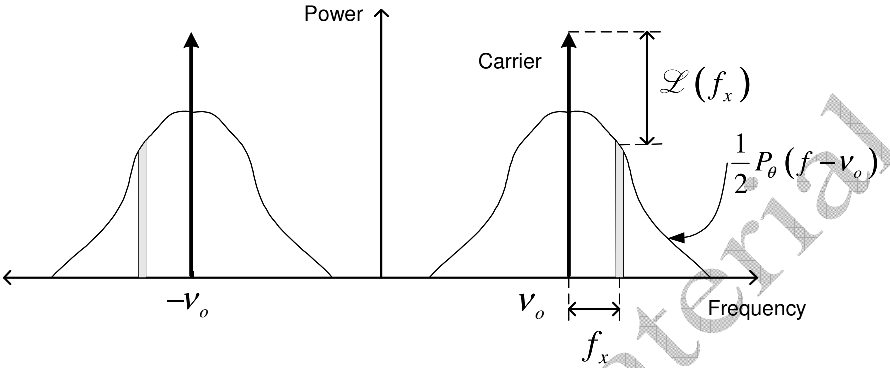

Figure 4-17 Resultant two-sided power spectral density from (4.65), and the single-sideband-to-carrier ratio $\mathscr{L}(f)$.

Both $\mathscr{L}(f)$ and $P_\theta(f)$ are *two-sided power spectral densities*, being defined for positive as well as negative frequencies.

The use of one-sided versus two-sided power spectral densities is a frequent point of confusion in the literature. Some PSDs are formally defined only as a one-sided density. Two-sided power spectral densities are used throughout this text (aside from the formal definitions for some quantities given in Section 4.6.1) because they naturally occur when the Wiener-Khintchine relationship is utilized.

4.6.1 Phase Noise Spectrum Terminology

A minimum amount of standardized terminology has been used thus far in this chapter to characterize phase noise quantities. In this section, several of the more important formal definitions that apply to phase noise are provided.

A number of papers have been published which discuss phase noise characterization fundamentals [34]–[40]. The updated recommendations of the IEEE are provided in [41] and those of the CCIR in [42]. A collection of excellent papers is also available in [43].

In the discussion that follows, the nominal carrier frequency is denoted by ν_o (Hz) and the frequency-offset from the carrier is denoted by f (Hz) which is sometimes also referred to as the *Fourier frequency*.

One of the most prevalent phase noise spectrum measures used within industry is $\mathscr{L}(f)$ which was encountered in the previous section. This important quantity is defined as [44]:

> $\mathscr{L}(f)$: The normalized frequency-domain representation of phase fluctuations. It is the ratio of the power spectral density in one phase modulation sideband, referred to the carrier frequency on a spectral density basis, to the total signal power, at a frequency offset f. The units[30] for this quantity are Hz^{-1}. The frequency range for f ranges from $-\nu_o$ to ∞. $\mathscr{L}(f)$ is therefore a two-sided spectral density and is also called *single-sideband phase noise*.

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[29] It implicitly assumed that the units for $\mathscr{L}(f)$, dBc/Hz or rad^2/Hz, can be inferred from context.

[30] Also as rad^2/Hz.

$$z_i = p_i \exp\left(\frac{\alpha}{2}\Delta p\right) \tag{4B.10}$$

A minimum of one filter section per frequency decade is recommended for reasonable accuracy. A sample result using this method across four frequency decades using 3 and 5 filter sections is shown in Figure 4B-3.

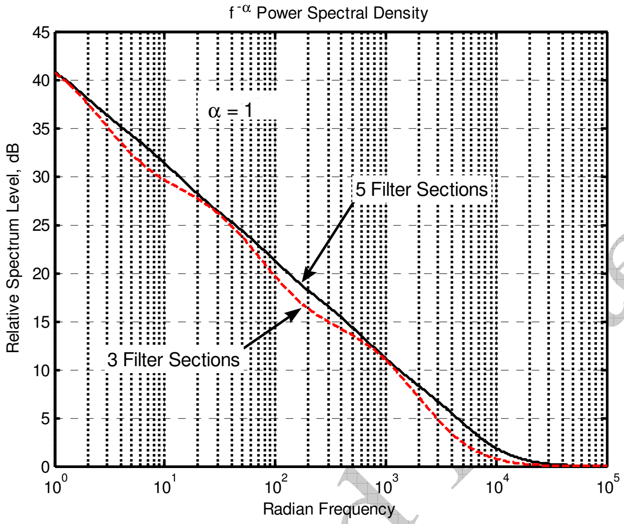

Figure 4B-3 $1/f$ noise creation using recursive $1/f^2$ filtering method[4] with white Gaussian noise.

$1/f^{\alpha}$ Noise Generation Using Fractional-Differencing Methods

Hosking [6] was the first to propose the *fractional differencing* method for generating $1/f^{\alpha}$ noise. As pointed out in [3], this approach resolves many of the problems associated with other generation methods. In the continuous-time-domain, the generation of $1/f^{\alpha}$ noise processes involves the application of a nonrealizable filter to a white Gaussian noise source having $s^{-\alpha/2}$ for its transfer function. Since the z-transform equivalent of $1/s$ is $H(z) = (1 - z^{-1})^{-1}$, the *fractional digital filter* of interest here is given by

$$H_{\alpha}(z) = \frac{1}{\left(1 - z^{-1}\right)^{\alpha/2}} \tag{4B.11}$$

A straightforward power series expansion of the denominator can be used to express the filter as an infinite IIR filter response that uses only integer-powers of z as

$$H_{\alpha}(z) \approx \left[1 - \frac{\alpha}{2}z^{-1} - \frac{\frac{\alpha}{2}\left(1 - \frac{\alpha}{2}\right)}{2!}z^{-2} - \ldots\right]^{-1} \tag{4B.12}$$

in which the general recursion formula for the polynomial coefficients is given by

4 Book CD:\Ch4\u13070_recursive_flicker_noise.m.

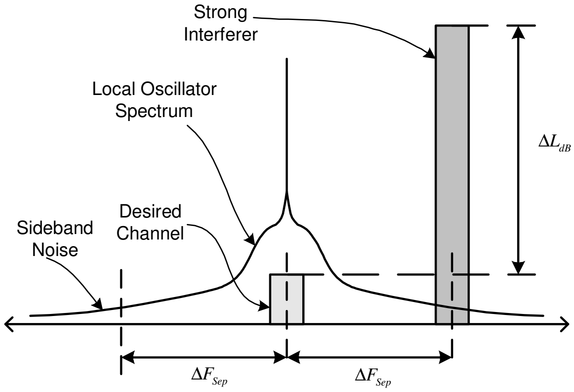

Figure 5-9 Strong interfering channels are heterodyned on top of the desired receive channel by local oscillator sideband noise.

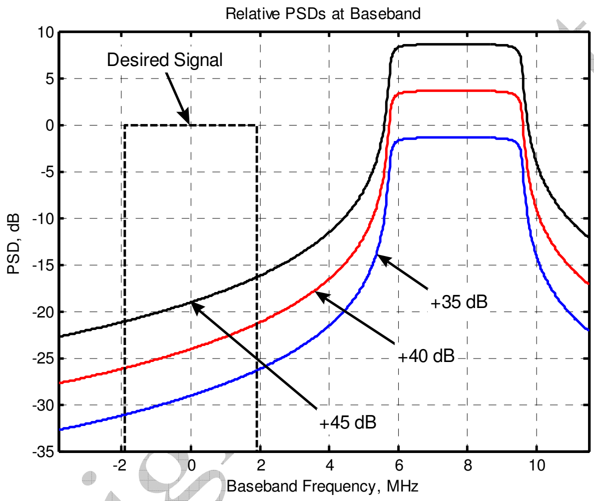

Figure 5-10 Baseband spectra[10] caused by reciprocal mixing between a strong interferer that is offset 4*B* Hz higher in frequency than the desired signal and stronger than the desired signal by the dB amounts shown.

The first term in (5.28) $2BL_{Floor}$ is attributable to the ultimate blocking performance of the receiver as discussed in Section 5.3. The resultant output SNR versus input SNR is given by

$$SNR_{out} = \left[\frac{1}{SNR_{in}} + \frac{\sigma_{MFX}^2}{2BL_{IQ}} \right]^{-1} \tag{5.29}$$

It is worthwhile to note that the interfering spectra in Figure 5-10 are not uniform across the matched-filter frequency region [–*B*, *B*]. Multicarrier modulation like OFDM (see Section 5.6) will potentially be affected differently than single-carrier modulation such as QAM (see Section 5.5.3) when the interference spectrum is not uniform with respect to frequency.

The result given by (5.29) is shown for several interfering levels versus receiver input SNR in Figure 5-11.

[10] Book CD:\Ch5\u13157_rx_desense.m. Lorentzian spectrum parameters: $L_o = -90$ dBc/Hz, $f_c = 75$ kHz, $L_{Floor} = -160$ dBc/Hz, $B = 3.84/2$ MHz.

of 3° rms phase noise is shown in Figure 5B-8. The tail probability is worse than the exact computations shown in Figure 5-17 but the two results otherwise match very well.

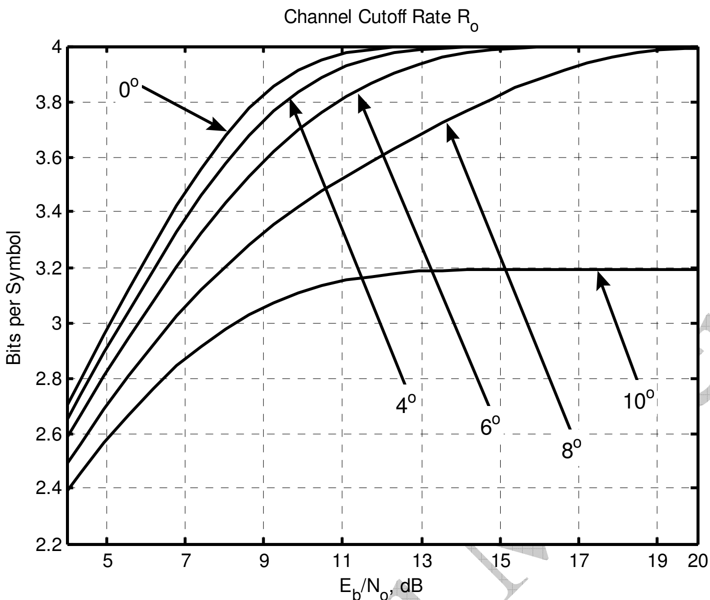

Figure 5B-6 Channel cutoff rate,[7] R_o, for 16-QAM with static phase errors as shown, from (5B.16).

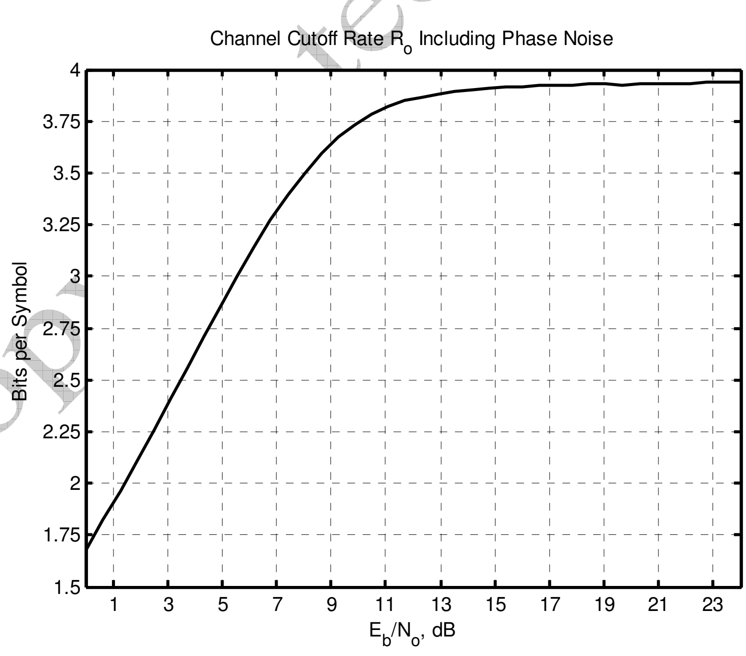

Figure 5B-7 R_o for[8] 16-QAM versus E_b/N_o for 5° rms phase noise from (5B.18) (to accentuate loss in R_o even at high SNR values).

[7] Book CD:\Ch5\u13176_rolo.m.
[8] Ibid.

required, however, because the offset current will introduce its own shot-current noise contribution, and the increased duty-cycle of the charge-pump activity will also introduce additional noise and potentially higher reference spurs. Single-bit Δ-Σ modulators are attractive in this respect because they lead to the minimum-width phase-error distribution possible.

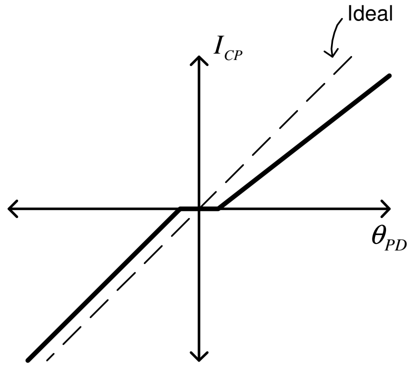

Figure 8-70 Charge-pump (i) dead-zone and (ii) unequal positive versus negative error gain.

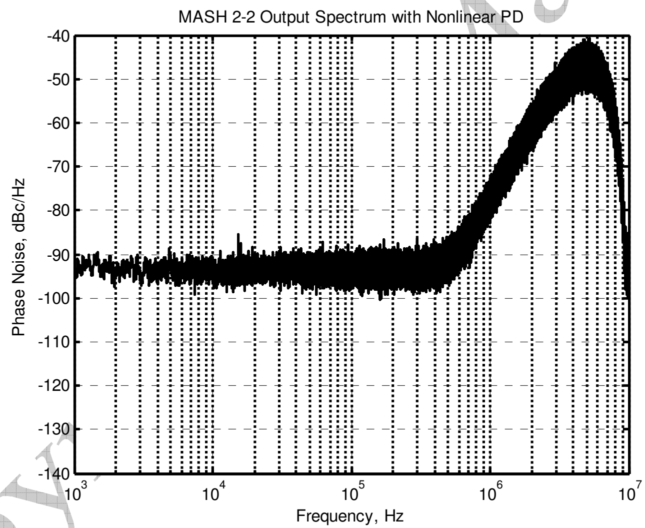

Figure 8-71 Phase error power spectral density[48] for the MASH 2-2 Δ-Σ modulator shown in Figure 8-55 with $M = 2^{22}$, $P = M/2 + 3{,}201$, and 2% charge-pump gain imbalance. Increased noise floor and discrete spurs are clearly apparent compared to Figure 8-56.

Classical random processes theory can be used to provide several useful insights about nonlinear phase detector operation. In the case of unequal positive-error versus negative-error phase detector gain, the memoryless nonlinearity can be modeled as

$$\theta_{pd} = \phi_{in} + \alpha\left(\phi_{in} > 0\right)\phi_{in} \tag{8.39}$$

where α represents the additional gain that is present for positive phase errors. The instantaneous phase error due to the modulator's internal quantization creates a random phase error sequence that can be represented by

48 Book CD:\Ch8\u12735_MASH2_2_nonlinear.m.

the sampling-point within each symbol-period after the datalink signal has been fully acquired. In the example results that follow, the data source is assumed to be operating at 1 bit-per-second, utilizing square-root raised-cosine pulse-shaping with an excess bandwidth parameter $\beta = 0.50$ at the transmitter. The eye-diagram of the signal at the transmit end is shown in Figure 10-15. The ideal matched-filter function in the CDR is closely approximated by an $N = 3$ Butterworth lowpass filter having a –3 dB corner frequency of 0.50 Hz like the filter used in Section 10.4. The resulting eye-diagram at the matched-filter output is shown in Figure 10-16 for $E_b/N_o = 25$ dB.

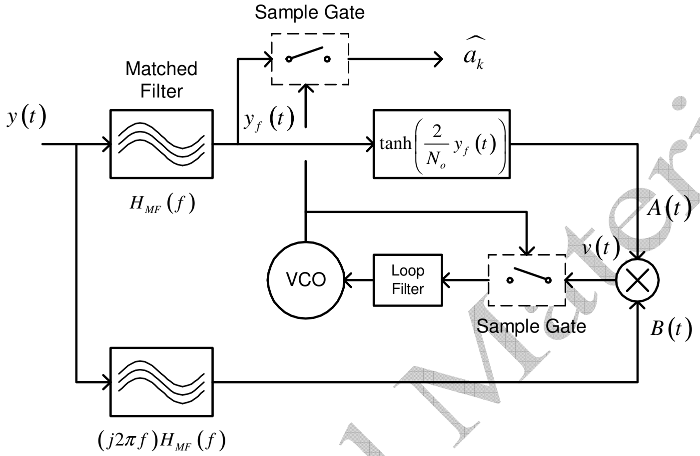

Figure 10-14 ML-CDR implemented with continuous-time filters based on the timing-error metric given by (10.21).

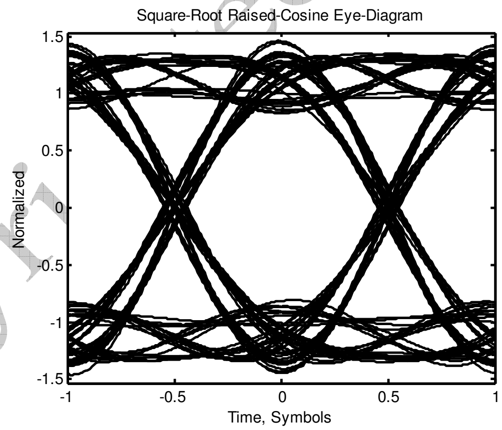

Figure 10-15 Eye diagram[15] at the data source output assuming square-root raised-cosine pulse shaping with an excess bandwidth parameter $\beta = 0.50$.

A clear understanding of the error metric represented by $v(t)$ in Figure 10-14 is vital for understanding how the CDR operates. The metric is best described by its *S-curve* behavior versus input E_b/N_o as shown in Figure 10-17. Each curve is created by setting the noise power spectral density N_o for a specified E_b/N_o value with $E_b = 1$, and computing the average of $v(kT_{sym} + \varepsilon)$ for $k = [0, K]$ as the timing-error ε is swept across $[0, T_{sym}]$. The slope of each S-curve near the zero-error steady-state tracking value determines the linear gain of the metric that is needed to compute the closed-loop bandwidth, loop stability margin, and other important quantities. For a given input SNR,

[15] Book CD:\Ch10\u14004_ml_cdr.m.

the corresponding S-curve has only one timing-error value ε_o for which the error metric value is zero and the S-curve slope has the correct polarity. As the gain value changes with input E_b/N_o, the closed-loop parameters will also vary. For large gain variations, the Haggai loop concept explored in Section 6.7 may prove advantageous.

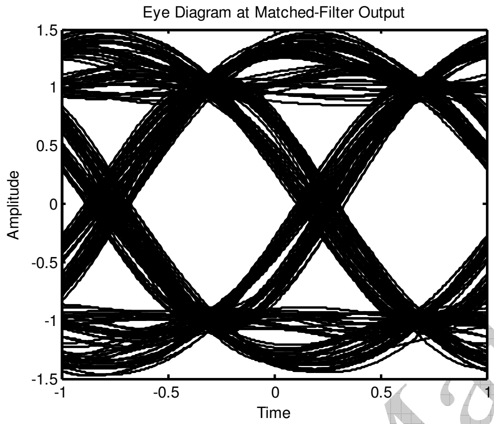

Figure 10-16 Eye diagram[16] at the CDR matched-filter output for E_b/N_o = 25 dB corresponding to the data source shown in Figure 10-15 and using an N = 3 Butterworth lowpass filter with BT = 0.50 for the approximate matched-filter.

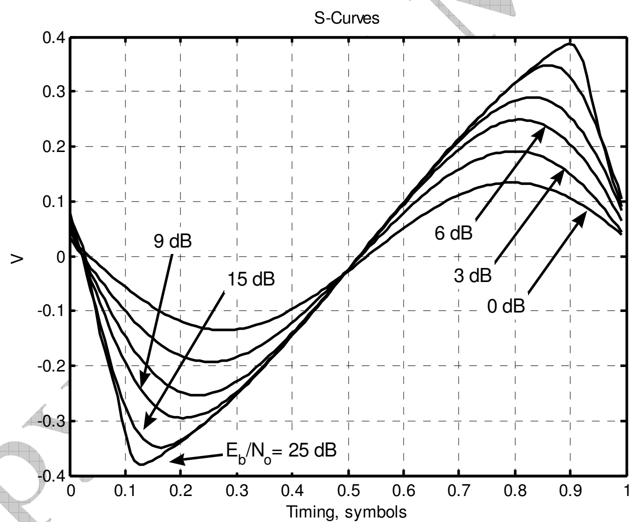

Figure 10-17 S-curves[17] versus E_b/N_o corresponding to Figure 10-16 and ideal ML-CDR shown in Figure 10-14. E_b = 1 is assumed constant.

A second important characteristic of the timing-error metric is its variance versus input E_b/N_o and static timing-error ε. For this present example, this information is shown in Figure 10-18. The variance understandably decreases as the input SNR is increased, and as the optimum time-alignment within each data symbol is approached. The variance of the recovered data clock σ_{clk}^2 can be closely estimated in terms of the tracking-point voltage-error variance from Figure 10-18 denoted by σ_{ve}^2 (V^2), the slope (i.e., gain) of the corresponding S-curve (K_{te}, V/UI) from Figure 10-17, the symbol rate F_{sym} (= $1/T_{sym}$), and the one-sided closed-loop PLL bandwidth B_L (Hz) as

[16] Ibid.
[17] Ibid.